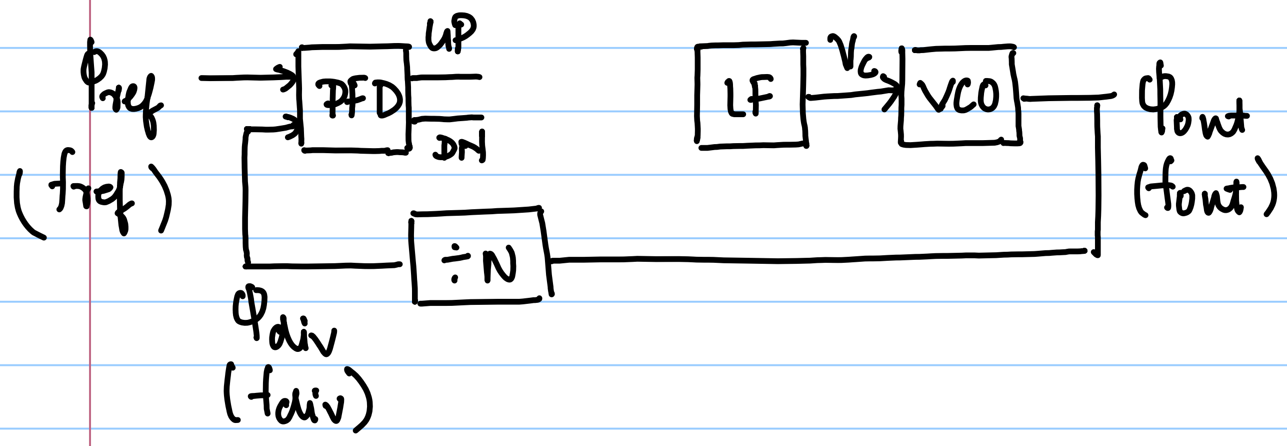
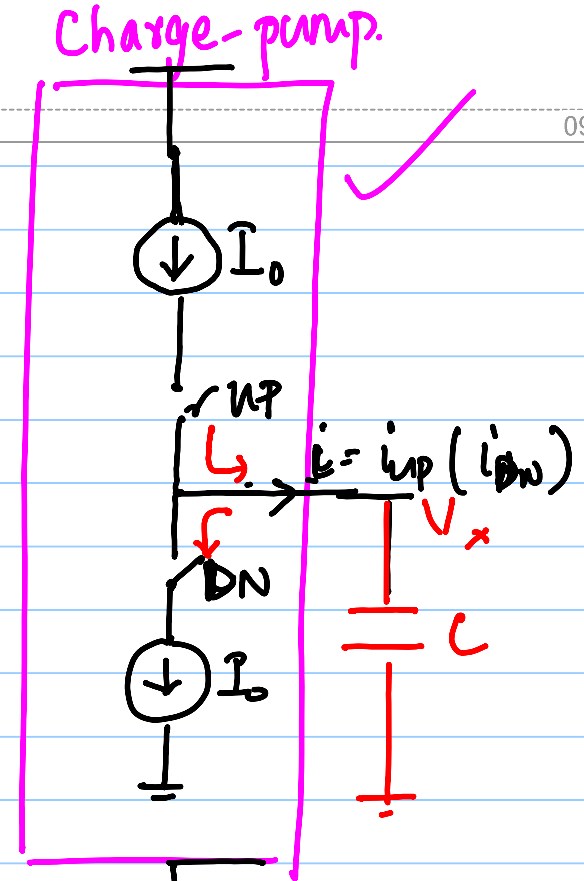
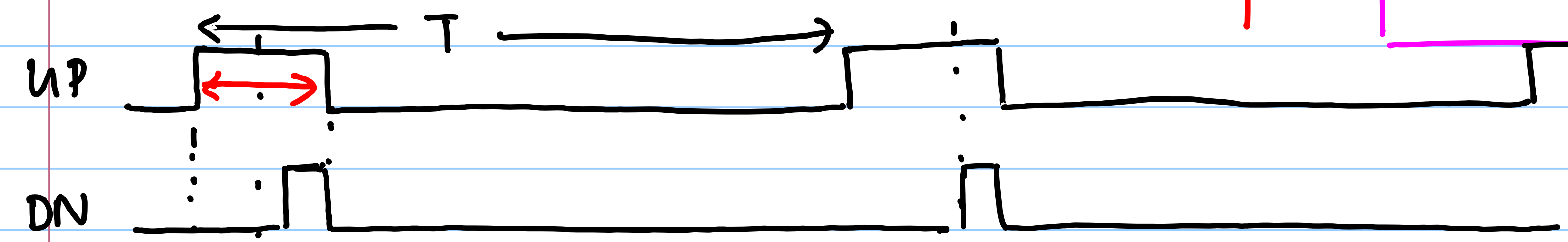
Note Title — 09-10-2020

