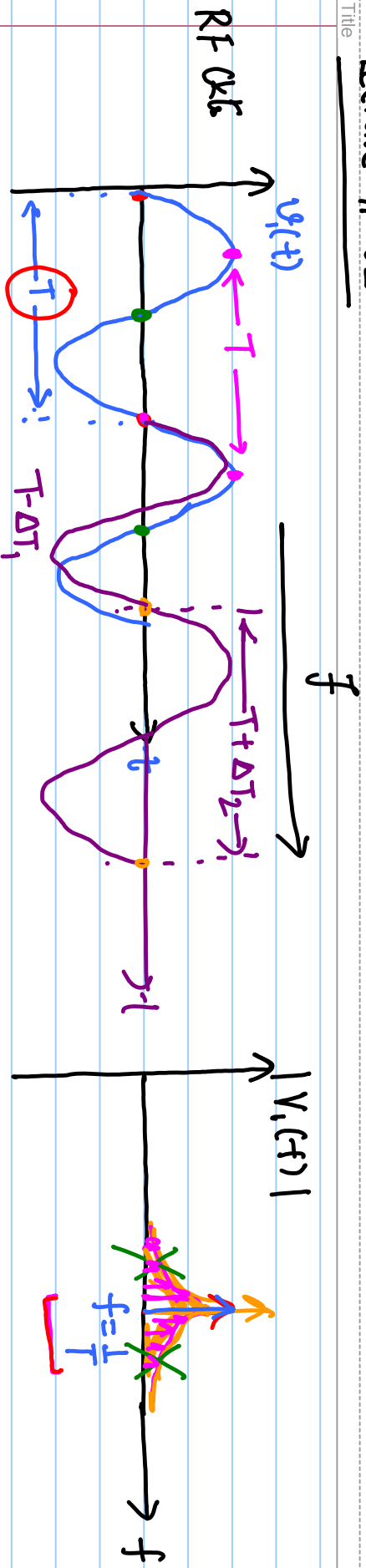


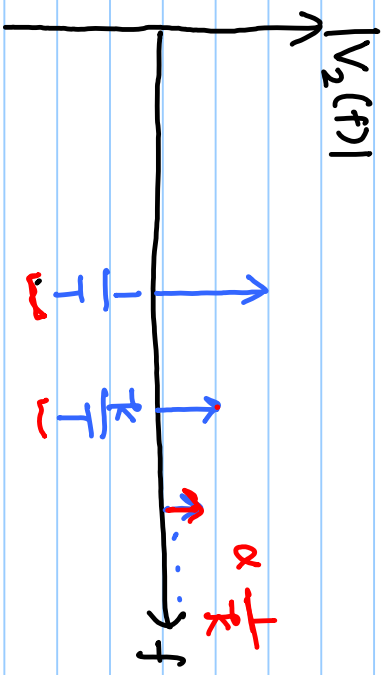
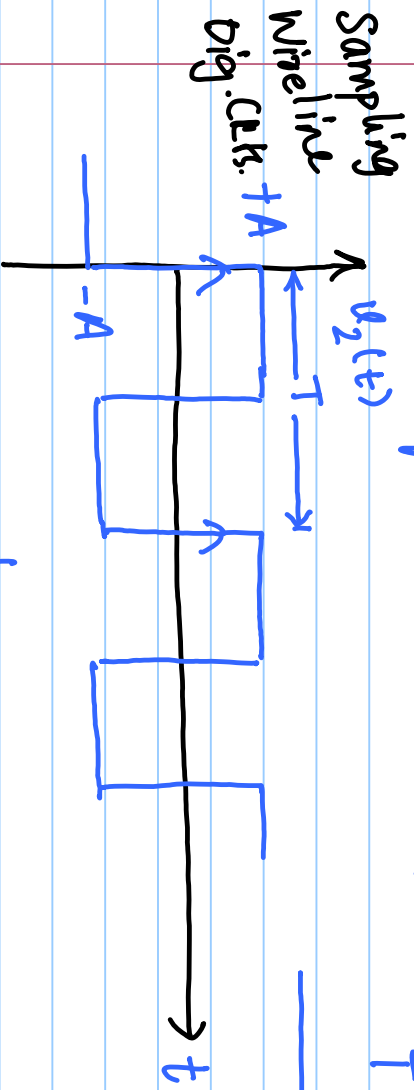
Lecture # 02



