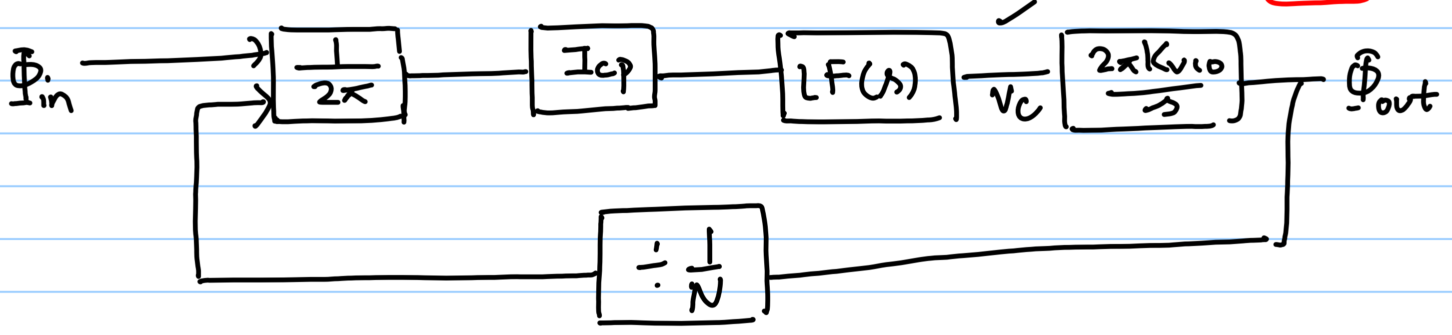
Note Title 04-09-2018

Lecture #16



$\Delta f_{out} = K_{vco} \Delta V_c$

Application

1. Noise → ω_u, Power
2. Stability → Φ_m

ω_u, Φ_m

$$Li(s) = \frac{1}{2\pi} \, I_{cp} \, \frac{(1+s/\omega_z)}{(1+s/\omega_{p3})} \, \frac{1}{s(C_1+C_2)} \, \frac{2\pi K_{vco}}{s} \, \frac{1}{N}$$

I_{cp}

R

C_1

C_2

K_{vco}: ✓

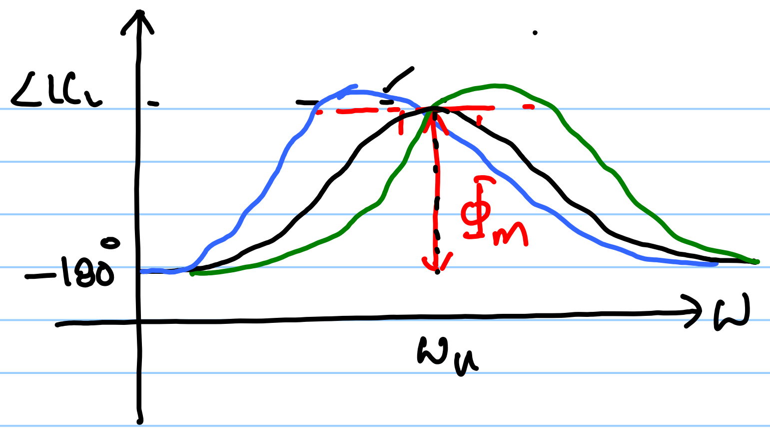
$$\omega_z = \frac{1}{RC_1} ✓$$

$$\omega_{p3} = \frac{1}{\frac{RC_1C_2}{C_1+C_2}}$$

$$\frac{\omega_z}{\omega_{p3}} = \left[\frac{C_1+C_2}{C_2}\right]^{-1} = \left[1+\frac{C_1}{C_2}\right]^{-1}$$

$$= \frac{1/\cancel{R}\cancel{C_1}}{1/\cancel{R}\cancel{C_1}C_2/C_1+C_2} = \frac{C_2}{C_1+C_2}$$