Lecture #16

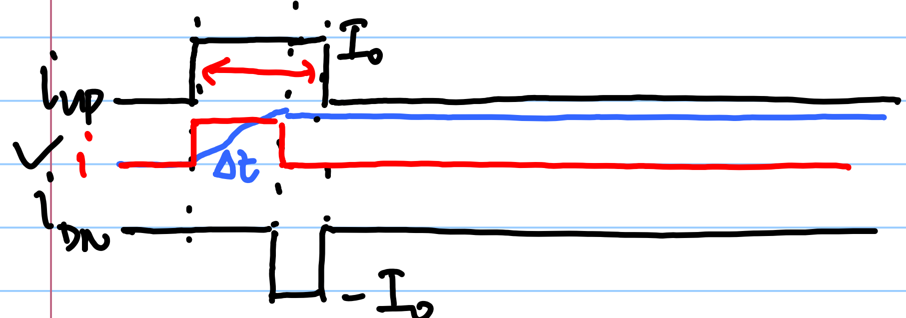




$$\overline{V_{PD}} = \overline{UP - DN} = \frac{\phi_{er}}{2\pi}$$



$$\Delta t := \frac{T}{2\pi} \cdot \phi_{er}$$



$$V_x(nT) = V_x(\overline{n-1}\,T) + \left(\int i \cdot dt\right)\frac{1}{C}$$

$$= V_x(\overline{n-1}\,T) + \frac{1}{C} I_0 \cdot \Delta t$$

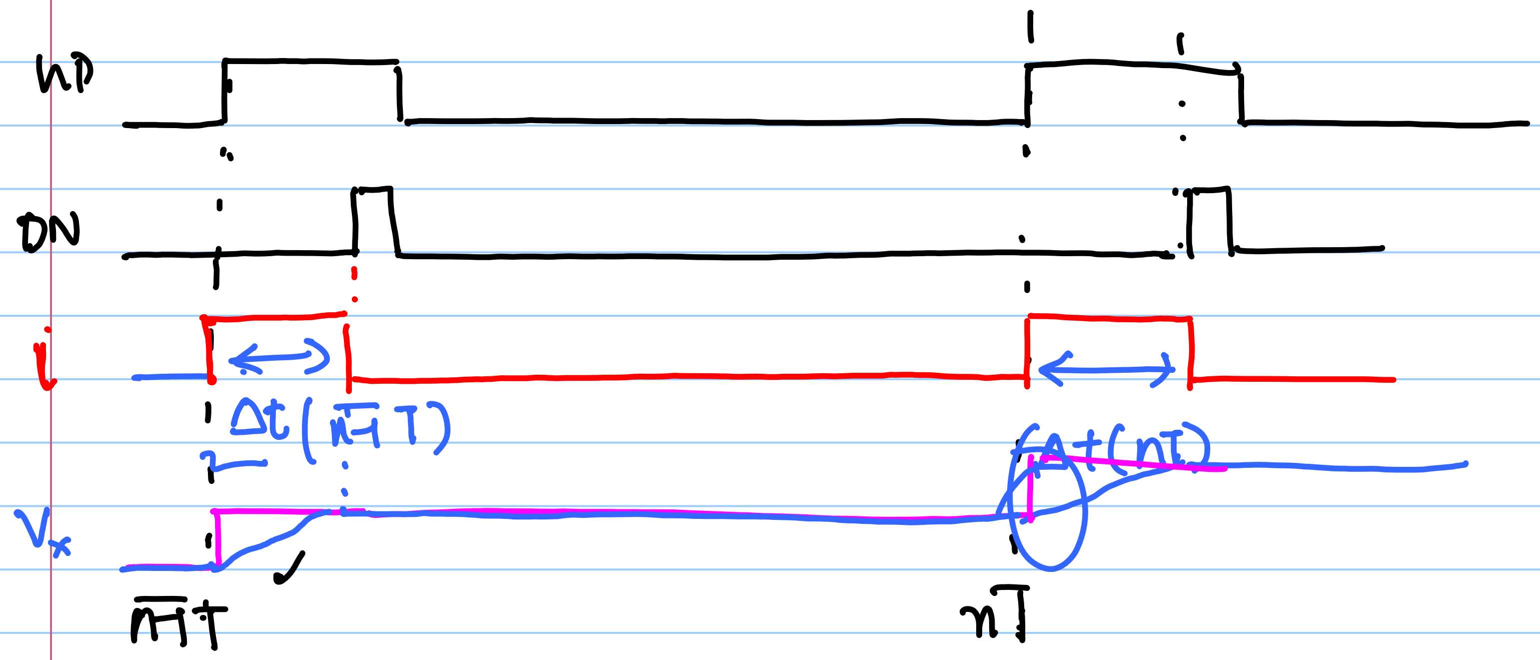
$$V_x(nT) = \underline{V_x(\overline{n-1}\,T)} + \frac{I_0 \cdot \Delta t}{C}$$

