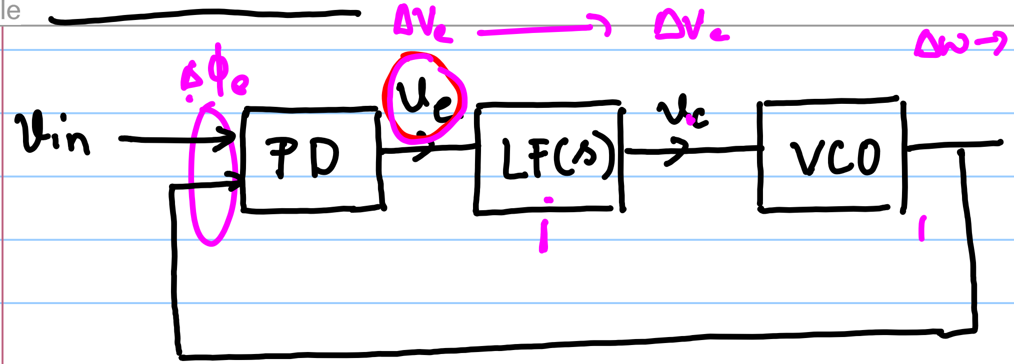
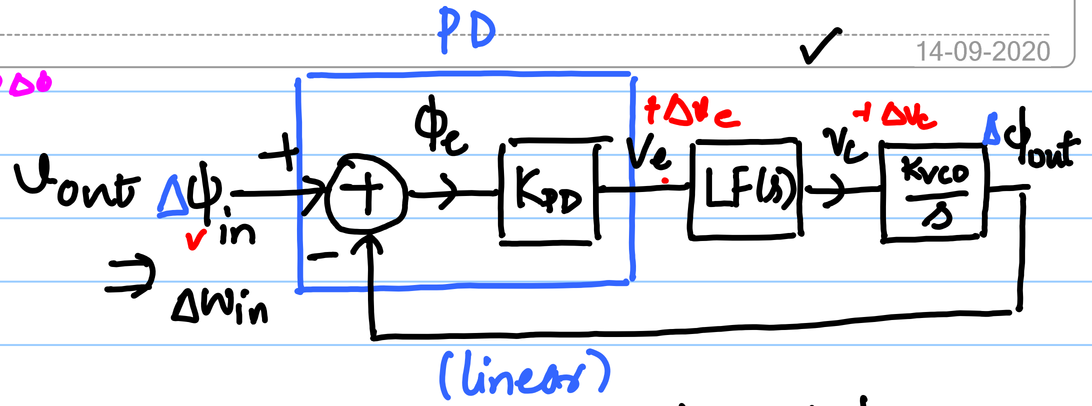
Note Title 14-09-2020

Lecture # 04



Block Diagram of PLL



Small-signal model

$v_{in} = a_{in} \sin(\omega_{in} t)$

$v_{out} = a_{out} \cos(\omega_o t)$

✓ $\phi_{in}(t) = \int \omega_{in}\, dt$

$\phi_o(t) = \int \omega_o\, dt$

$\phi_e(t) = \phi_{in}(t) - \phi_o(t)$

$\omega_o(t) = \omega_{free} + K_{VCO} v_c(t)$; $[K_{VCO}] = \frac{rad/s}{V}$

$\frac{V_c(s)}{V_e(s)} = LF(s)$ ✓

$V_e \propto \phi_e$

- PLL is locked

$\omega_o(t) = \omega_{in}(t)$ ✓

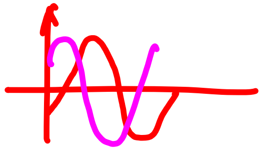
$\frac{d\phi_e(t)}{dt} = 0$ ✓

- $\frac{\Delta V_e}{\Delta \phi_e} = K_{PD} = \frac{dV_e}{d\phi_e}$ ✓

- $\frac{\Delta \omega_o}{\Delta V_c} = K_{VCO}$

- $\Delta\phi_{out}(s) = \frac{\Delta\omega_o}{s} = \frac{K_{VCO}}{s} \cdot \Delta V_c(s)$

