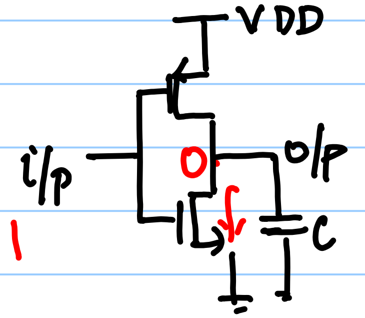
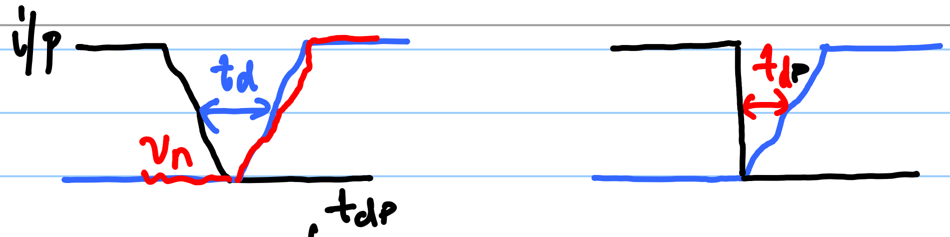
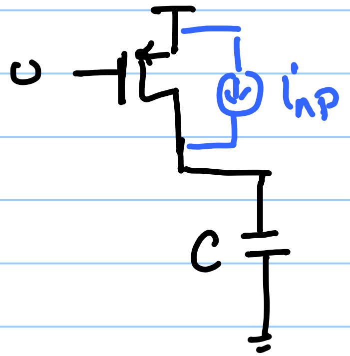
Note Title 13-11-2020

Lecture # 34





$$\frac{1}{C}\int_0^{t_{dp}} (I_p + i_{np})\, dt = \frac{VDD}{2} \checkmark$$

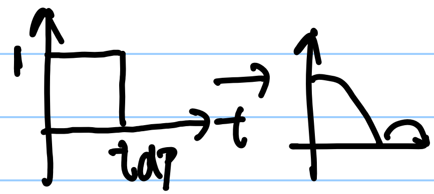


$$\frac{I_p t_{dp}}{C} + \frac{1}{C}\int_0^{t_{dp}} i_{np}\, dt = \frac{VDD}{2}$$

$$\langle t_{dp} \rangle = \frac{C\,VDD}{2 I_p}$$

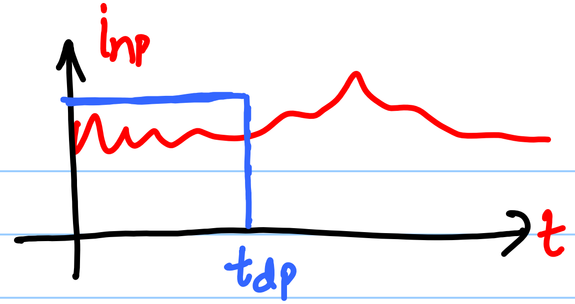
$$\checkmark\ \sigma_{t_{dp}}^2 = \frac{1}{I_p^2}\left\langle \left(\int_0^{t_{dp}} i_{np}\, dt \right)^2 \right\rangle$$

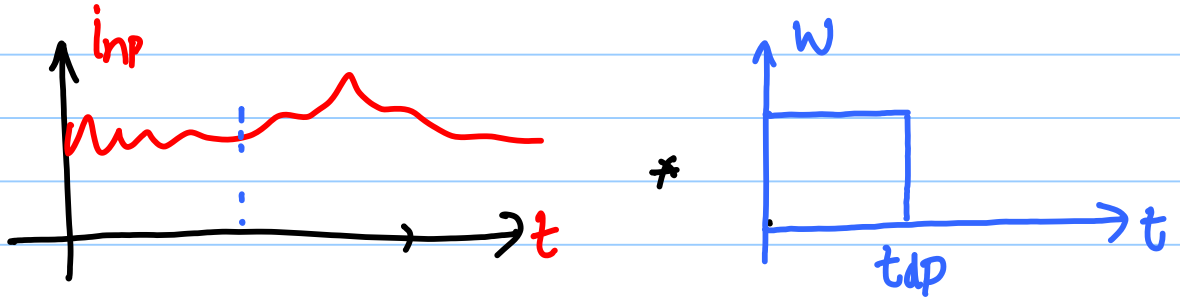
$$\frac{1}{I_p^2}\int_0^{t_{dp}} i_{np}\cdot dt = \frac{1}{I_p^2}\int_0^{\infty} i_{np}(x) \times w(t_{dp}-x)\, dx$$



$$S_{t_{dp}}(f) = \frac{t_{dp}^2}{I_p^2} \operatorname{sinc}^2(f t_{dp})\, S_{i_{np}} \checkmark$$

