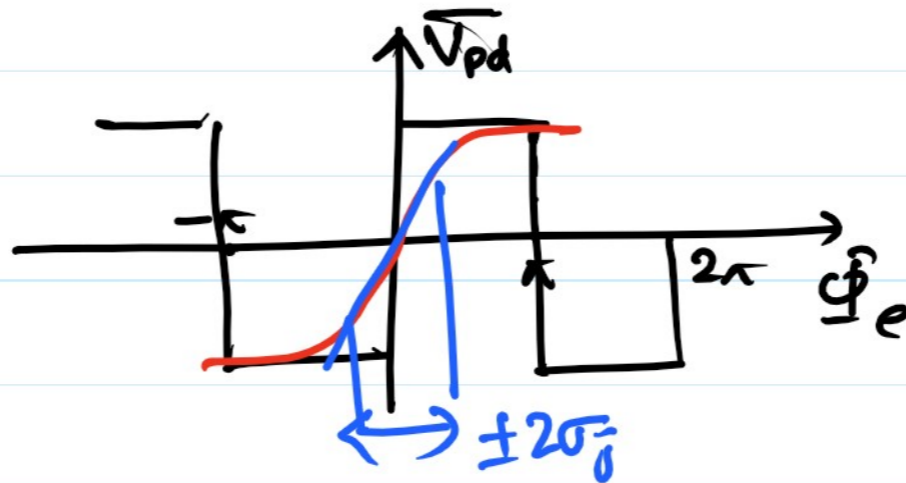
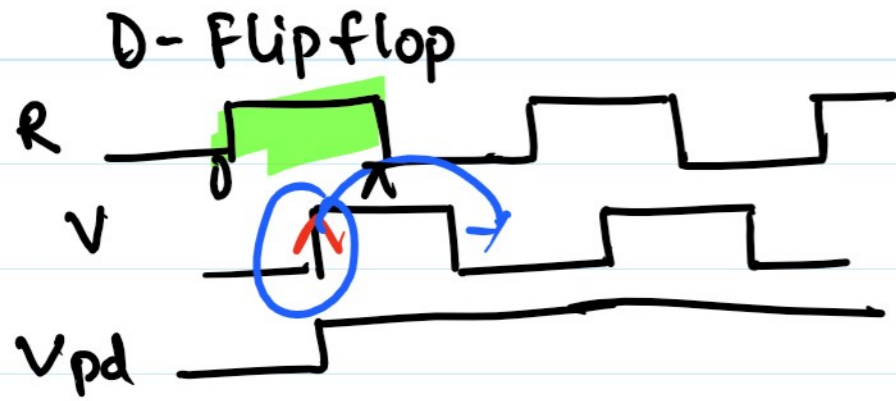
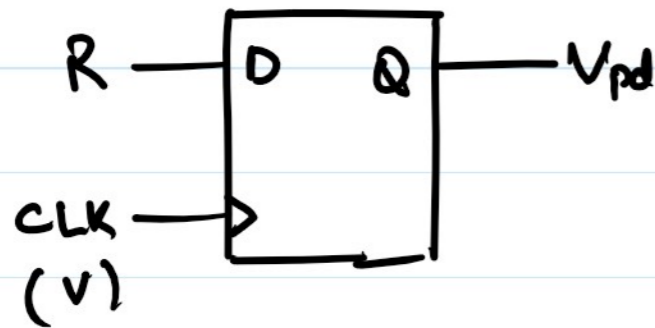


