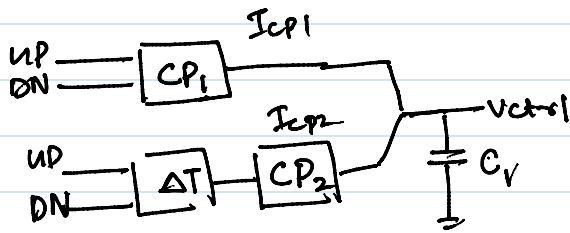


Loop Filter.



$$\begin{aligned}
 V_{ctrl}(s) &= I_{cp} \times \frac{1}{sC_1} + I_{cp} e^{-s(\Delta T)} \frac{1}{sC_1} \\
 &= \frac{1}{sC_1} [I_{cp1} + I_{cp2} (1 - s\Delta T)] \\
 &= \frac{1}{sC_1} (I_{cp1} + I_{cp2}) \left[1 - \frac{I_{cp2} \Delta T}{I_{cp1} + I_{cp2}} s \right] \\
 &= \frac{1}{sC_1} \underbrace{(I_{cp1} - I_{cp2})} \left[1 + \frac{I_{cp2} \Delta T}{I_{cp1} - I_{cp2}} s \right]
 \end{aligned}$$

$$I_{cp1} - I_{cp2} > 0,$$

For +ve $\Delta\phi_{err}$, CP_1 pushes current into cap.
 CP_2 drains current from cap.

$$V_{ctrl}(s) = \frac{I_{cp}}{sC_1} \left[1 + \frac{I_{cp2} \cdot \Delta T \cdot s}{I_{cp}} \right]$$

$$\omega_z = \frac{1}{\frac{I_{cp2}}{I_{cp}} \cdot \Delta T}$$

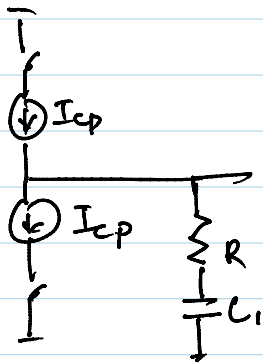
$$\text{Eq: } \Delta T = \frac{1}{2\pi \times 10^6} \approx 0.16 \mu\text{s}$$

Digital Phase Locked Loop (DPLL)

- 1.) Loop filter occupies a lot of area
 - large area for passive cap.
 - large area for active cap. + leakage current.

2) Mismatch in CP or UP/DN current sources.

3) Digital logic is invariant to DVT variations.



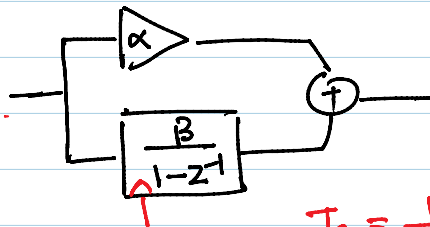
$$I_{cp} \left(R + \frac{1}{sC_1} \right) \Rightarrow I_{cp} \left[R + \frac{1}{C_1} \frac{T_s}{2} \frac{(1+z^{-1})}{1-z^{-1}} \right]$$

Bilinear trans. $s = \frac{2}{T_s} \frac{1-z^{-1}}{1+z^{-1}}$

T_s is sampling freq.)

$$LF(z) = I_{cp} \left[R + \frac{\frac{T_s}{2C} (1+z^{-1})}{1-z^{-1}} \right]$$

$$= \frac{I_{cp}}{1-z^{-1}} \left[\left(R + \frac{T_s}{2C} \right) + \left(\frac{T_s}{2C} - R \right) z^{-1} \right] \checkmark$$



$$LF(z) = \alpha + \frac{\beta}{1-z^{-1}}$$

$$= \frac{(\alpha + \beta) - \alpha \cdot z^{-1}}{1-z^{-1}}$$

$T_s = \frac{1}{f_{ref}}$

$$\alpha + \beta = I_{cp} \cdot \left(R + \frac{T_s}{2C} \right)$$

$$\alpha = I_{cp} \left(R - \frac{T_s}{2C} \right)$$

$$\Rightarrow \beta = I_{cp} \cdot \frac{T_s}{C}$$

$$\frac{\alpha}{\beta} = \frac{R - \frac{T_s}{2C}}{T_s/C} = \frac{RC}{T_s} - \frac{1}{2}$$

$$LG(s) = \frac{I_{cp}}{2\pi} (R + 1/sC) \times \frac{K_{VCO}}{s}$$

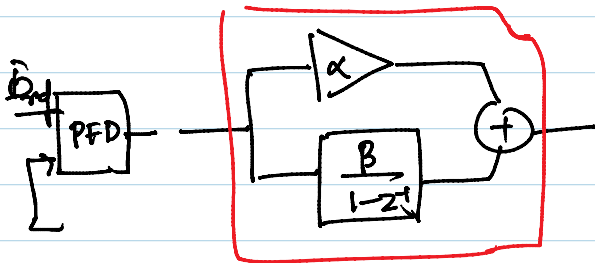
$$= \frac{I_{cp}}{2\pi s^2 C} \cdot K_{VCO}$$

$$\hat{\phi}_m = \tan^{-1}(w_u \cdot RC) = \tan^{-1}\left(\frac{w_u}{w_z}\right) \Rightarrow \frac{1}{w_z} = \frac{1}{w_u} \tan(\hat{\phi}_m)$$

