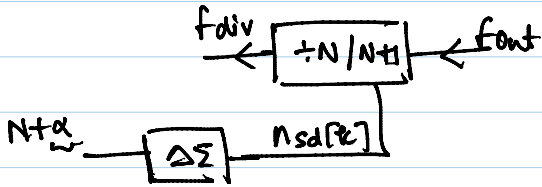


$$x[k] = n_{sd}[k] - q[k]$$

$$n_{fcw}[k] - q[k-1] = x[k]$$

$$n_{sd}[k] = n_{fcw}[k] + q[k] - q[k-1]$$

$$N_{sd}(z) = n_{fcw}(z) + q(z)(1-z^{-1})$$



$$f_{out} = (N+\alpha) f_{ref}$$

$n_{fcw}[k]$  : 10 bit i/p.  $\Rightarrow f_{out} = N f_{ref} + \frac{f_{ref}}{2^{10}} \times p$   
 $0 < p < 2^{10}$        $\frac{p}{2^{10}} = \alpha$

$p = 4$  ;  $0 < \alpha < 2^{10}$

$n_{fcw}[k] = 4$

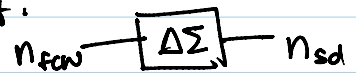
$f_{out} = (N + \frac{4}{2^{10}}) f_{ref}$

$n_{sd}[k] = n_{fcw}[k] + (q[k] - q[k-1])$

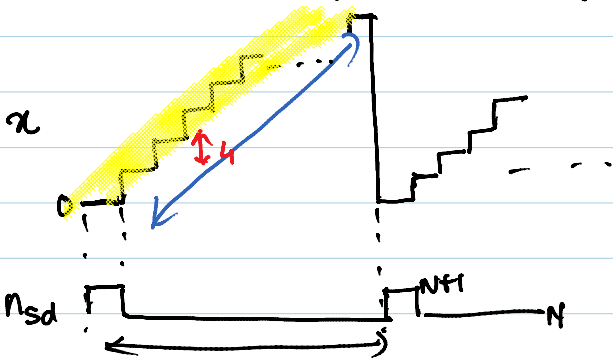
$n_{sd}[0] = 0, q[0] = 0, x[k] = 0$

$x[1] = n_{fcw}[1] + q[0] = 4$

$x[2] = n_{fcw}[2] + q[1] = 8$



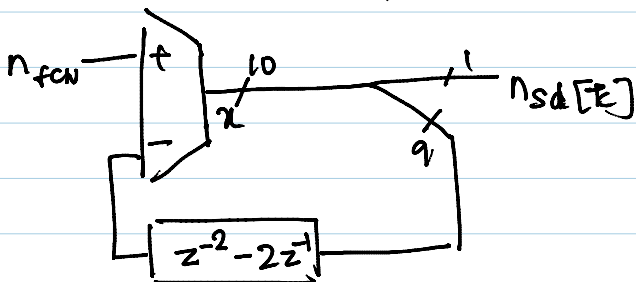
$N_{sd}(z) = n_{fcw}(z) + (1-z^{-1})q$



$f_{out} = N f_{ref} + \frac{f_{ref}}{2^{10}}$

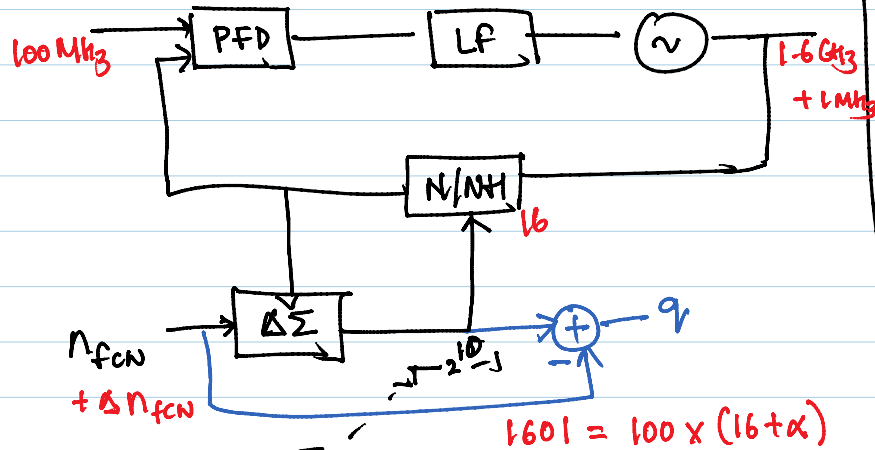
$n_{fcw} = 1$

$NFF(z) = (1-z^{-1})^2$   
 $= 1 + z^{-2} - 2z^{-1}$



Frac-N PLL

$$\left[ \frac{z^{-2} - 2z^{-1}}{z^2 - 2z^{-1}} \right]$$



### Frac-N PLL

$$1) \Delta F_{min} = \frac{f_{ref}}{2^N} \approx 100 \text{ kHz}$$

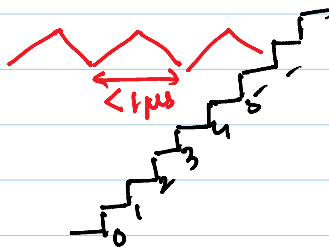
$$2) \text{Min Spur freq.} \geq \frac{f_{ref}}{2^N}$$