$$\frac{\omega_{p3}}{\omega_z} = 1+\frac{C_1}{C_2}$$



$$\angle LG = -180^\circ + \tan^{-1}\left(\frac{\omega_u}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p3}}\right)$$

$$\Phi_m = \tan^{-1}\left(\frac{\omega_u}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p3}}\right)$$

$$\left.\frac{d\Phi_m}{d\omega}\right|_{\omega=\omega_u} = 0 \Rightarrow \left.\frac{1}{1+\left(\frac{\omega}{\omega_z}\right)^2} \times \frac{1}{\omega_z} - \frac{1}{1+\left(\frac{\omega}{\omega_{p3}}\right)^2} \times \frac{1}{\omega_{p3}}\right|_{\omega=\omega_u} = 0$$

$$\frac{\omega_z}{\omega_z^2 + \omega_u^2} - \frac{\omega_{p3}}{\omega_{p3}^2 + \omega_u^2} = 0$$

$$\omega_u = \sqrt{\omega_z . \omega_{p3}}$$

$$\omega_u = \sqrt{\frac{1}{RC_1} \times \frac{1}{\frac{RC_1C_2}{C_1+C_2}}} = \frac{1}{RC_1}\sqrt{\frac{C_1+C_2}{C_2}} = \omega_z \underbrace{\sqrt{\frac{C_1}{C_2}+1}}$$

$$\frac{\omega_u}{\omega_z} = \sqrt{1+\frac{C_1}{C_2}}$$

ω

$$\frac{\omega_u}{\omega_{p3}} = \frac{\omega_z\sqrt{1+C_1/C_2}}{\omega_z\left(1+\frac{C_1}{C_2}\right)} = \frac{1}{\sqrt{1+\frac{C_1}{C_2}}}$$

$$\hat{\Phi}_m = \tan^{-1}\left(\frac{\omega_u}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p3}}\right)$$

$$= \tan^{-1}\left(\sqrt{1+\frac{C_1}{C_2}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{1+\frac{C_1}{C_2}}}\right)$$

$$= \tan^{-1}(x) - \tan^{-1}\left(\frac{1}{x}\right)$$

$$\tan(\hat{\Phi}_m) = \frac{x-\frac{1}{x}}{1+x\times\frac{1}{x}} = \frac{x-\frac{1}{x}}{2}$$

$$\frac{C_1}{C_2} = f(\hat{\Phi}_m)$$

$\checkmark$

✓ $\frac{C_1}{C_2}$

✓ $\omega_z = \omega_u \Big/ \sqrt{1+\frac{C_1}{C_2}} = \frac{1}{RC_1} \xrightarrow{R} C_1 \xrightarrow{\frac{C_1}{C_2}} C_2$