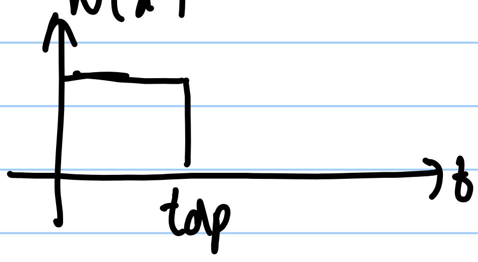
$$\int_0^{t_{dp}} i_{np}\, dt \quad ✓$$

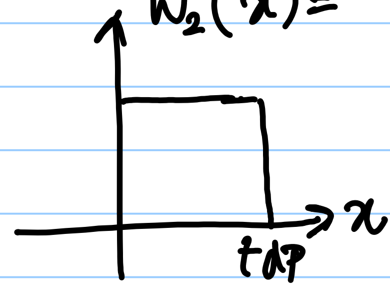




$$y(t) = i_{np}(t) * w(t) \quad = \int_0^{\infty} i_{np}(x)\; w(t-x)\; dx$$

$W_2(x) = w(t_{dp} - x)$





$W_1(x) : \quad w(-x)$

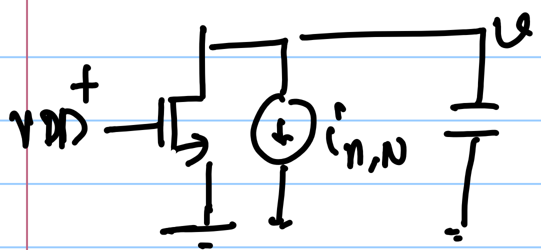
$W_2(x) : \quad W_1(x - t) = w(t - x)$

Wiener - Khinchine Theorem

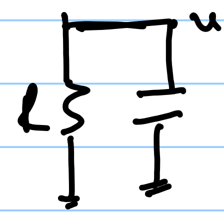
$$\sigma_{tdp}^2 = \int_0^\infty S_{tdp}\,.df = \frac{t_{dp}}{\pi I_p^2} \underbrace{S_{i_{n},p}} \underbrace{\int_0^\infty \frac{\sin^2 x}{x^2}\,dx}_{\pi/2}$$

$$\sigma_{tdp}^2 = \frac{4kT\gamma_p\, t_{dp}}{I_p\,(VDD - |V_{tp}|)}$$

$$S_{in,p} = 4kT\gamma_p \frac{2I_p}{VDD - |V_{tp}|}$$

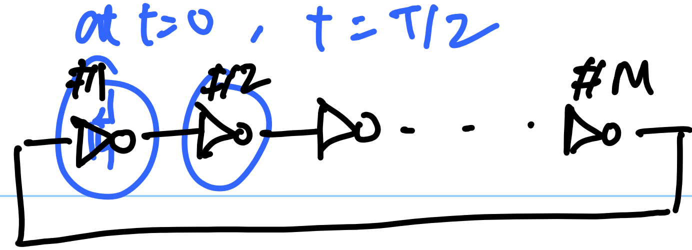


$$\langle v_n^2 \rangle = \underbrace{\frac{kT}{C}}$$



$$\sigma_{tdp}^2 = \frac{4kT\gamma_p\,.t_{dp}}{I_p(VDD-|V_{tp}|)} + \frac{\langle v_n^2\rangle}{(I_p/C)^2}$$

$$= \frac{4kT\gamma_p\,.t_{dp}}{I_p(VDD-|V_{tp}|)} + \frac{kTC}{I_p^2}$$



$$f_0 = \frac{1}{M(t_{dp}+t_{dN})}$$

$$= \frac{2}{MCVDD}\left(\frac{1}{I_N}+\frac{1}{I_p}\right)^{-1}$$

$$\langle t_{dp}\rangle = \frac{CVDD}{2I_p} \checkmark$$

$$\langle t_{dN}\rangle = \frac{CVDD}{2I_N} \checkmark$$

$$f_0 = \frac{I/C}{MVDD} \quad ; \quad I_N = I_p = I$$

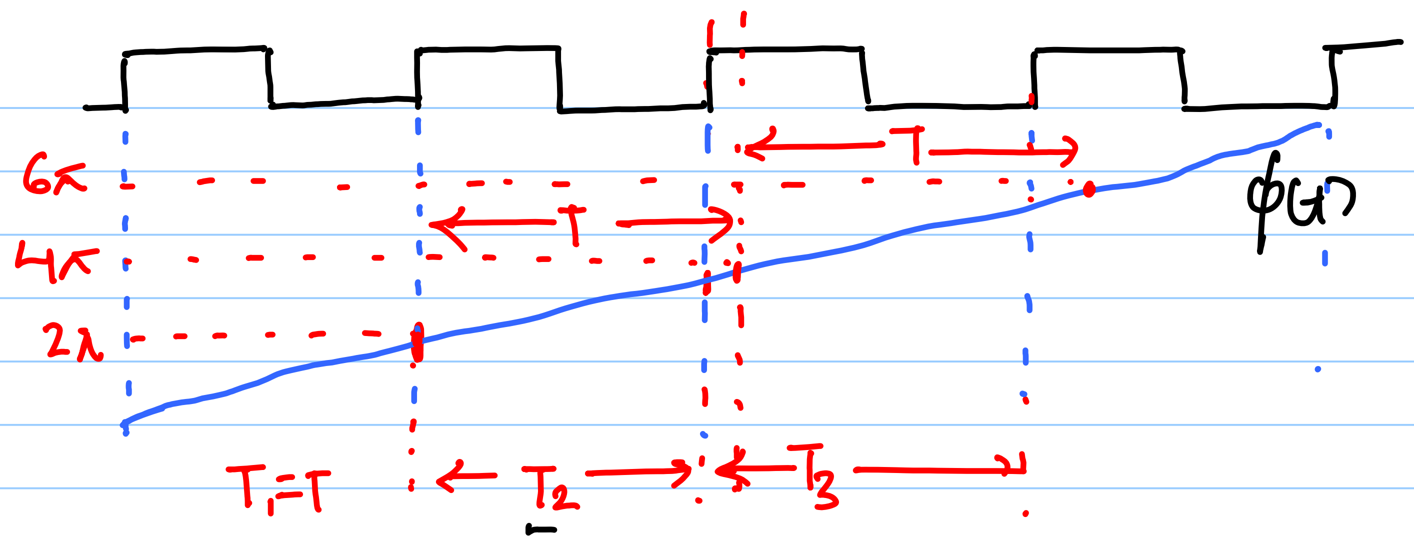
$$T = M\,(t_{dp}+t_{dN})$$ time period.