Eq: 900 MHz  $\rightarrow$  925 MHz

$$\Delta F = 200 \text{ kHz}$$

$$900 \cdot 200 \text{ MHz}$$

$$900 \cdot 201 \text{ MHz}$$

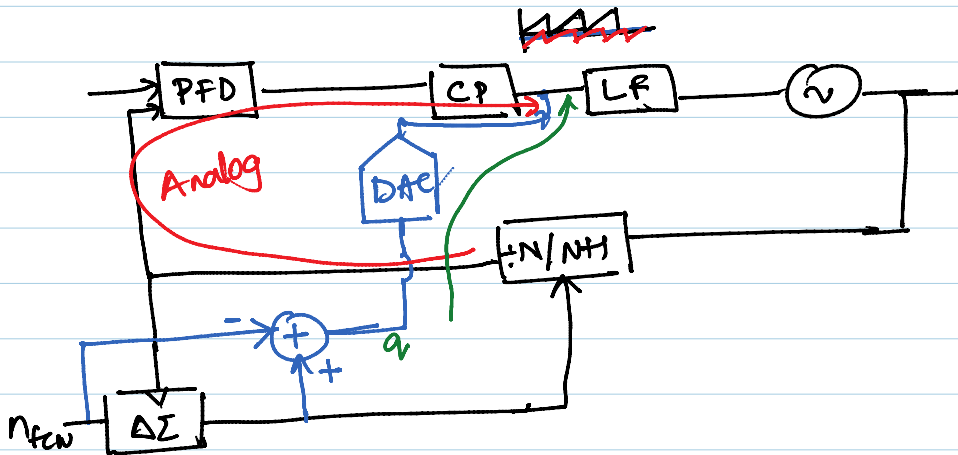


$$f_{ref} = 100 \text{ MHz}$$

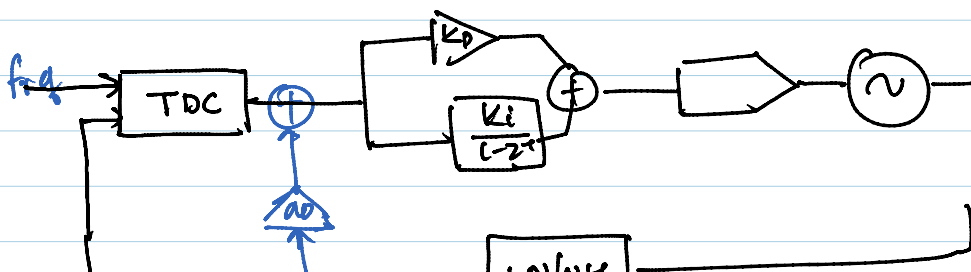
$$\frac{f_{ref}}{2^{10}} \approx \frac{100 \text{ MHz}}{1000} = 100 \text{ kHz}$$