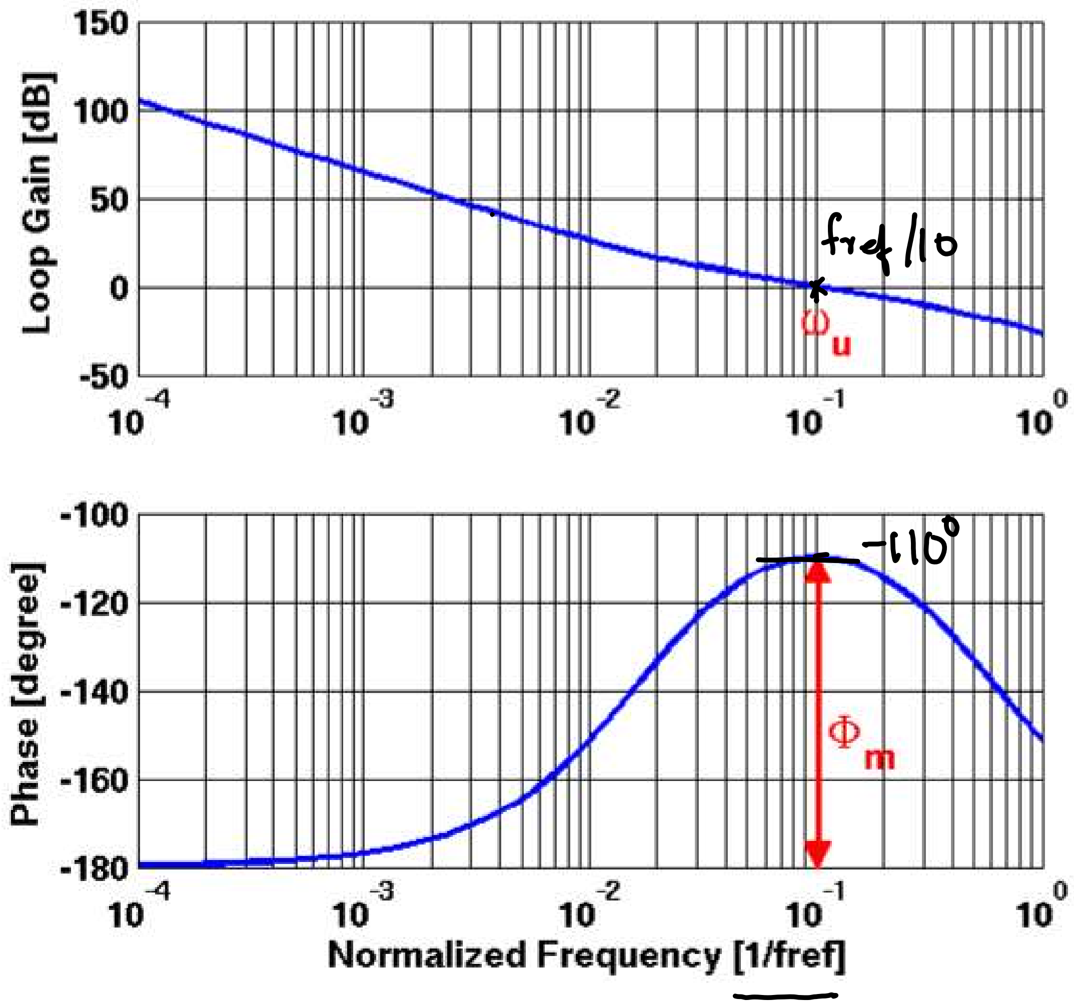
$\uparrow$ Noisy

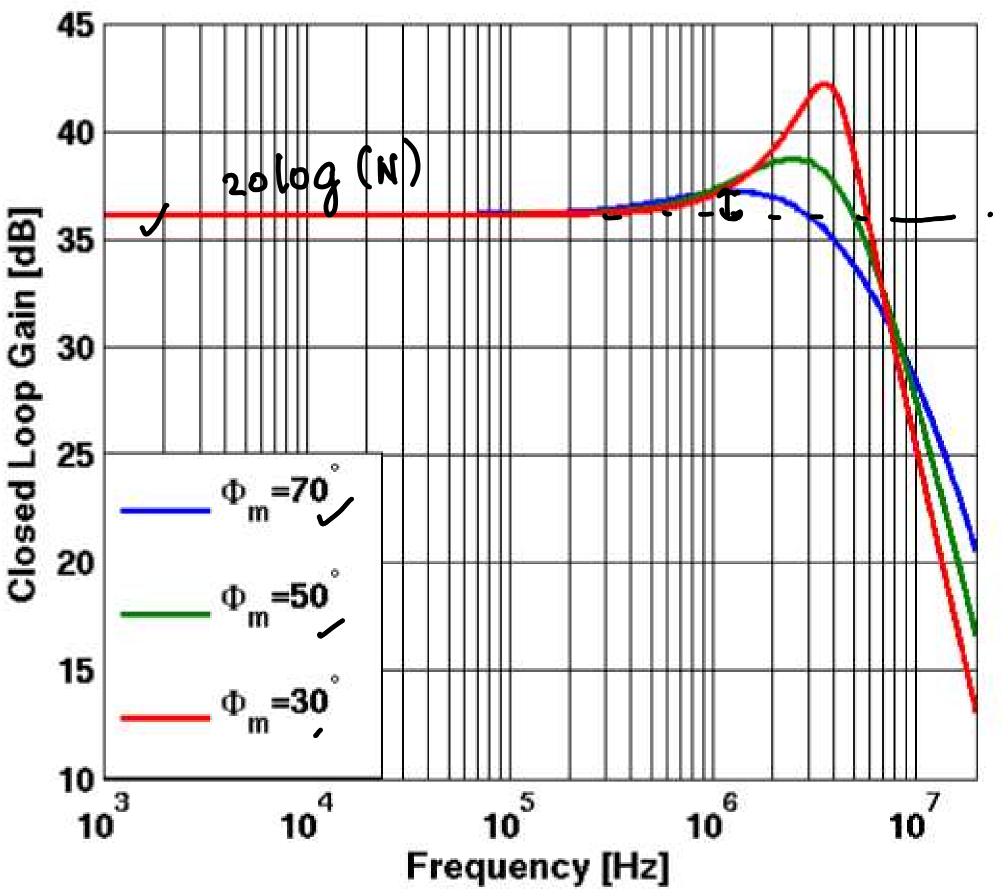
$\omega_u = \sqrt{\omega_z \cdot \omega_{p3}}$