Phase error detector



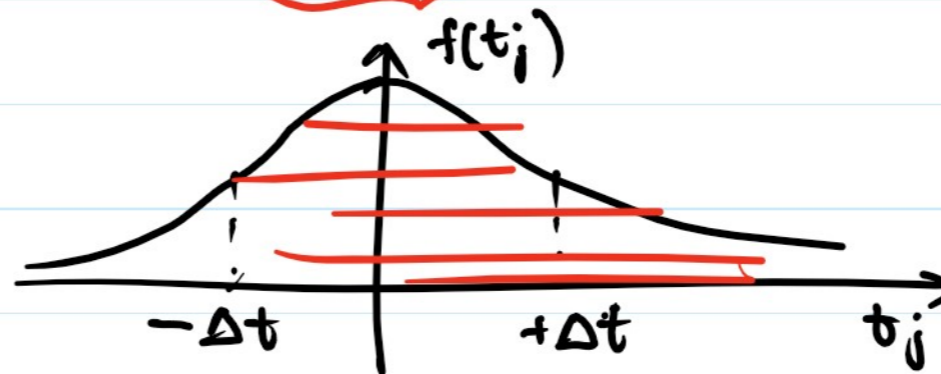
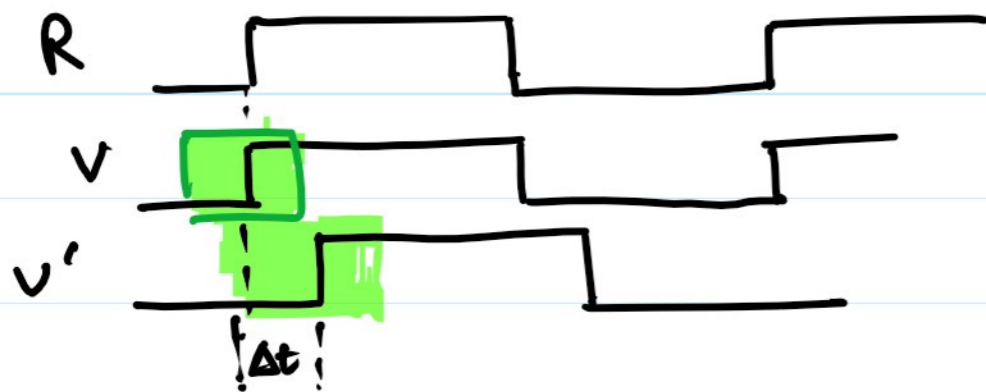
$$\frac{d(\bar{V}_{pd})}{d(\hat{\phi}_e)}$$

Noise on clock is Gaussian $N(0, \sigma)$



$$\bar{V}_{pd} = +1 P(\Delta t + t_j \geq 0) - 1 P(\Delta t + t_j < 0)$$

$$= P(t_j \geq -\Delta t) - P(t_j < -\Delta t)$$



$$\bar{V}_{pd} = 1 - 2P(t_j < -\Delta t)$$

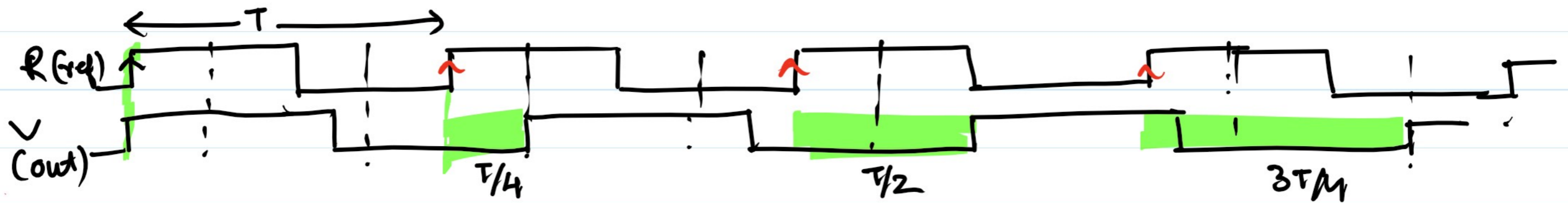
$$V_{pd} = 1 - 2P(t_j < -\Delta t)$$

$$= 1 - 2 \int_{-\infty}^{-\Delta t} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_j} \exp\left(-\frac{t_j^2}{2\sigma^2}\right) dt_j$$

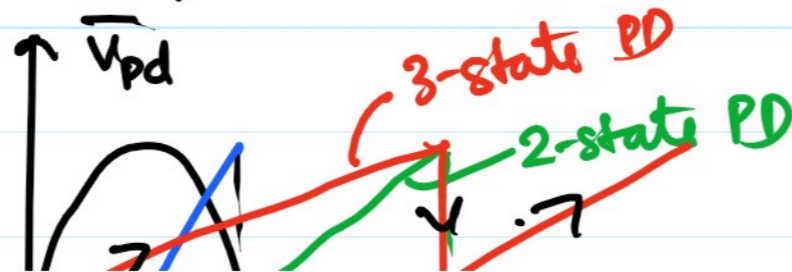
$$\frac{d(\bar{V}_{pd})}{d(\Delta t)} \approx -\frac{2}{\sqrt{2\pi}} \frac{1}{\sigma_j} (-1) \exp\left(-\frac{\Delta t^2}{2\sigma^2}\right)$$

$$\left. \frac{d(\bar{V}_{pd})}{d(\Delta t)} \right|_{\Delta t=0} = \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_j}$$

PDs Multiplier, sample & hold, EXOR, 2-state PD, 3-state PFD, D-Flip flop.