$$\Delta V = \frac{I_0 \cdot \Delta t}{C} = \frac{\Delta q}{C}$$

$$x[n+1] = x[n] + \alpha \cdot \underbrace{u[n]}_{nT}$$

$$V_x(nT) = V_x(\overline{n-1}\,T) + \frac{I_0}{C}\,\Delta t(\overline{n-1}\,T)$$

$$V_x(z) = z^{-1} V_x(z) + \frac{I_0}{C}\, z^{-1}\, \Delta t(z)$$



$$V_x(z)\,(1 - z^{-1}) = \frac{I_0}{C}\,\Delta t(z) \cdot z^{-1}$$

$$z = e^{sT}$$

$$V_x(z) = \frac{I_0}{C}\,\Delta t(z)\,\frac{1}{z-1}$$

$$V_x(s) = \frac{I_0}{C}\,\Delta t(s)\,\frac{1}{e^{sT}-1}$$

$$V_x(s) = \frac{I_0}{C}\,\frac{\Delta t(s)}{1+sT-1}$$

$$= \frac{I_0}{C}\,\frac{\Delta t(s)}{sT}$$

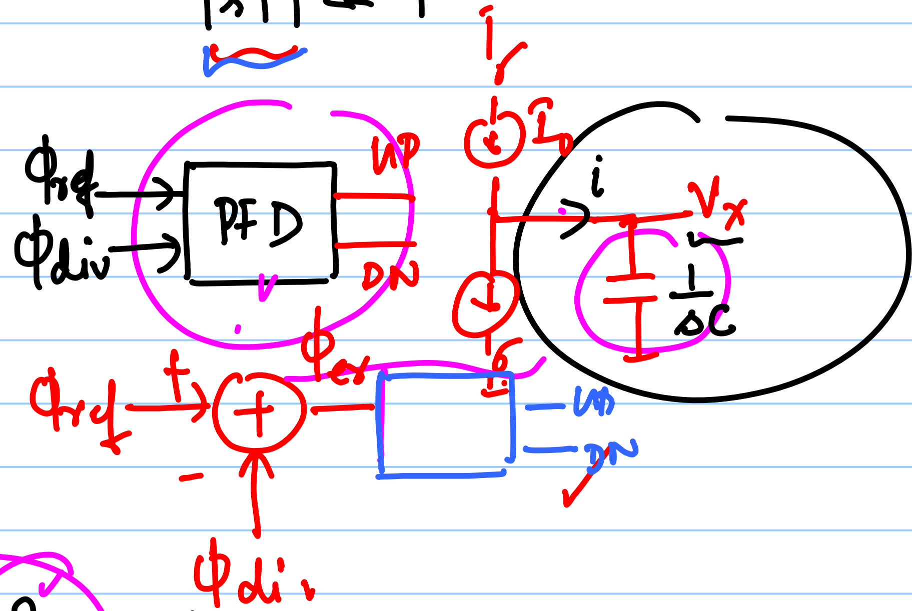
$$= \frac{I_0}{2\pi s C}\cdot\underbrace{\frac{2\pi\cdot\Delta t(s)}{T}}$$