$$u_1(t) = A \sin(\omega_0 t)$$

$$= A \sin\left(2\pi \times \frac{1}{T} t\right)$$

freq. of sine wave, $f = \frac{1}{T}$

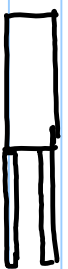


$$u_2(t) = \sum a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$\omega_0 = \frac{2\pi}{T}$$

$$0 < f = \frac{1}{T} \leq 20-30 \text{ GHz}$$

1. Crystal Oscillators.



Piezo-electric devices

→ Voltage / Current / signal.

sinusoidal

- Periodicity depends on the geometry of crystal.

$f_{\text{crystal}} \leq 100 \text{ MHz}$ - Frequency of oscillations has least

variation. $E[\Delta T^2]$ is minimum,

ΔT_{max} is also limited, $E[\Delta T] = 0$

Time period of sine waves

$$v_1(f) :$$

$$\sqrt{T}$$

$$\sqrt{T + \Delta T_1}$$

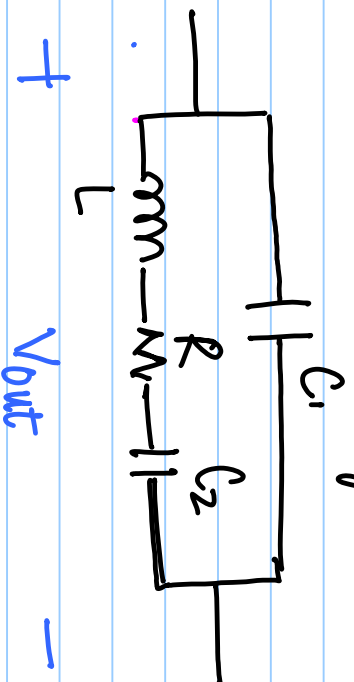
$$\sqrt{T + \Delta T_2}$$

$$\sqrt{T - \Delta T_3}$$

ΔT is a random variable

$$\text{Avg. time period} = E[T] = \frac{1}{N} \sum_{i=0}^{N-1} T_i = T + E[\Delta T] = T$$

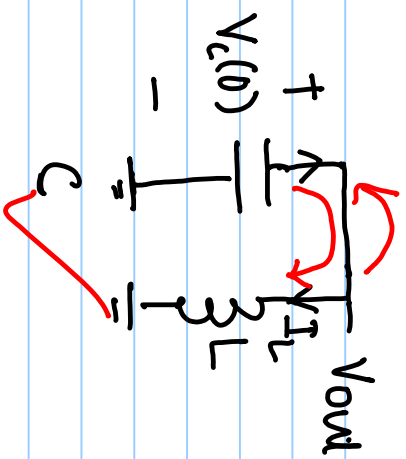
Electrical model of a crystal osc.



- V_{out} is sinusoidal nature.