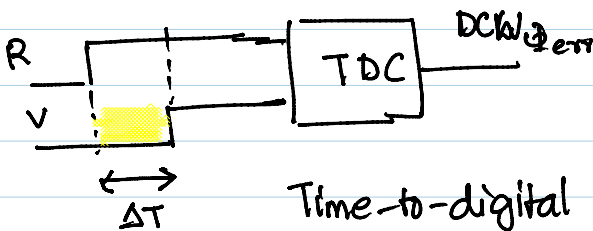
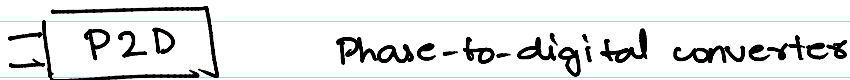
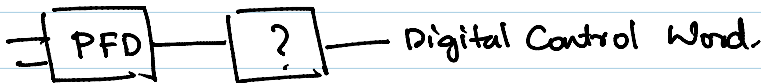
$$|L| = 1 \Rightarrow w_u = \frac{I_{cp} \cdot K_{vco} \cdot R}{2\pi}$$

$$\frac{\alpha}{\beta} = \frac{F_{ref}}{w_z} - \frac{1}{2} = \frac{F_{ref}}{w_u} \cdot \tan(\hat{\phi}_m) - \frac{1}{2}$$

$$\frac{\alpha}{\beta} = \frac{F_{ref}}{w_u} \cdot \tan(\hat{\phi}_m) - \frac{1}{2}$$



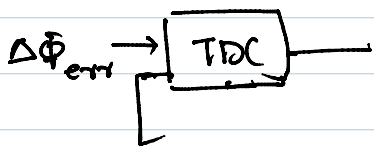
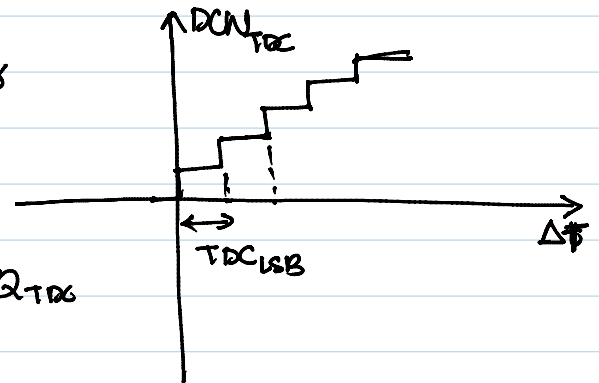
Loop-filter



Time-to-digital converter

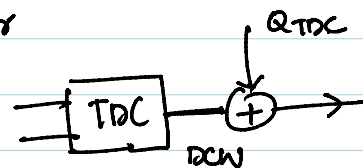
$$\Delta\hat{\phi}_{err} = 2\pi \cdot \frac{\Delta T}{T_{ref}}$$

$$\Delta T = \underbrace{\left[\frac{\Delta T}{TDC_{LSB}} \right]}_{DCW} \cdot TDC_{LSB} + Q_{TDC}$$

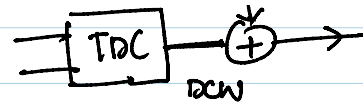


$$\Delta F = \frac{\Delta\hat{\phi}_{err}}{2\pi} \cdot T_{ref} = \frac{T_{ref}}{2\pi} \cdot \Delta\hat{\phi}_{err}$$

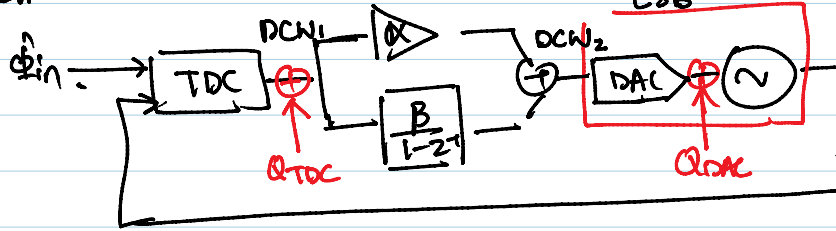
$$= \left[\frac{\Delta T}{TDC_{LSB}} \right] \cdot TDC_{LSB}$$



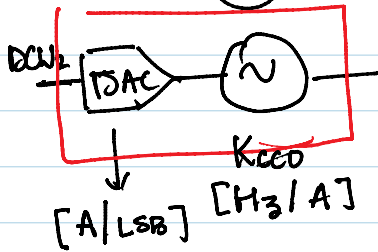
$$= \left[\frac{\Delta T}{TDC_{LSB}} \right] \cdot TDC_{LSB}$$



$$\frac{\hat{\phi}_{out}}{\hat{\phi}_{in}} = LC(z) = \frac{T_{ref}}{2\pi} \times \frac{1}{TDC_{LSB}} \times \left(\alpha + \frac{\beta}{1-z^{-1}} \right) \frac{K_{DCO}}{s}$$



$$v_{ctrl} \sim \hat{\phi}_{out} = \frac{K_{VCO}}{s}$$



$$K_{DCO} [Hz/LSB]$$