$$e^{sT} = 1 + sT + \frac{(sT)^2}{2!} + \cdots$$

$$\approx 1 + sT$$

$$|sT| \ll 1$$

$$\boxed{V_x(s) = \frac{I_0}{2\pi}\,\frac{1}{sC}\,\phi_{er}(s)}$$

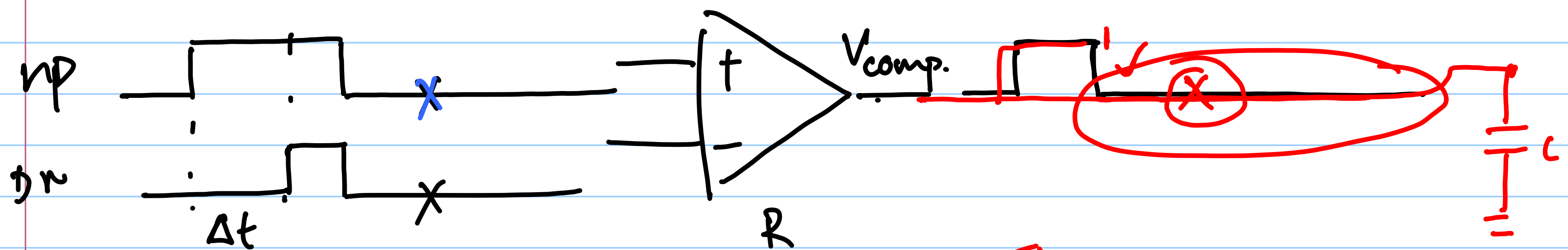


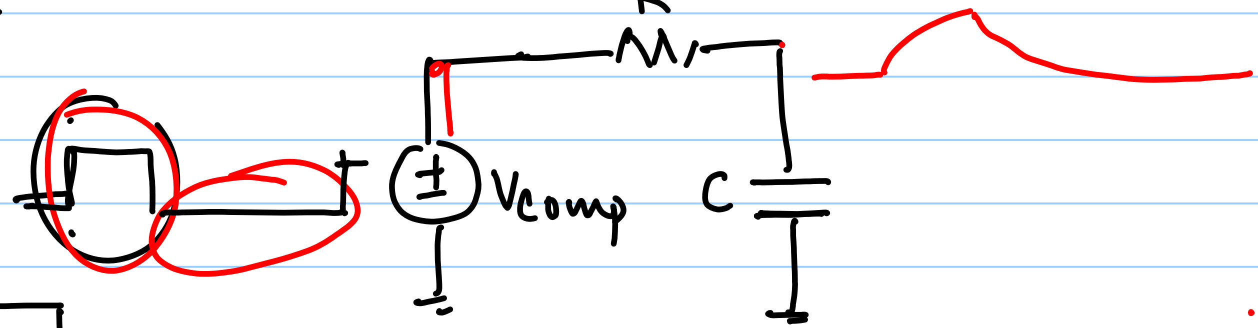
$$V_x(s) = i(s)\,\frac{1}{sC}$$

