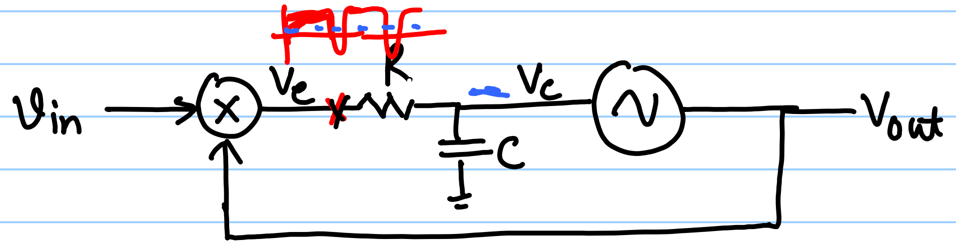
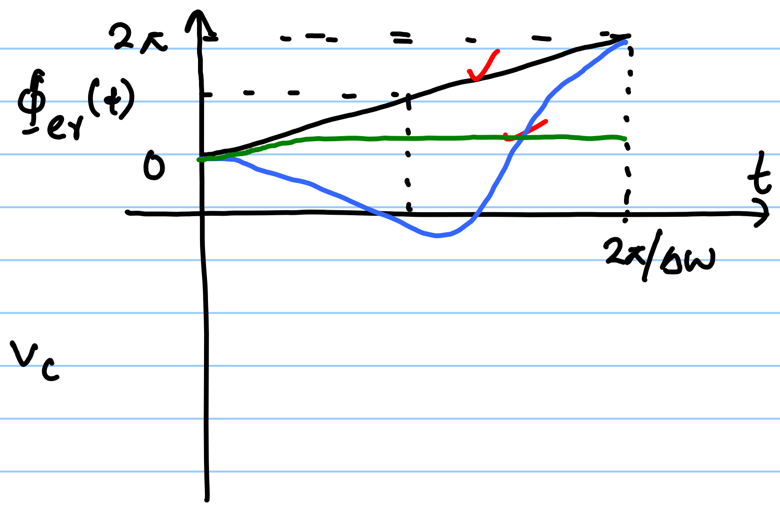
Note Title 06-08-2018

Lecture #4





$v_{in} = \sin(\omega_{in} \cdot t)$

$V_{out} = \cos(\omega_{out} \cdot t)$

$\omega_{out} = \omega_{free} + 2\pi K_{vco} \cdot V_c$

$t = 0, \quad V_c = 0$

$\omega_{out} = \omega_{free}$

$$V_e = \frac{1}{2}\left[\underbrace{\sin(\overline{\omega_{in}+\omega_{out}}\, t)} + \sin(\underbrace{\overline{\omega_{in}-\omega_{out}}}\, t)\right]$$

$$\underline{\Phi_e(t)} = \int (\omega_{in} - \omega_{out})\, dt$$

$$= \int (\omega_{in} - \omega_{free})\, dt - \int 2\pi K_{vco} \cdot V_c \cdot dt$$

$$= \underline{\Delta\omega(0)\, t} - \underbrace{2\pi K_{vco} \int V_c\, dt}_{=0}$$

$$v_e = \frac{1}{2}\sin(\Phi_e(t))$$

$$= \frac{1}{2}\sin\left(\Delta\omega(0).\,t - 2\pi K_{vco}\int v_c . dt\right)$$