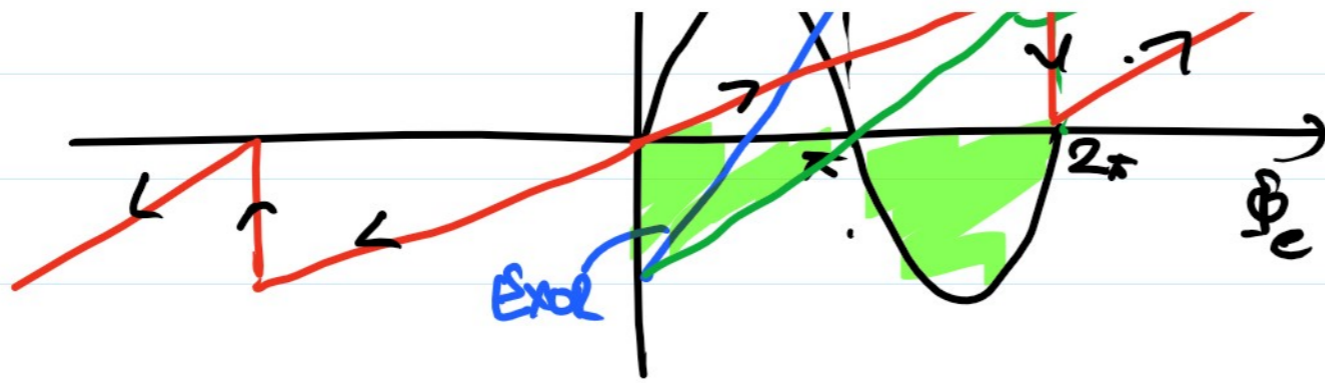
$$f_R = \frac{1}{T}, \quad f_V = \frac{1}{T + T/4} = \frac{4}{5T}$$