$$f \propto \frac{1}{\sqrt{LC}}$$

$$\sqrt{LC}$$



at $t=0$, $V_{out} = V_c(t)$

$$I_L = 0$$

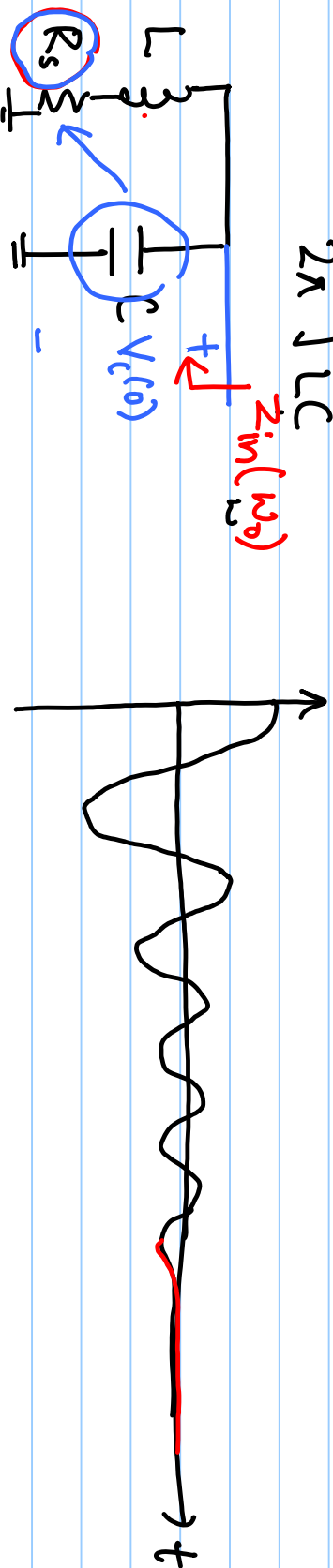
$$-C \frac{dV_{out}}{dt} = I_L$$

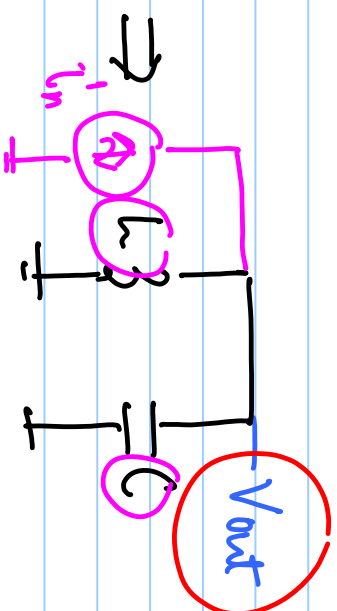
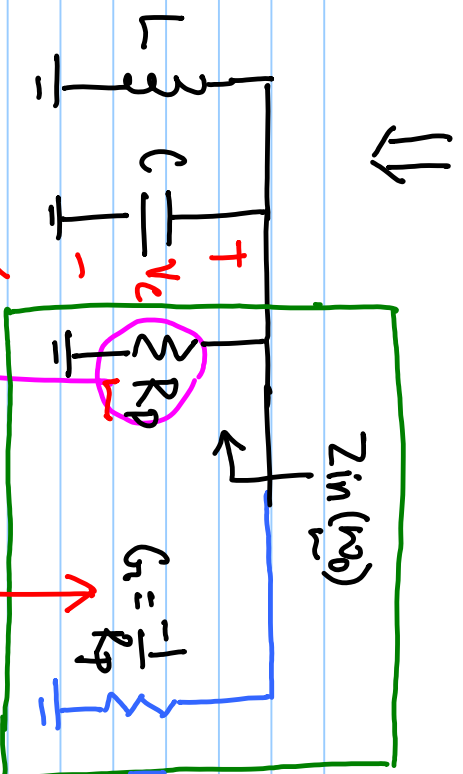
$$V_{out} = L \frac{dI_L}{dt}$$

$$V_{out} = \sin(\omega_0 t)$$

$$2\pi f_0 = \omega_0 = \frac{1}{\sqrt{LC}}$$

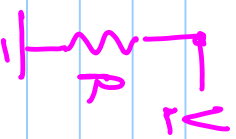
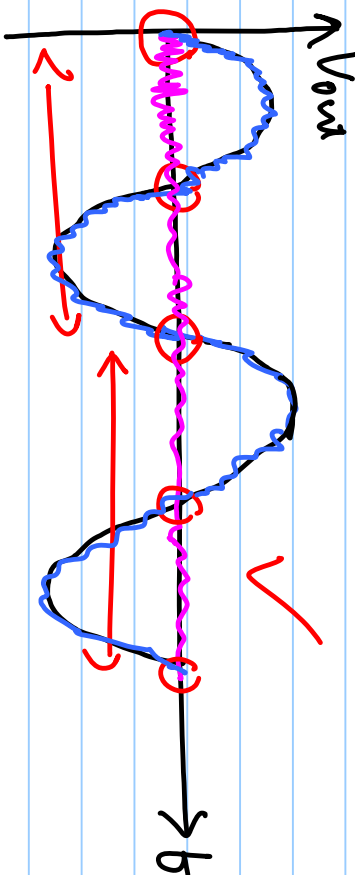
$$f_0 = \frac{1}{2\pi \sqrt{LC}}$$





Active Circuits.