$|LG(\omega_u)| = 1$

$$\Rightarrow \frac{K_{VCO}}{N} \frac{1}{\omega_u^2 (C_1+C_2)} \frac{\sqrt{1+(\omega_u/\omega_z)^2}}{\sqrt{1+(\omega_u/\omega_{p3})^2}} \times I_{cp} = 1$$

$I_{cp} =$


Loop Gain [dB]
150
100
50
0
-50
fref/10
ω_u
Phase [degree]
-100
-120
-140
-160
-180
-110°
Φ_m
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
Normalized Frequency [1/fref]

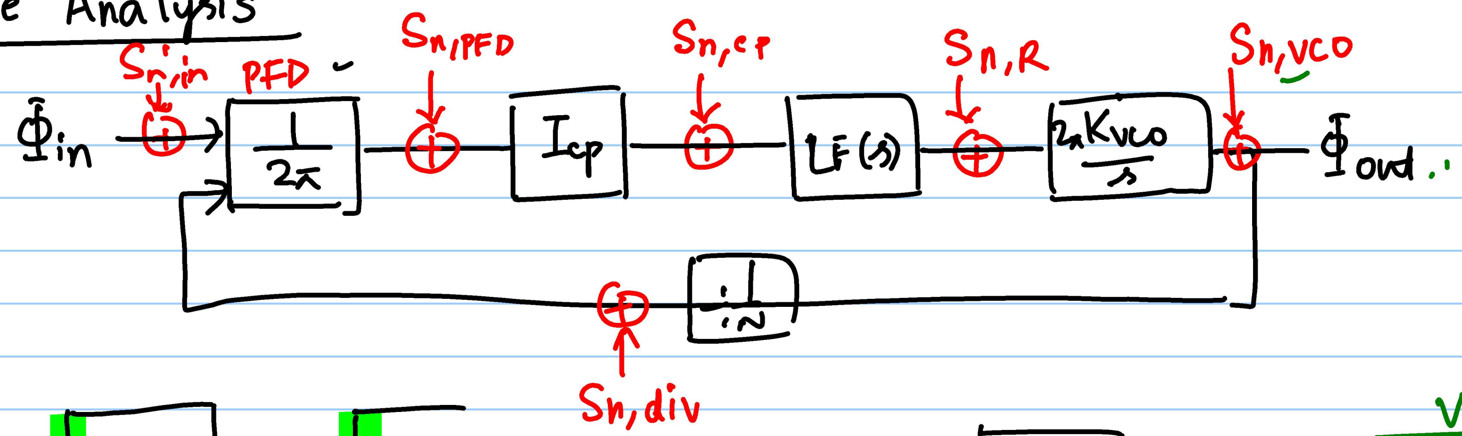


$$\frac{\Phi_{out}}{\Phi_{in}} = \frac{N \cdot LG(s)}{1 + LG(s)}$$

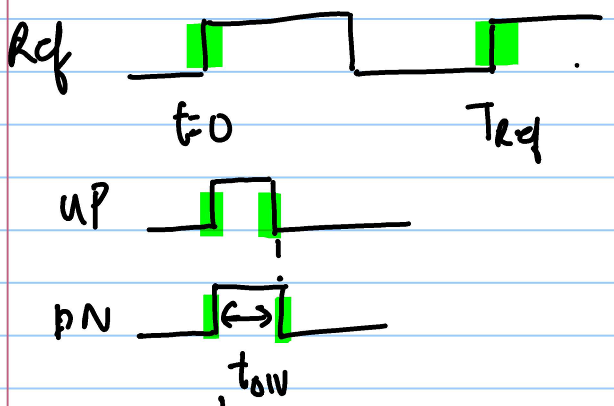