$$\sigma_\tau^2 = M\left(\sigma_{tdp}^2 + \sigma_{tdN}^2\right)$$ σ_τ : period jitter

$$= M\left(\frac{4kT\gamma_p \cdot t_{dp}}{I_p(VDD-|V_{tp}|)} + \frac{KTC}{I_p^2} \quad \frac{4kT\gamma_p \cdot t_{dN}}{I_N(VDD-|V_{tp}|)} + \frac{KTC}{I_N^2}\right)$$

$$\checkmark\ \sigma_\tau^2 = \frac{kT}{If_0}\left(\frac{2}{Vdd-V_t}(\gamma_N+\gamma_p) + \frac{2}{VDD}\right) \checkmark \quad ; \quad I_p = I_N = I$$

$$V_{tn} = |V_{tp}| = V_t$$



$$\tau_i = \frac{1}{2\pi f_0}\left(\phi(t_{i+1}) - \phi(t_i)\right) = \frac{1}{2\pi f_0}\Delta\phi_i$$

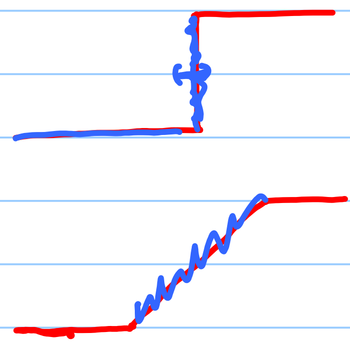
$$S_{\Delta\phi}(f) = S_\phi(f)\left|1 - e^{-j2\pi f/f_0}\right|^2 = 4S_\phi(f)\sin^2(\pi f/f_0)$$

$$S_\tau(f) = S_\phi(f)\frac{\sin^2(\pi f/f_0)}{(\pi f_0)^2}$$

$$\sigma_\tau^2 = \int_0^\infty S_\tau(f)\,df = \int_0^\infty S_\phi(f)\frac{\sin^2(\pi f/f_0)}{(\pi f_0)^2}\,df$$

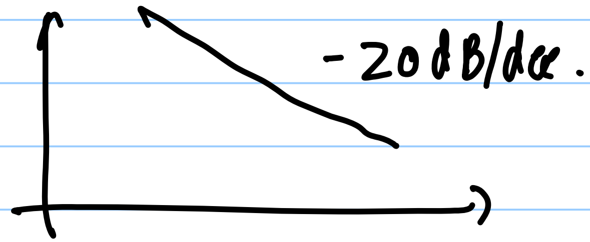
$$\mathcal{L}(f) = \frac{S_\phi(f)}{2} = \frac{S_w}{f^2} \; ; \; f \text{ is offset from } f_0$$

$$\sigma_\tau^2 = \int \frac{2S_w}{f^2} \frac{\sin^2(\pi f/f_0)}{(\pi f_0)^2} \, df = \frac{S_w}{f_0^3}$$



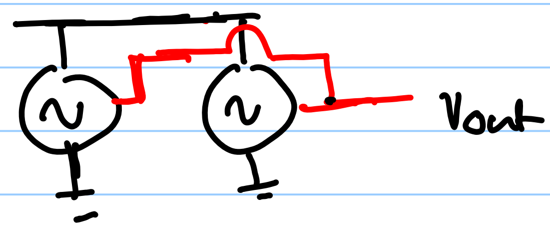
$$\mathcal{L}(f) = \sigma_\tau^2 \frac{f_0^3}{f^2}$$

$$\mathcal{L}(f) = \frac{2kT}{I}\left(\frac{1}{V_{dd}-V_t}(\gamma_N+\gamma_P) + \frac{1}{VDD}\right)\left(\frac{f_0}{f}\right)^2$$



$$\propto \frac{1}{f^2}$$

$$\propto \frac{1}{VDD \times I} \Rightarrow 10\log_{10}\left(\frac{1}{2}\right)$$



$$\propto f_0^2$$