$$\frac{U_N^2}{\Delta F} = 4kTR$$

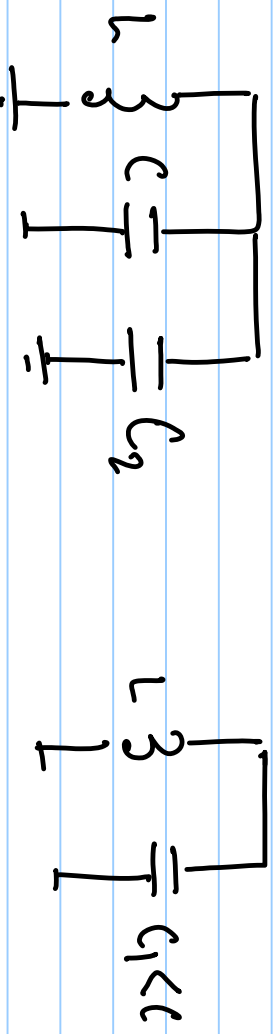


- We can realize higher frequency clock source with L & C.
- We added noise to clock source.
- Output frequency varies with L & C.

$$M_0 = 1 \text{ Grad/s} = \frac{1}{\sqrt{10^{-9} \times 10^{-9}}} = \frac{1}{\sqrt{1.05 \times 10^{-9}} \approx 1.0001 \times 10^{-9}}$$

\downarrow \downarrow
 $1 \mu\text{H}$ $1 \mu\text{F}$
+0.05 nH +0.1 pF

- Tunable inductor or capacitor to vary frequency.



Crystal

-|0|-

$\leq 100 \text{ MHz}$

Least Variation in T

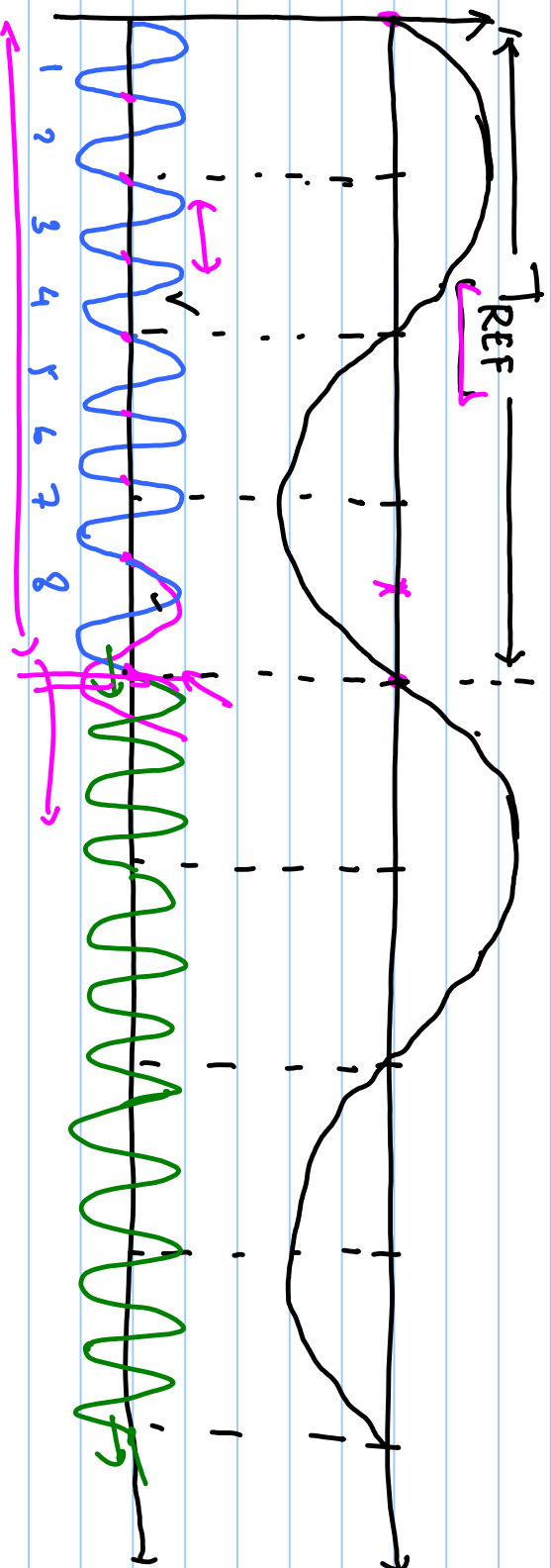
No tunability.