$$\approx N \frac{LG}{LG}$$

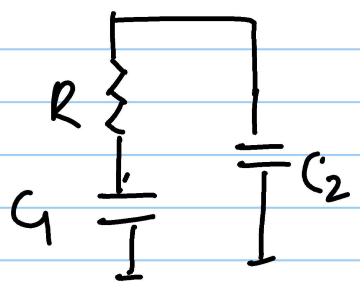
$$\boxed{\omega_u \lesssim \frac{\omega_{ref}}{10}}$$

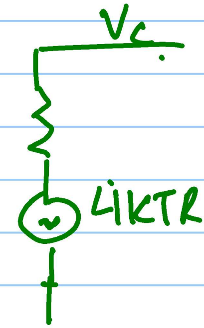
Noise Analysis

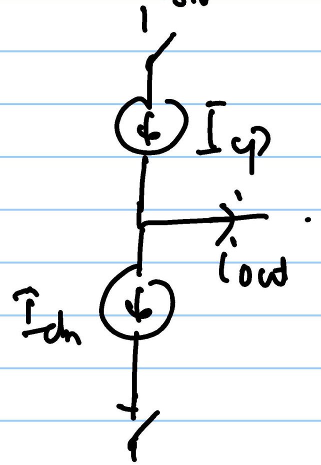


$\hat{\Phi}_{n,out}$



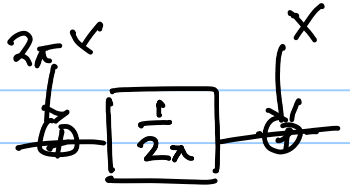






$$i_{out} = i_{n,up} - i_{n,dn}$$

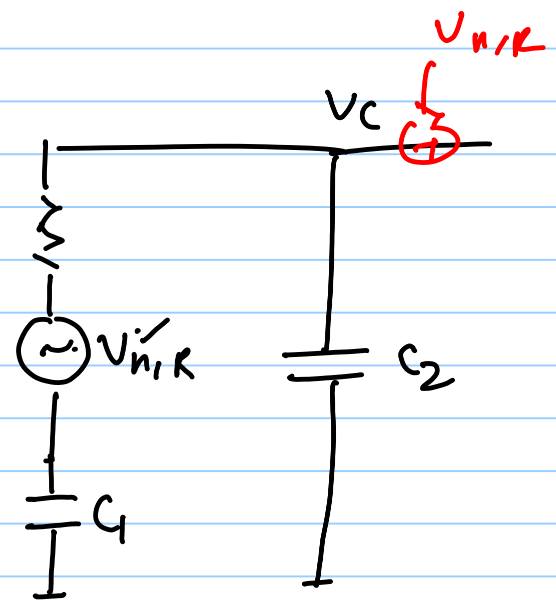
Noise Transfer Functions



$$NTF_{in} = \frac{\Phi_{n,out}}{\Phi_{n,in}} = 1 \times \frac{N \cdot LG}{1+LG},$$