$$f_{out} = 1.6 \text{ GHz} + n \times 1 \text{ MHz}$$

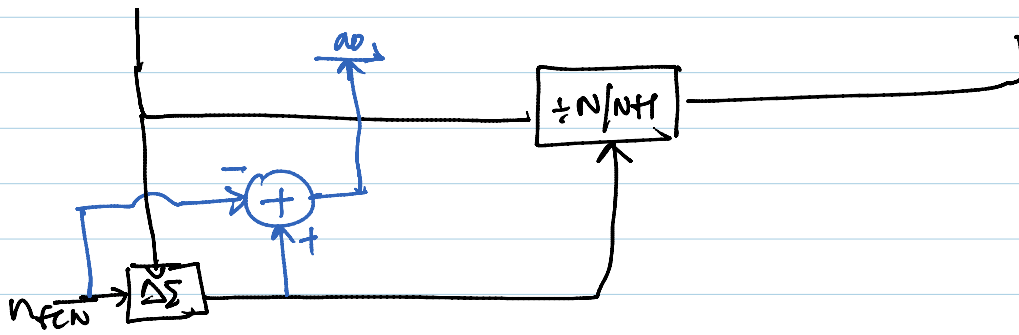
$$N_{fcw} = 0, 10, 20, 30 \dots$$



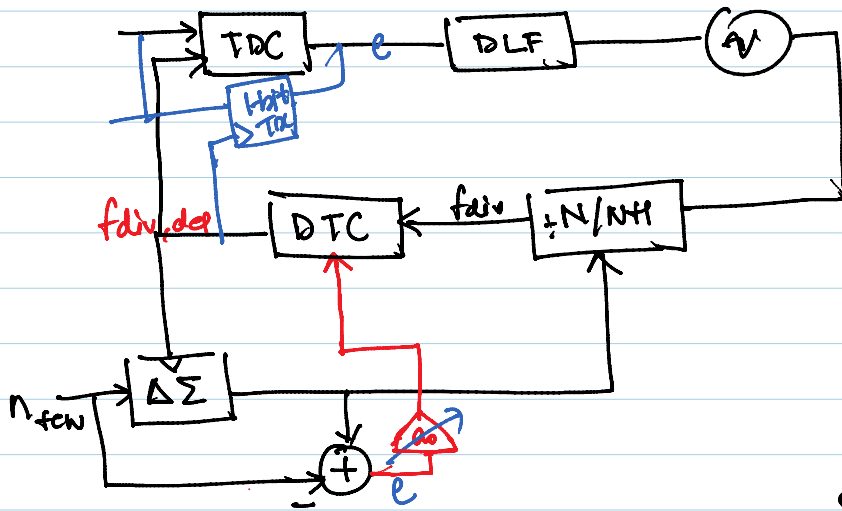
- 1) Mismatch between two paths for noise cancellation
- 2) Non-linearity/resolution of DAC limit cancellation of quantization noise



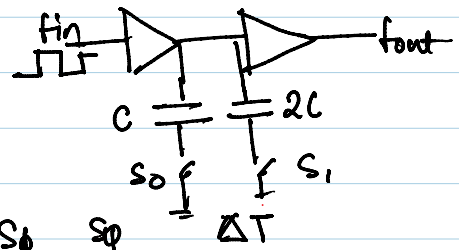
- 1) TDC resolution is limited by inverter delay.



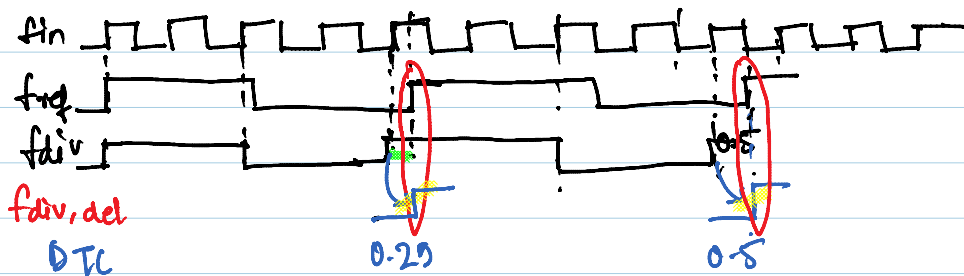
- delay -
- 2) TDC non-linearity
  - 3) Quantization noise



TDC: Time to digital converter  
 DTC: Digital to time converter



| \$S_0\$ | \$S_1\$ | \$\Delta T\$          |
|---------|---------|-----------------------|
| 0       | 0       | \$\Delta t_{offset}\$ |
| 0       | 1       | \$+t_d\$              |
| 1       | 0       | \$+2t_d\$             |
| 1       | 1       | \$+3t_d\$             |

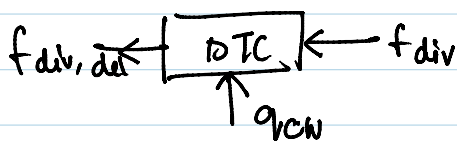


| \$q\$ | \$n_{sd}\$ |
|-------|------------|
| 0     | 0          |
| 0.25  | 0          |
| 0.50  | 0          |
| 0.75  | 0          |
| 0     | 1          |

$f_{out} = 2.5 \text{ GHz}$        $T = 400 \text{ ps}$

TDC:  $\Delta t_d = 5 \text{ ps}$

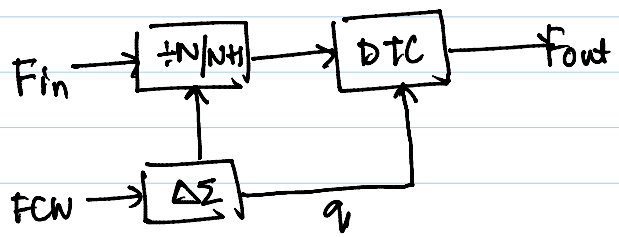
DTC:  $\Delta t_d = 0.5 \text{ ps}$



Range of DTC: 400 ps

- 1) PVT variation for DTC
- 2) DTC gain must be calibrated
- 3) DTC non-linearity

$q_{rcw}$  (10-bit) ;  $t_d = 0.4 \mu s$



Fractional-N Divider