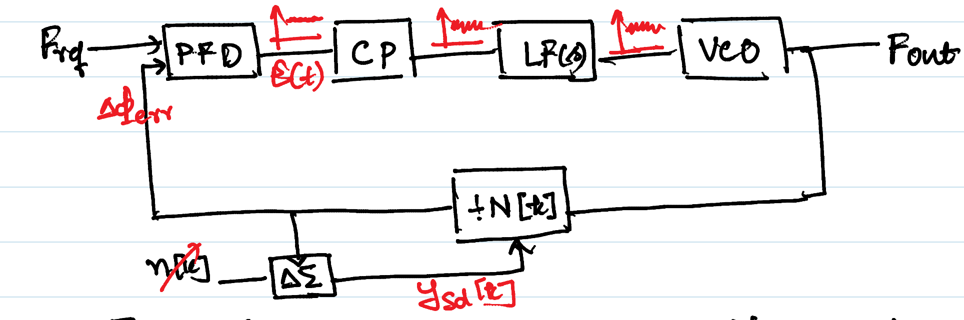
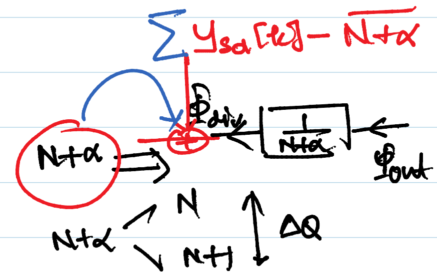
Lecture 40

Monday, April 10, 2017
10:59 AM

Fractional-N PLL



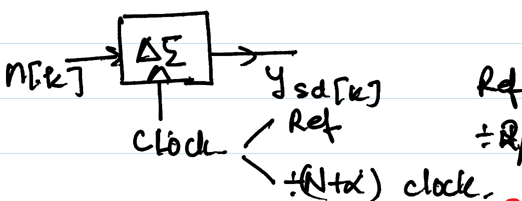


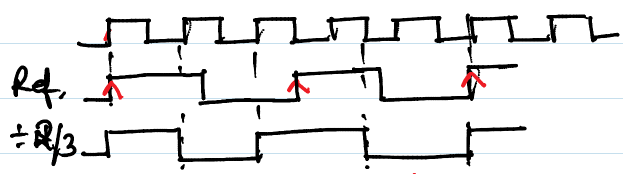
$F_{out} = (N+\alpha) F_{ref}, \quad 0 < \alpha < 1, \quad N = 1, 2, 3 \ldots$

Int-N PLL	Frac-N PLL
1) Constant N[k]	Varying N[k]
2) N[k] is integer	$N[k] = N+\alpha, \quad 0<\alpha<1$
3) In steady state $\Phi_{in} - \Phi_{div} = 0$	In steady state $\Phi_{in} - \Phi_{div} \neq 0$

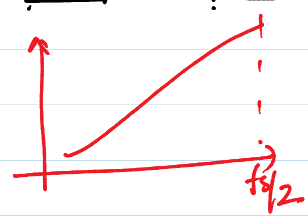
$$LG(s) = \frac{1}{2\pi} I_{cp} \frac{(R + 1/sC_1)}{1} \frac{K_{vco}}{sN} \qquad \text{Int-N PLL}$$

$$LG(s) = \frac{1}{2\pi} I_{cp} (R + 1/sC_1) \frac{K_{vco}}{\underbrace{s N(t)}} \quad (N+\alpha)$$





$OSR = \frac{f_s}{2 f_{BW}}$



Loop gain analysis of Frac-N PLL.

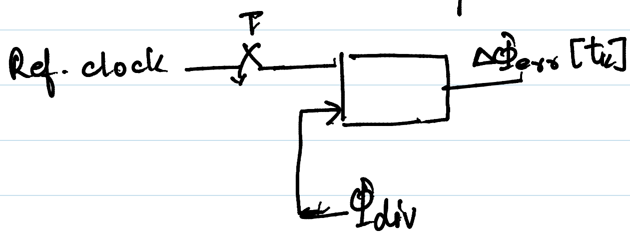
1) t_k: time instant for rising edge on ref. clock

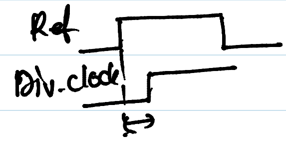
$t_k - t_{k-1} = T$ (ref. time period).