$$NTF_{PFD} = 2\pi \frac{N \cdot LG}{1+LG},$$

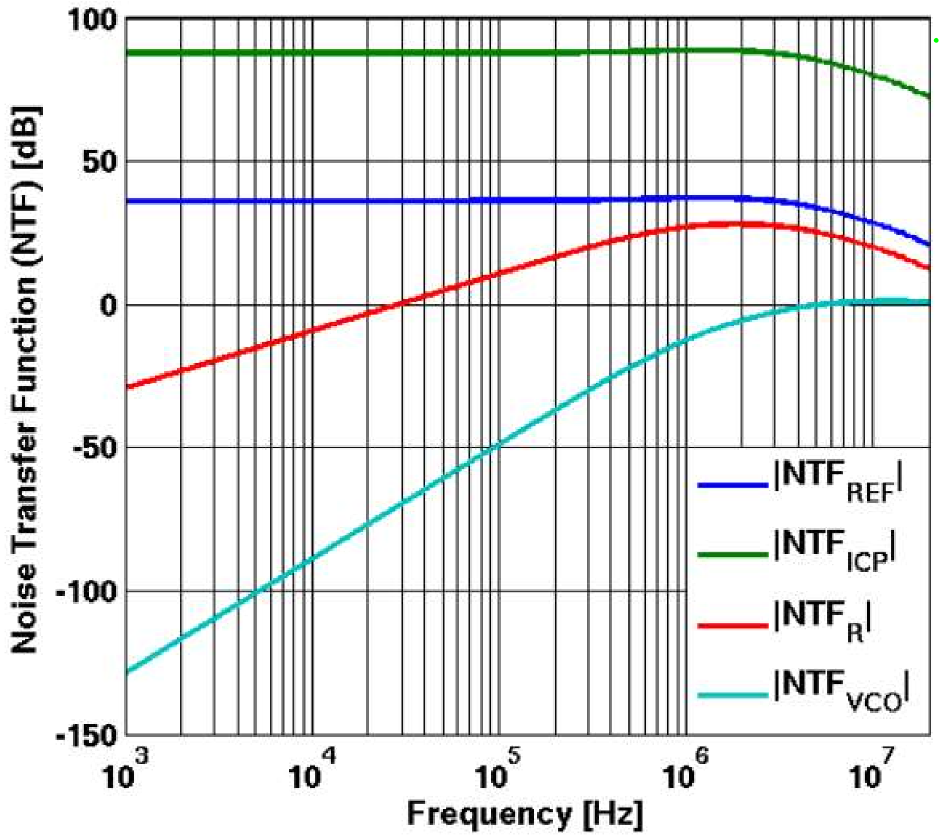
$$NTF_{CP} = \frac{2\pi}{I_{cp}} \frac{N \cdot LG}{1+LG},$$



$$NTF_R = \frac{\Phi_{n,out}}{V_{n,R}} = \frac{V_{n,c}}{V_{n,R}} \times \frac{\Phi_{n,out}}{V_{n,c}},$$

$$= \frac{1/sC_2}{R + 1/sC_2 + 1/sC_1} \times \boxed{\frac{2\pi K_{VCO}/s}{1+LG}} \quad \cdot s^2 \cdot \boxed{\frac{LG}{1+LG}}$$