$v_{in} = a_{in} \sin(\omega_{in} t + \phi_{in}(0))$



$v_{out} = a_o \quad \cos(\omega_o t + \phi_o(0))$

$v_{in} \times v_{out}$

$= \frac{a_{in} a_o}{2} \cdot 2 \sin(\omega_{in} t + \phi_{in}(0))$
$\cdot \cos(\omega_o t + \phi_o(0))$

$= \frac{a_{in} a_o}{2} \Big[\sin(\overline{\omega_{in} + \omega_o}\, t + \phi_{in}(0) + \phi_o(0))$
$+ \sin(\overline{\omega_{in} - \omega_o}\, t + \phi_{in}(0) - \phi_o(0)) \Big]$

$= \frac{a_{in} a_o}{2} \Big[\sin(\underline{\omega_{in} + \omega_o}\, t + \_ \; .)$
$+ \sin(\Delta\omega . t + \phi_{er}(0)) \Big]$

$\omega_{in} = \omega_o \times 1.001$

$\Rightarrow \omega_{in} + \omega_o = 2.001 \omega_o$ #1 filter

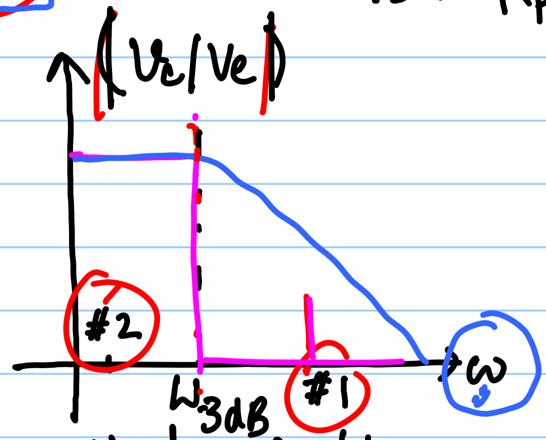
$\omega_{in} - \omega_o = 0.001 \omega_o$ #2 ✓

Gain in forward path

$K_{PD} \, LF(s) \, \frac{K_{VCO}}{s} = LG(s)$

$\frac{\Delta\phi_o}{\Delta\phi_{in}} = \frac{LG(s)}{1 + 1 \cdot LG(s)}$

$\frac{\Delta\phi_o}{\Delta\phi_{in}} = H(s) = \frac{K_{PD} \, LF(s) \, K_{VCO}}{s + K_{PD} \, LF(s) \, K_{VCO}}$



Magnitude of phase-transfer fn w.r.t freq.

Phase of phase-transfer fn. w.r.t freq.

$$V_e = \frac{a_{in}a_o}{2}\left[\sin(\overline{\omega_{in}+\omega_o}\,t + \phi_{in}(0)+\phi_o(0)) + \sin(\Delta\omega(t) + \phi_{er}(0))\right]$$

$$V_c = \frac{a_{in}a_o}{2}\sin(\Delta\omega(t) + \phi_{er}(0)) \quad V = \frac{a_{in}a_o}{2}\sin\left(\Delta\omega(0)\cdot t - \int K_{VCO} V_c(t)\,dt - \phi_{er}(0)\right)$$

at $t=0$

$$= \frac{a_{in}a_o}{2}\sin(\Delta\omega(0)\cdot t - K_{VCO} V_{c,st}\cdot t + \phi_{er,st1} + \phi_{er}(0)$$

$$\phi_{er,st} = \phi_{er,st1} + \phi_{er}(0)$$

$$\omega_{in}(t) = \omega_{in}(0)$$

$$\omega_o(t) = \omega_{free} + K_{vco}\cdot V_c(t)$$

$$\omega_{in}(t) - \omega_o(t) = \omega_{in}(0) - \omega_{free} - K_{vco}\cdot V_c(t)$$