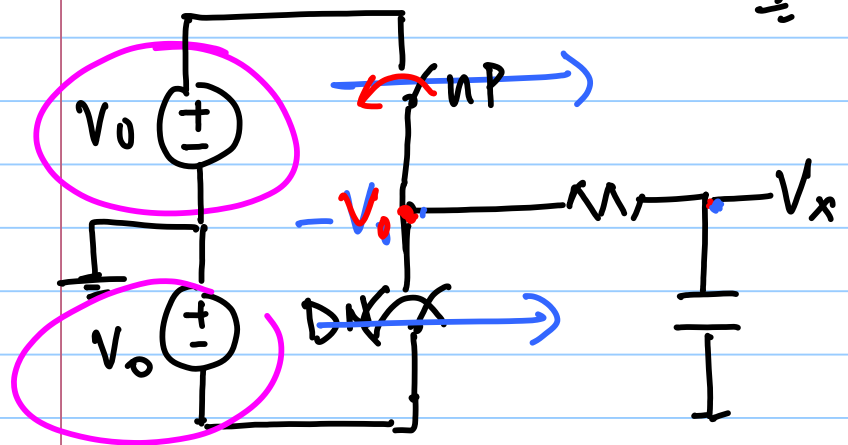
$$= \frac{I_0}{2\pi}\,\phi_{er}\,\frac{1}{sC} = \phi_{er}\,\frac{I_0}{2\pi}\,\frac{1}{sC}$$

$$\frac{V_x(s)}{\phi_{e_T}(s)} = \frac{I_0}{2\pi}\frac{1}{sC}$$

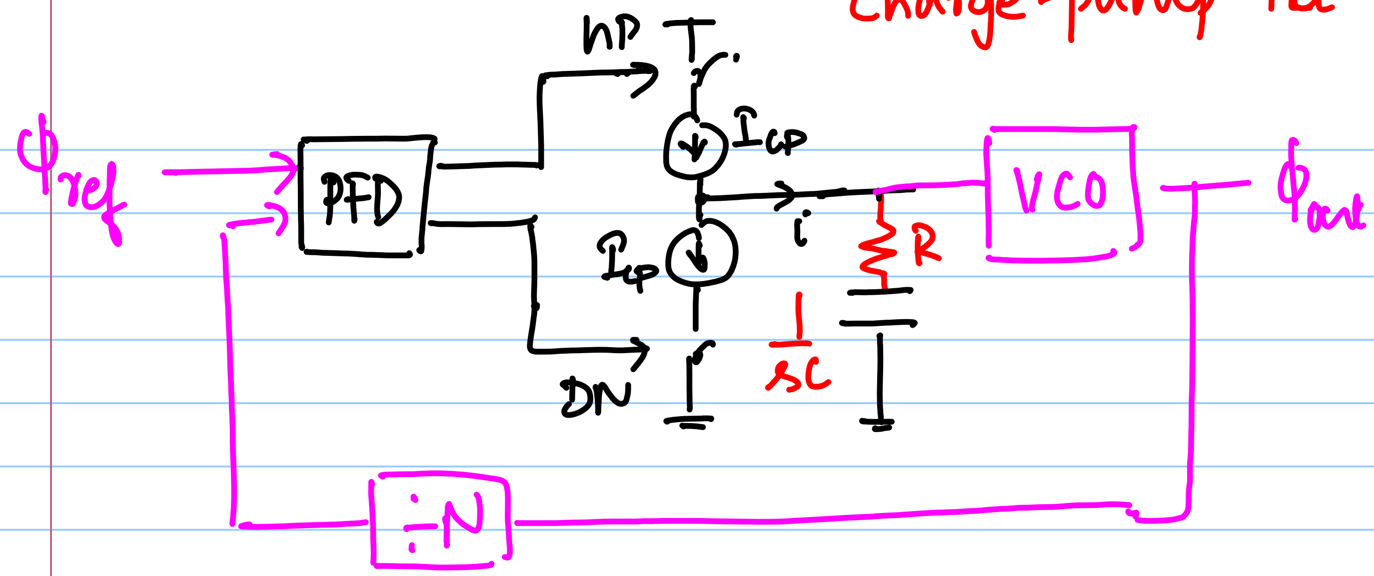
$$= \frac{1}{2\pi} \cdot I_0 \cdot \frac{1}{sC}$$