$$\frac{\Phi_{n,out}}{\Phi_{n,VCO}} = \frac{1}{1+LG}$$

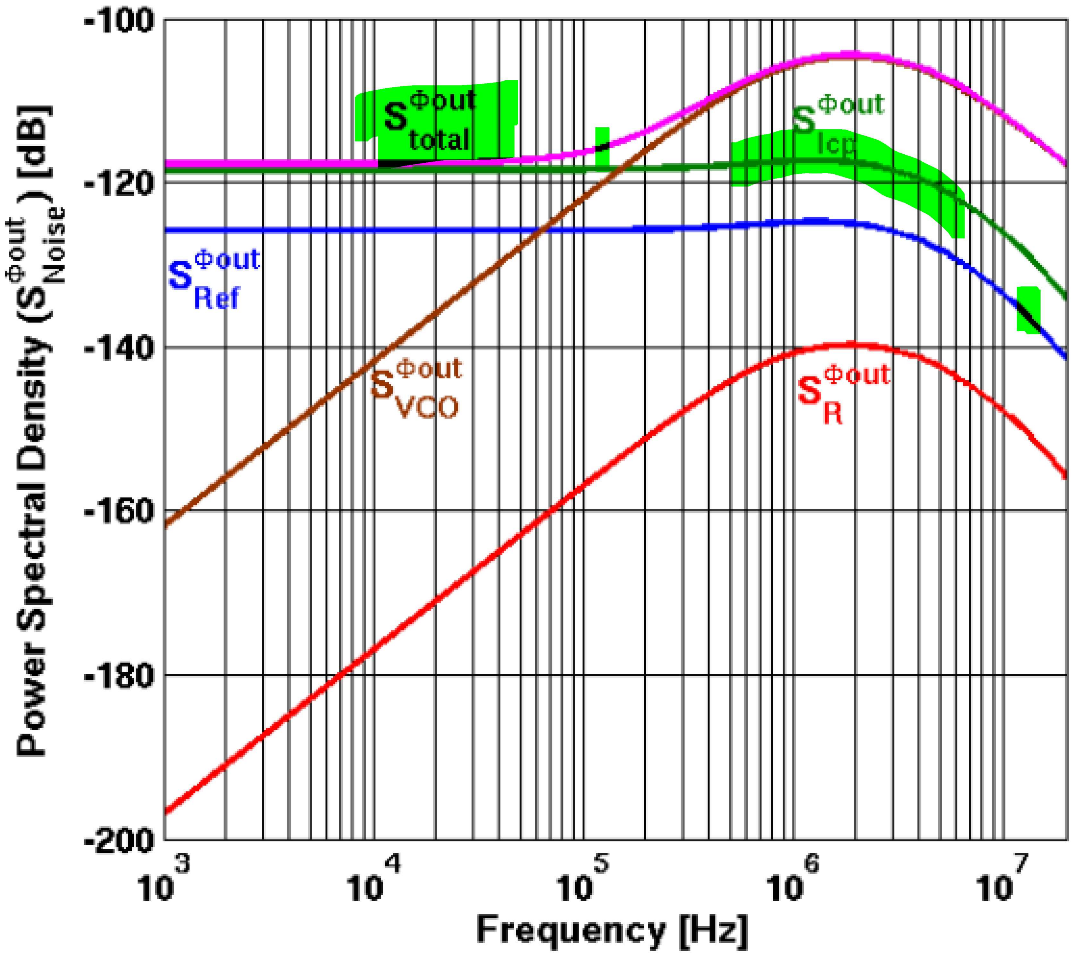


$$S^{out}_{n,ref} = S_{n,ref} \times |NTF_{ref}|^2$$

$$S^{out}_{n,cp} = S_{n,cp} \times |NTF_{cp}|^2$$

$$\vdots$$

$$S^{out}_{n,total} = S^{out}_{n,ref} + S^{out}_{n,cp} + \cdots + S^{out}_{n,vco} + \cdots$$



Power Spectral Density ($S^{\Phi out}_{Noise}$) [dB]
-100
-120
-140
-160
-180
-200
10^3
10^4
10^5
10^6
10^7
Frequency [Hz]
$S^{\Phi out}_{total}$
$S^{\Phi out}_{Icp}$
$S^{\Phi out}_{Ref}$
$S^{\Phi out}_{VCO}$
$S^{\Phi out}_{R}$