$t_k + \Delta t_k$: time instant for rising edge on divided clock.

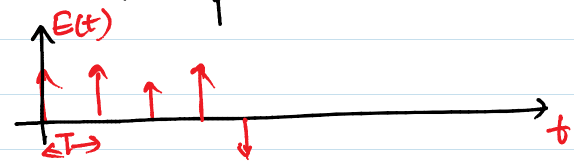
Phase error at i/p of PFD, $\Delta\Phi_{err}(k) = \frac{2\pi}{T}\left[(t_k+\Delta t_k) - t_k\right]$

$\Delta\Phi = 2\pi \cdot \frac{\Delta t}{T}$





$\Delta\phi = 2\pi \frac{\Delta t}{T}$



2) $\Phi_{vco}(t) = 2\pi \int K_{vco} \cdot V_{ctrl}\, dt$

$= \underbrace{2\pi f_{nom} t} + \underbrace{\Phi_{out}(t)}$

3).

$\Phi_{vco}(t_k+\Delta t_k) - \Phi_{vco}(t_{k-1}+\Delta t_{k-1}) = 2\pi N[k-1]$ ✓

$= 2\pi f_{nom}(t_k+\Delta t_k - t_{k-1} - \Delta t_{k-1}) + \Phi_{out}(t_k+\Delta t_k) - \Phi_{out}(t_{k-1}+\Delta t_{k-1})$

$= 2\pi f_{nom}\underbrace{(t_k - t_{k-1})}_{T} + 2\pi f_{nom}(\Delta t_k - \Delta t_{k-1}) + \Phi_{out}(t_k+\Delta t_k) - \Phi_{out}(t_{k-1}+\Delta t_{k-1})$

$= 2\pi f_{nom} \cdot N_{nom}$ + " + " ✓

$2\pi f_{nom}(\Delta t_k - \Delta t_{k-1}) = 2\pi(\underbrace{N[k-1] - N_{nom}}_{n[k-1]}) - [\Phi_{out}(t_k+\Delta t_k) - \Phi_{out}(t_{k-1}+\Delta t_{k-1})]$

Sum over, $k = 1, 2 \ldots k$

$$2\pi f_{nom}(\Delta t_k - \Delta t_0) = 2\pi \sum_{m=1}^{k} n[m-1] - \left[\Phi_{out}(t_k+\Delta t_k) - \Phi_{out}(0)\right]$$

$t=0, \quad \Delta t_0 = 0, \quad \Phi_{out}(0) = 0$

$$2\pi f_{nom} \cdot \Delta t_k = 2\pi \sum_{m=1}^{k} n[m-1] - \Phi_{out}(t_k+\Delta t_k)$$

$$2\pi f_{nom}\cdot \Delta t_k = 2\pi \sum_{m=1} n[m-1] - \Phi_{out}(t_k+\Delta t_k)$$

$$\Delta t_k = \frac{1}{f_{nom}} \sum_{m=1}^{k} n[m-1] - \frac{\Phi_{out}(t_k+\Delta t_k)}{2\pi f_{nom}}$$

$$\Phi_{div}[k] = 2\pi\cdot\frac{\Delta t_k}{T}$$

$$\Rightarrow \quad \Phi_{div}[k] = \frac{2\pi}{T\cdot f_{nom}} \sum_{m=1}^{k} n[m-1] - \frac{\cancel{2\pi}\,1}{T\cdot f_{nom}} \Phi_{out}(t_k+\Delta t_k)$$

$$\Phi_{div}[k] = \frac{1}{N_{nom}}\, 2\pi \sum_{m=1}^{k} n[m-1] - \frac{1}{N_{nom}} \Phi_{out}(t_k+\Delta t_k)$$

Z-transform $\Rightarrow$

$$\Phi_{div}(z) = \frac{1}{N_{nom}}\, \frac{2\pi\cdot z^{-1}}{1-z^{-1}} \times n[z] - \frac{1}{N_{nom}} \Phi_{out}(z)$$