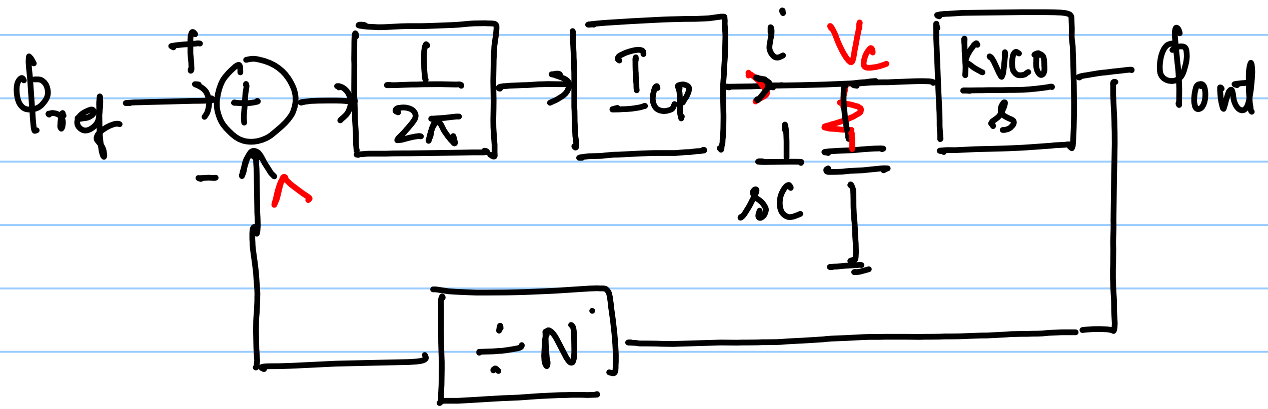


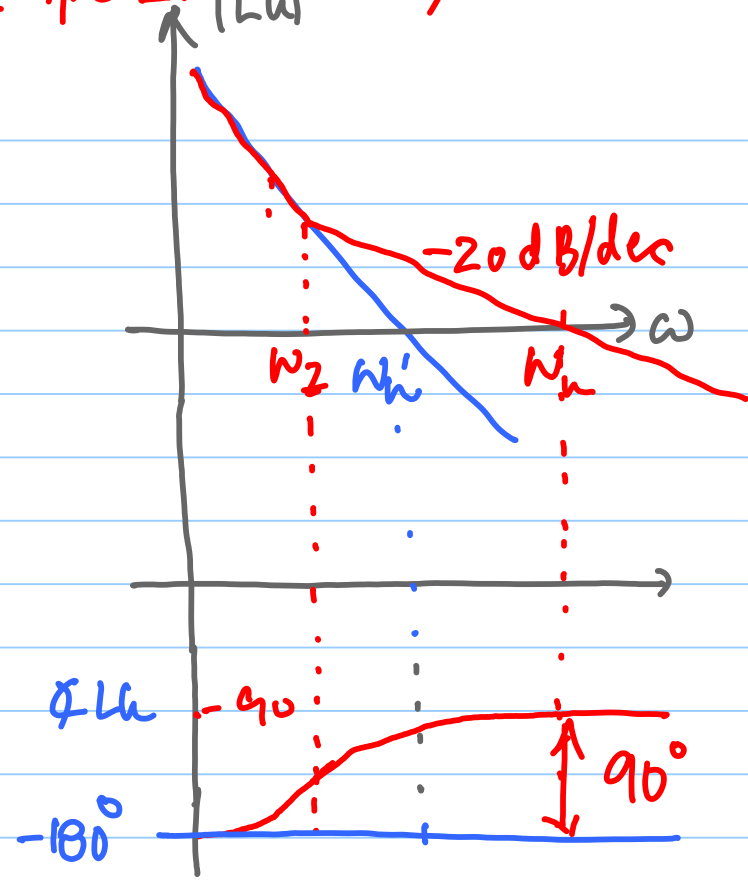




Charge-pump PLL (Type-II, Order 2)







$$LG(s) = \frac{1}{2\pi} \, I_{cp} \, \frac{1}{sC} \, \frac{K_{VCO}}{s} \, \frac{1}{N} \rightarrow \frac{I_{cp} K_{VCO}}{2\pi s N}\left(R + \frac{1}{sC}\right)$$

$$= \frac{I_{cp} K_{VCO}}{2\pi \, s^2 C N}$$

$$\rightarrow \frac{I_{cp} K_{VCO}}{2\pi s^2 N C}(1 + sRC)$$

$\omega_z = 1/RC$