LC Oscillator

- frequency as desired.

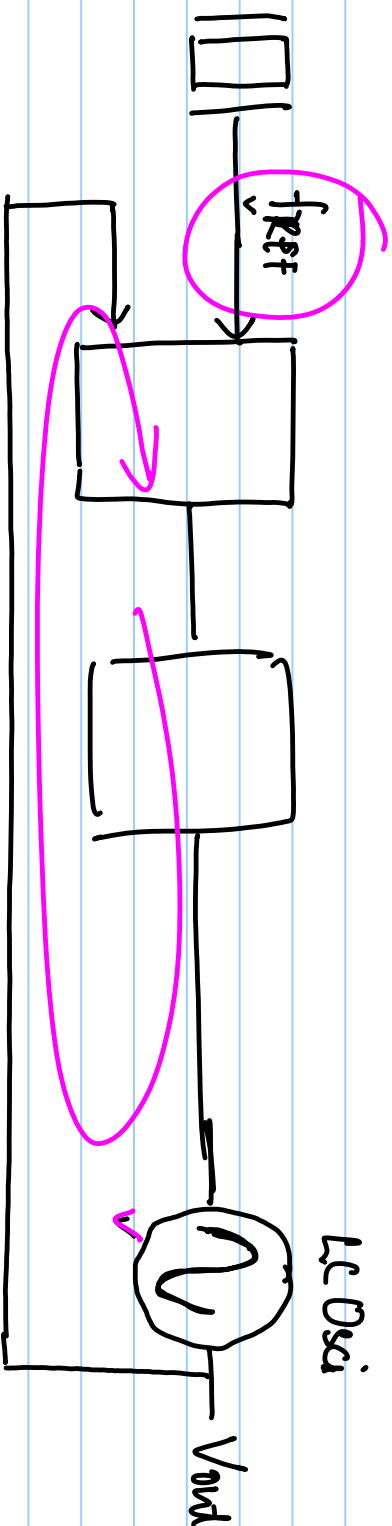
- More variation in T

- Tunability as desired.



Choose LC , $f_0 = \frac{8}{T_{REF}} = 8 f_{REF}$

We desire $f_0 = 8 f_{REF}$



Phase / Frequency locked loop.

$$V_{req}(t) = \sin(\omega_{req}t)$$

$$\phi_{out}(t) = \int \omega_{out} dt$$

$$V_{out}(t) = \sin(\omega_{out}t)$$

$$= \int \omega_{out} \cdot dt$$

$$= \sin(\underbrace{\omega_0 t}_{\text{fixed}} + \underbrace{\phi_n(t)}_{\text{Variation}})$$

fixed Variation.

Phase of o/p signal :

$$\phi_{out}(t) = \omega_0 t + \phi_n(t) =$$

Phase of i/p signal :

$$\phi_{in}(t) = \omega_{req} t \quad \checkmark$$

Assume $\omega_0 = \omega_{req}$,

$$\phi_{out}(t) = \omega_{req} t + \phi_n(t) \quad \checkmark$$

$$\left. \begin{array}{l} \phi_{in}(t) = \phi_n(t) - \phi_{out}(t) \\ \omega_{req} = \omega_0 \end{array} \right\} = (\omega_{req} - \omega_0)t - \phi_n(t)$$

Frequency of output sine wave, $\omega_{out} =$

$$\frac{d\phi_{out}(t)}{dt} \quad \checkmark$$

$$\omega_{out} = \omega_0 + \frac{d\phi_n(t)}{dt}$$

$$\omega_{out} = \omega_{ref} \quad \omega_{cont}$$

$$\omega_{out} \frac{d\phi_n(t)}{dt} = \omega_{ref}$$

$$\frac{d\phi_n(t)}{dt} = 0 \Rightarrow \phi_n(t) = 0$$

ω_{out} and ω_{ref} are locked in frequency and phase.

$$\omega_{out} = \omega_{ref}$$

$$\frac{d\phi_e}{dt} = 0 \Rightarrow \frac{d\phi_n(t)}{dt} = 0$$



$$W_{out} = W_{req}.$$

$$\frac{d\Phi_E}{dt} = 0$$