$$v_c \approx v_e$$

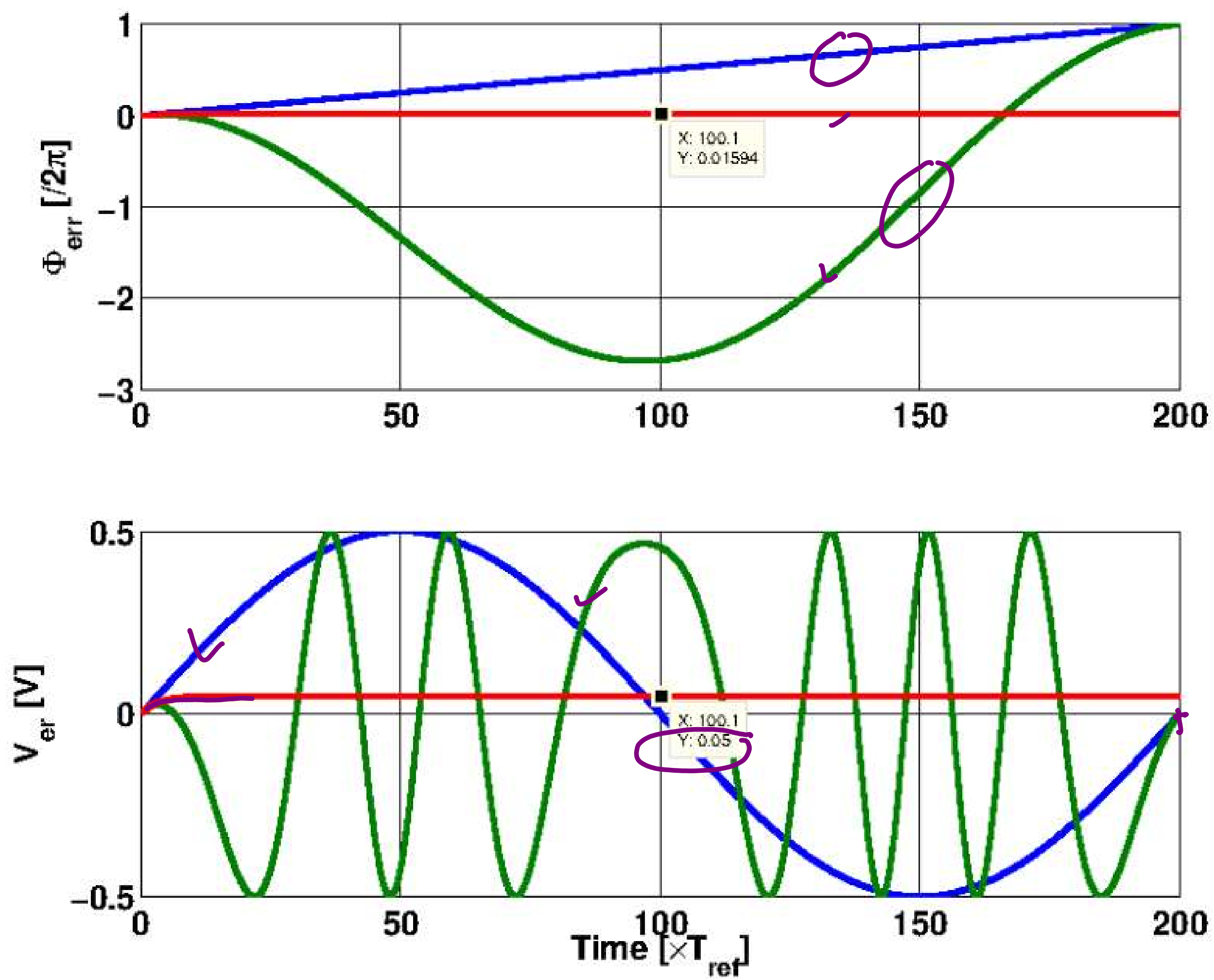
$$= \frac{1}{2}\sin(\Phi_{er}(t)) = \frac{1}{2}\sin(\Delta\omega(0)t)$$

$$\Phi_e(t) = \Delta\omega(0).t - 2\pi K_{vco}.K_{pd}\int_0^t \sin(\underbrace{\Phi_{er}(t)}_{\Delta\omega(0).t})\,dt$$

$$= \underbrace{\Delta\omega(0).\,t}_{} - \underbrace{\frac{2\pi K_{vco}.K_{pd}}{\Delta\omega(0)}\,\underbrace{(1-\cos(\Delta\omega(0).t))}_{0}}_{}$$

$$\frac{d\Phi_e(t)}{dt} = \Delta\omega(0) - 2\pi K_{vco}.K_{pd}\sin(\Phi_{er}(t)) = 0$$

$$\Rightarrow \boxed{\Delta\omega(0) = 2\pi K_{vco}.K_{pd}\sin(\Phi_{er}(\infty))}$$