$$= \Delta\omega(0) - K_{vco}\cdot V_c(t)$$

#1: $\Delta\omega(0) = 0$

$$\Delta\omega(t) = \underbrace{\Delta\omega(0)}_{=0} - K_{vco}\cdot V_c(t)$$

$$0 = 0 - K_{vco}\cdot V_c \Rightarrow V_c = 0$$

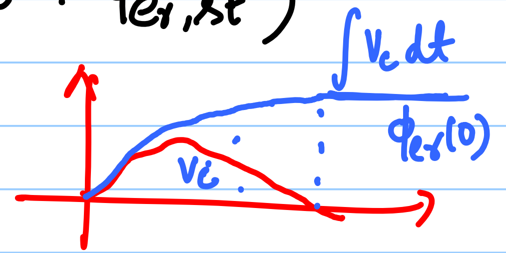
#2: $\Delta\omega(0) \neq 0$ (In steady state)

$$\omega_{in} = \omega_o \Rightarrow \Delta\omega(0) = K_{vco}\cdot V_{c,st} = K_{vco}\left(\lim_{t\to\infty} V_c\right)$$

$$V_{c,st} = \frac{\Delta\omega(0)}{K_{VCO}} \qquad \Delta\omega(0) = K_{VCO} \cdot V_{c,st}$$

$$V_{c,st} = \frac{a_{in} a_0}{2} \sin\left(\left(\Delta\omega(0) - K_{VCO} \cdot V_{c,st}\right) \cdot t + \phi_{er,st} \right)$$

$$= \frac{a_{in} a_0}{2} \sin(\phi_{er,st})$$



$$\phi_{er,st} = \sin^{-1}\left(\frac{2V_{c,st}}{a_{in} a_0} \right)$$

#1

$$V_c(t) = \frac{a_{in} a_0}{2} \sin\left(\phi_{er}(0) + \underbrace{\underset{=0}{\Delta\omega(0)} - \int_0^t K_{VCO} V_c(t) \cdot dt}_{=0} \right)$$

$$(\quad 0 \quad) \qquad \phi_{er}(0)$$

$$\sin(\phi_{er,st}) = \left. \frac{2V_{c,st}}{a_{in} a_0} \right|_{a_{in}, a_0 = 1} = 2V_{c,st}$$

$$\Rightarrow \quad -1 \le \sin(\phi_{er,st}) \le +1$$

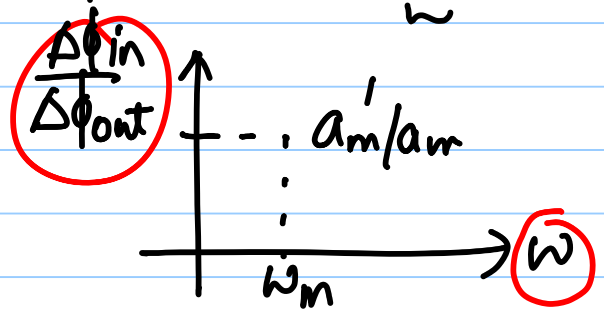
$$\frac{-1}{2} \le \frac{V_{c,st}}{V} \le \frac{1}{2}$$

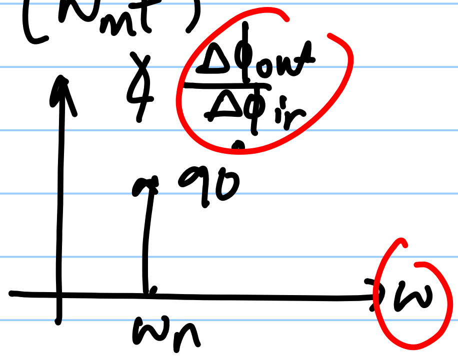
$$-\frac{1}{2} \le \frac{\Delta\dot{\omega}(0)}{K_{vco}} \le \frac{1}{2}$$

$$\frac{-K_{vco}}{2} \le \underbrace{\Delta\omega(0)} \le \frac{K_{vco}}{2}$$

$$\Delta\phi_{in} = a_m \sin(\omega_m t)$$

$$\Delta\phi_{out} = \underset{\sim}{a_m'} \cos(\omega_m t)$$





$$\left|\frac{a_m'}{a_m}\right|$$