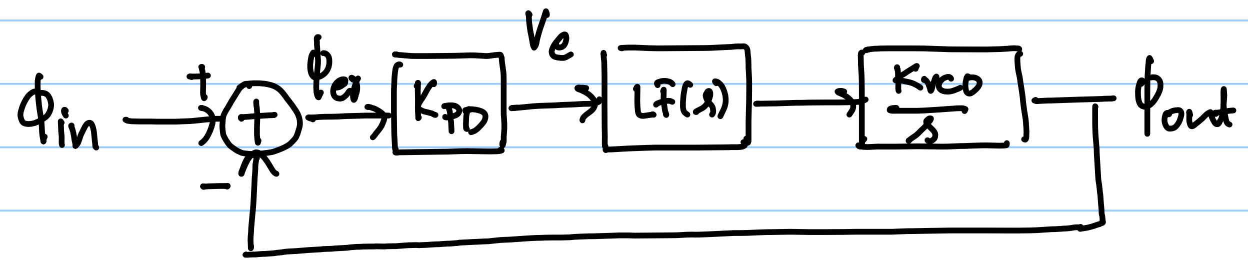
Lecture # 6

Note Title 17-09-2020



$$\Delta V_e = K_{PD}\Delta\phi_{er}$$

$$\Delta\omega_{in} = \Delta\omega(0)\, u(t)$$

$$\Delta\phi_{in} = \int \Delta\omega_{in}\cdot dt = \int_0^t \Delta\omega(0)\cdot dt$$

$$= \Delta\omega(0)\cdot t$$

$$\Delta\phi_{out}(t) = \Delta\phi_{in}(t) \Rightarrow \Delta\phi_{er} = 0$$

$$\frac{\Delta\phi_{out}(s)}{\Delta\phi_{in}(s)} = \frac{LG}{1+LG}$$

$$\frac{\Delta\phi_{er}(s)}{\Delta\phi_{in}(s)} = \frac{1}{1+LG}$$

$$\lim_{t\to\infty}\Delta\phi_{out}(t) = \lim_{s\to 0} s\cdot\frac{\Delta\omega(0)}{s^2}\,\frac{1}{1+\frac{s}{K_{PD}LF(s)K_{VCO}}}$$

$$\lim_{t\to\infty}\Delta\phi_{er}(t) = \lim_{s\to 0} \cancel{s}\cdot\frac{\Delta\omega(0)}{\cancel{s^2}}\,\frac{1}{\frac{\cancel{s}+K_{PD}LF(s)K_{VCO}}{\cancel{s}}}$$

$$= \frac{\Delta\omega(0)}{K_{PD}LF(0)\,K_{VCO}}$$

$$\Delta\omega_{in}(t) = \Delta\omega'(0)\cdot t\, u(t)$$

$$\lim_{t\to\infty}\Delta\phi_{er} = \lim_{s\to 0}\cancel{s}\cdot\frac{\Delta\omega'(0)}{s^{\cancel{2}}}\,\frac{1}{\frac{\cancel{s^2}+\cancel{s}K_{PD}LF(s)K_{VCO}}{\cancel{s}}}$$

$$= \lim_{s\to 0}\frac{\Delta\omega'(0)}{0+s\,K_{PD}LF(0)K_{VCO}}$$

$$LF(s) = \frac{A_{DC}}{1 + s/\omega_p} \Rightarrow$$

$$\Delta\phi_{in}(t) = \Delta\phi_{in}(0)\cdot u(t) \longrightarrow \lim_{t\to\infty} \Delta\phi_{er}(t) = \frac{\Delta\phi_{in}(0)}{1 + \frac{K_{PD}\,LF(0)\,K_{VCO}}{0}}$$

$$\Delta\omega_{in}(t) = \Delta\omega_{in}(0)\,u(t) \longrightarrow \lim_{t\to\infty} \Delta\phi_{er}(t) = \frac{\Delta\omega_{in}(0)}{0 + K_{PD}\,LF(0)\,K_{VCO}}$$

Loop filter, $LF(s) = \frac{N(s)}{D(s)}$

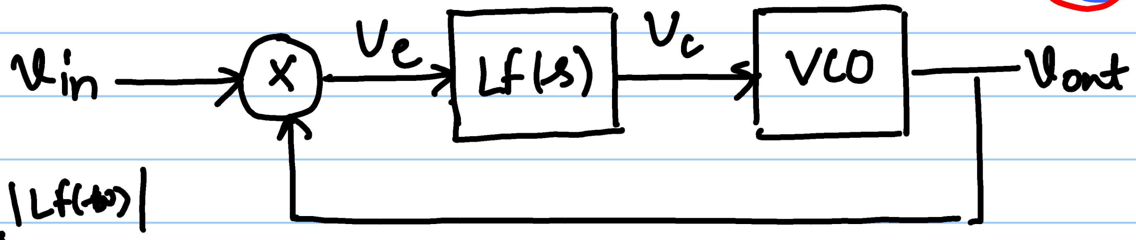
$$LG(s) = K_{PD}\,LF(s)\,\frac{K_{VCO}}{s}$$

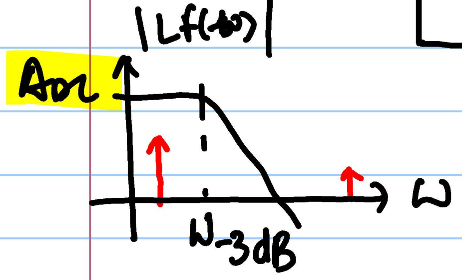
Type of PLL: # of integrators. Integrator has tf = $1/s$

Order of PLL: # of poles in PLL loop.