\bar{V}_{pd}

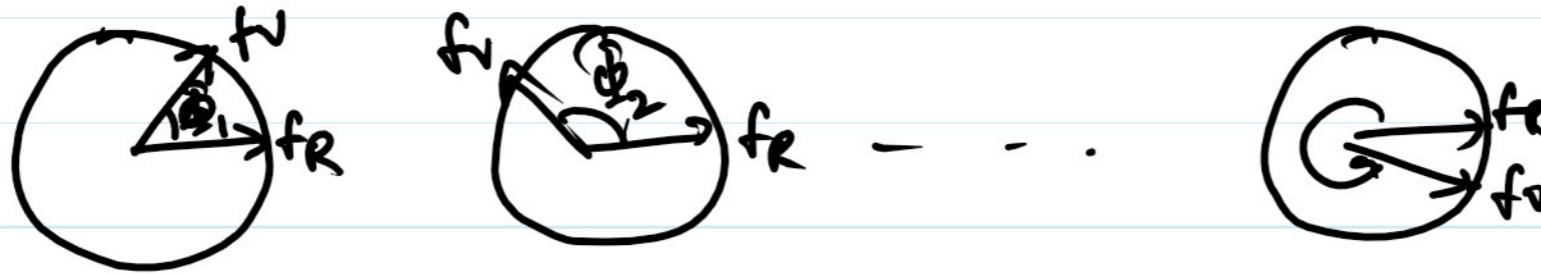
Sine

EXOR



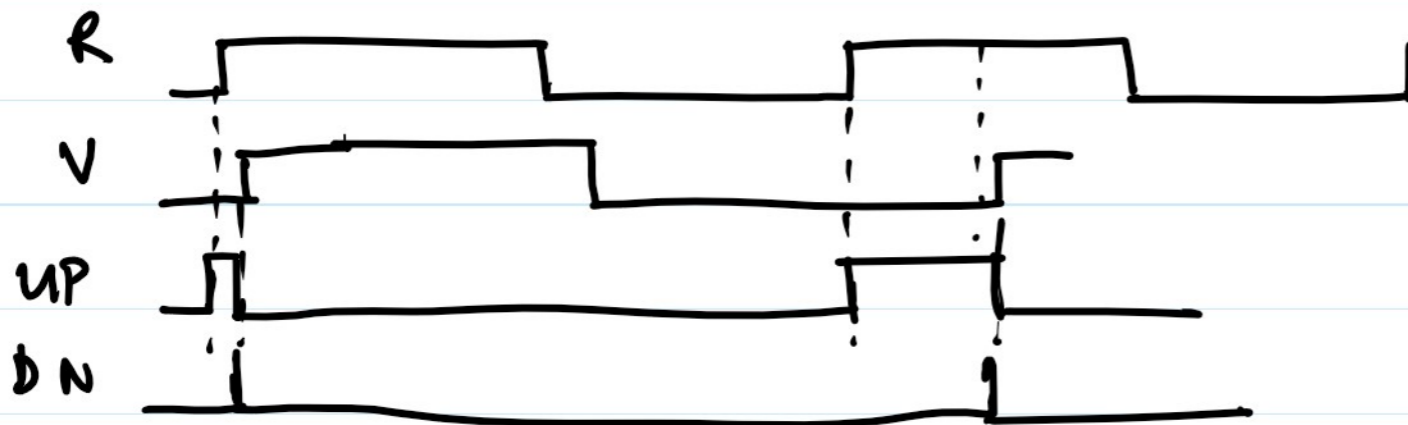
EXOR
2-stage PD

$$\phi_e = \int 2\pi (f_{in} - f_{out}) dt$$



$$\frac{\Delta\phi}{\text{cycle}} = \frac{2\pi (T_R - T_V)}{\max.(T_R, T_V)}$$

Gain of FD (frequency detector) using PFD



In one ref. period

$$\frac{\Delta t}{T} = \overline{V_{up} - V_{DN}}$$

$$\bar{V}_{FD} = \left[\frac{\Delta t}{T} + 2 \frac{\Delta t}{T} + \dots + N \frac{\Delta t}{T} \right] \times \frac{1}{N}$$

$$= \frac{\Delta t}{NT} (1+2+\dots+N)$$

$$= \frac{\Delta t}{NT} \frac{N(N+1)}{2}$$

$$= 0.5 \left(\frac{1}{N} + 1 \right)$$

$$= 0.5 \left(1 + \frac{\Delta t}{T} \right)$$

$$= 0.5 \left(1 + \frac{T_R - T_V}{T_R} \right)$$

$$= 1 - 0.5 \frac{T_V}{T_R}$$

$$\bar{V}_{FD} = \bar{V}_{up} - \bar{V}_{DN} = - \left[1 - 0.5 \frac{f_R}{f_V} \right]$$

$$= \frac{0.5 f_R}{f_V} - 1 \quad ; \quad f_V > f_R$$

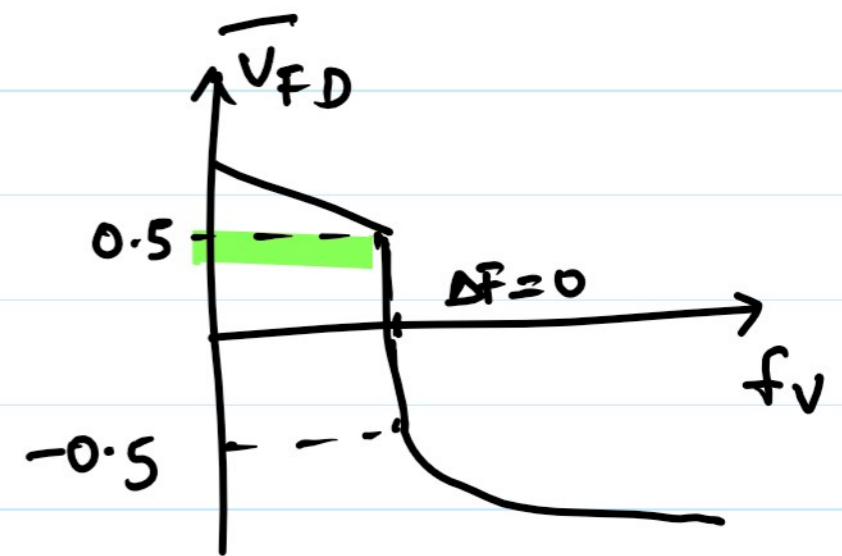
$$\bar{V}_{FD} = \bar{V}_{up} - \bar{V}_{DN} = 1 - \frac{0.5 f_V}{f_R} \quad ; \quad f_V < f_R$$

$$N(\Delta t) = T$$

T_R reference period

T_V o/p period

$$\Delta t = T_R - T_V \quad ; \quad T_R > T_V$$



$$\overline{V_{PD}} = \frac{f_v}{V_{LUP} - V_{DN}} = 1 - \frac{0.5 f_v}{f_R} ; f_v < f_R$$