$LF(s)$	1	$\frac{A_{DC}}{1+s/\omega_p}$	$\frac{(1+s/\omega_z)}{s\,(1+s/\omega_{p2})}$	$\frac{(1+s/\omega_{z1})(1+s/\omega_{z2})}{s^2}$
Type	1	1	2 / 2	3
Order	1	2	2 / 3	3

Type-II, Order-3





at t=0

$V_{in} = \sin(\omega_{in} t)$, $\omega_{in} = 1$ Grad/s

$V_{out} = \cos(\omega_0 t)$

$\omega_0 = 0.99$ Grad/s $+ K_{vco} \cdot V_c$

at $t=0$, $V_c = 0$

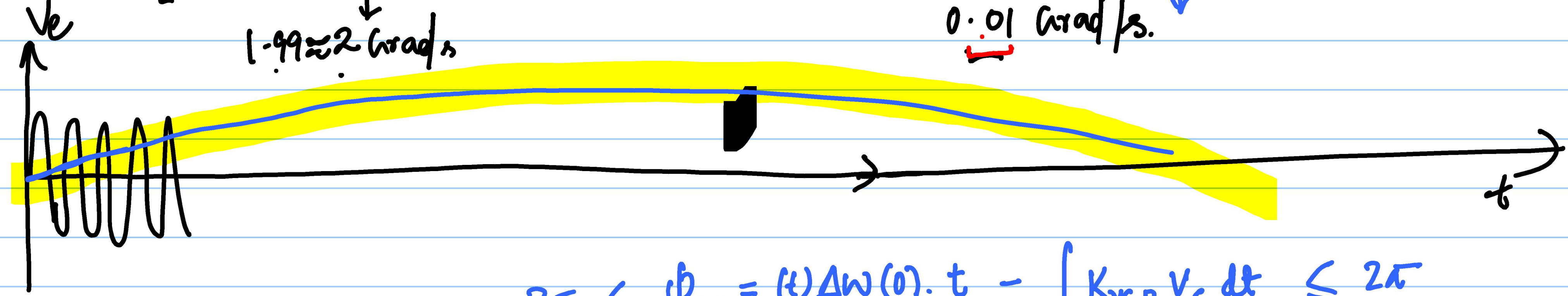
$V_e = V_{in} \times V_{out}$

$$= \frac{1}{2}\left[\sin\left(\overline{\omega_{in} + \omega_{free}}\, t + \int_0^t K_{vco} \cdot V_c \cdot dt\right) + \sin\left(\overline{\omega_{in} - \omega_{free}}\, t - \int K_{vco} \cdot V_c\, dt\right)\right]$$

$$= \frac{1}{2}\left[\sin\left(\overline{\omega_{in} + \omega_{free}}\, t + \int \quad \right) + \sin\left(\Delta\omega(0)\, t - \int K_{vco} \cdot V_c \cdot dt\right)\right]$$

$1.99 \approx 2$ Grad/s

0.01 Grad/s.

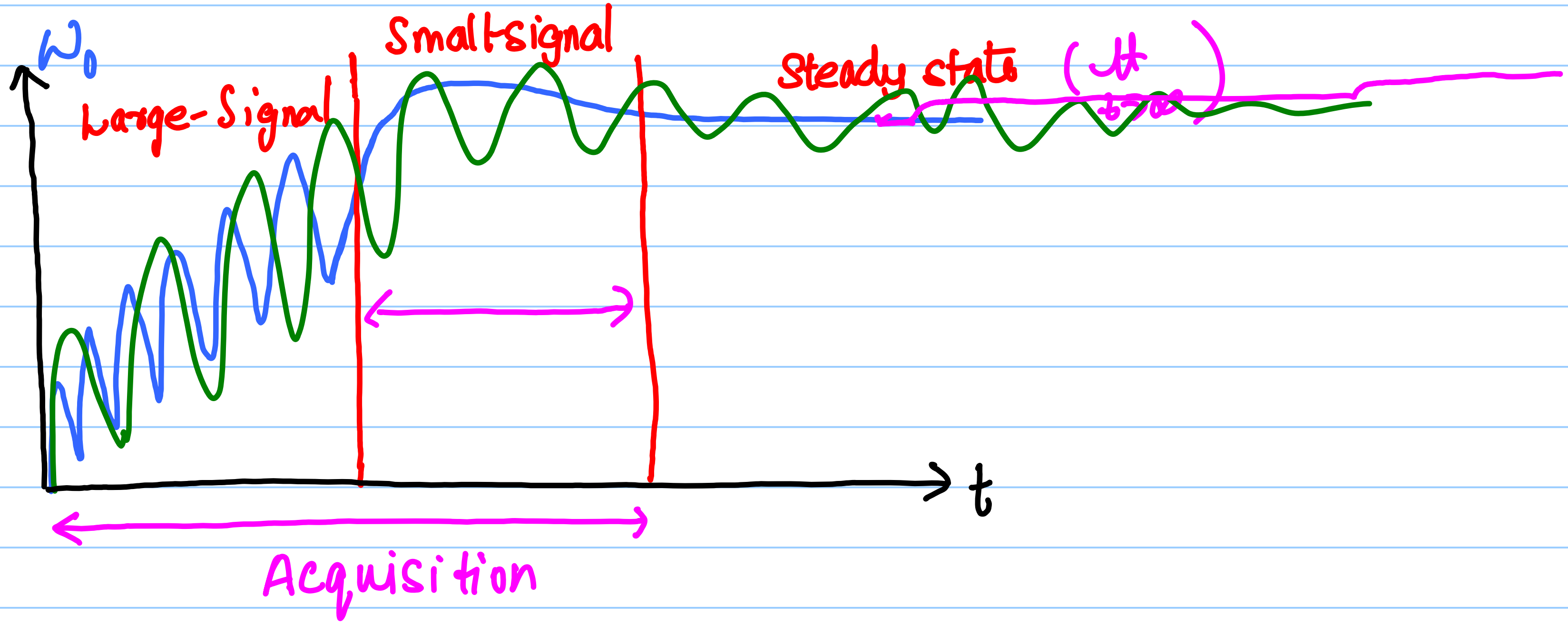


$-2\pi \leq \phi_{er} = (t)\Delta\omega(0) \cdot t - \int K_{vco} V_c\, dt \leq 2\pi$

$$V_c(t) \approx \frac{1}{2} \sin\left(\Delta\omega(0)t - \int K_{vco} \cdot V_c \, dt \right) = \frac{1}{2} \sin\left(\phi_{er}(t) \right) \checkmark$$

$$\omega_0 = \omega_{free} + K_{vco} \cdot V_c = \omega_{free} + K_{vco} \times \frac{1}{2} \sin(\phi_{er})$$

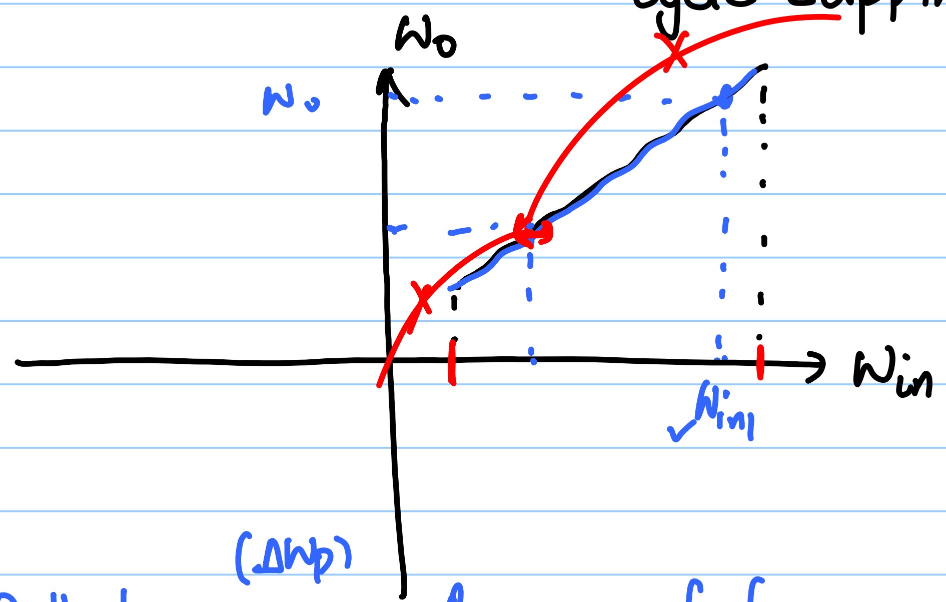
$$V_{out} = \sin\left(\omega_{free} \cdot t + \int K_{vco} \times \frac{1}{2} \sin(\phi_{er}) \, dt \right)$$



Acquisition Ranges.

Hold-in range ($\Delta\omega_H$) : The range of frequencies the PLL can stay in lock.

Lock-in range ($\Delta\omega_L$) : The range of frequencies the PLL locks without cycle slipping (without phase error exceeding 2π)



Pull-in range ($\Delta\omega_P$): Range of frequencies the PLL locks with or without cycle slipping.

$$\Delta\omega_H \geqslant \Delta\omega_P \geqslant \Delta\omega_L$$