$\Delta F = 5 MHz$

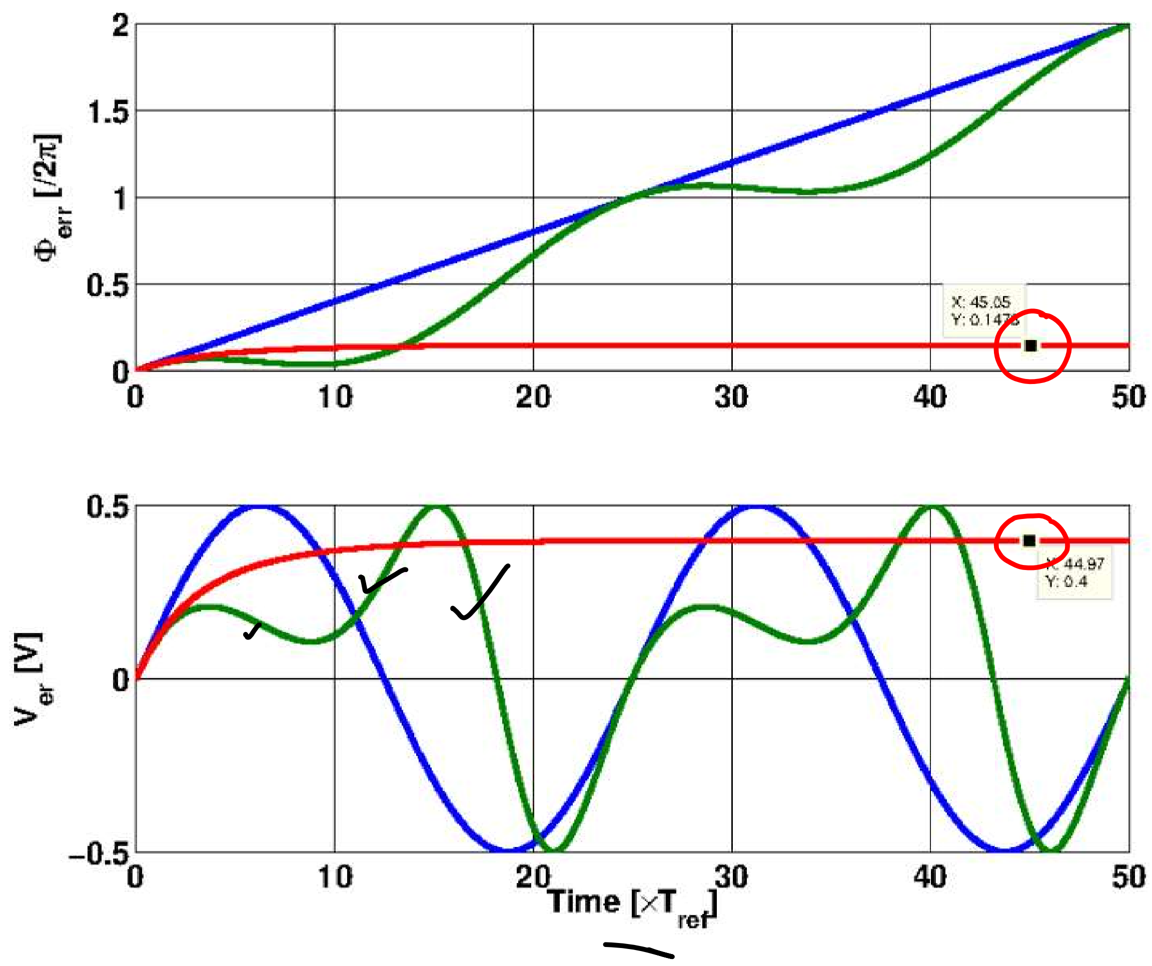
$2\pi K_{vco} = 2\pi \times 100 \; M\,rad/s$

$K_{pd} = 1/2$

$V_{er} = \sin(\Delta\omega . t)$

$\Delta\omega \leq 2\pi K_{vco} . K_{pd} \sin(\Phi_e)$

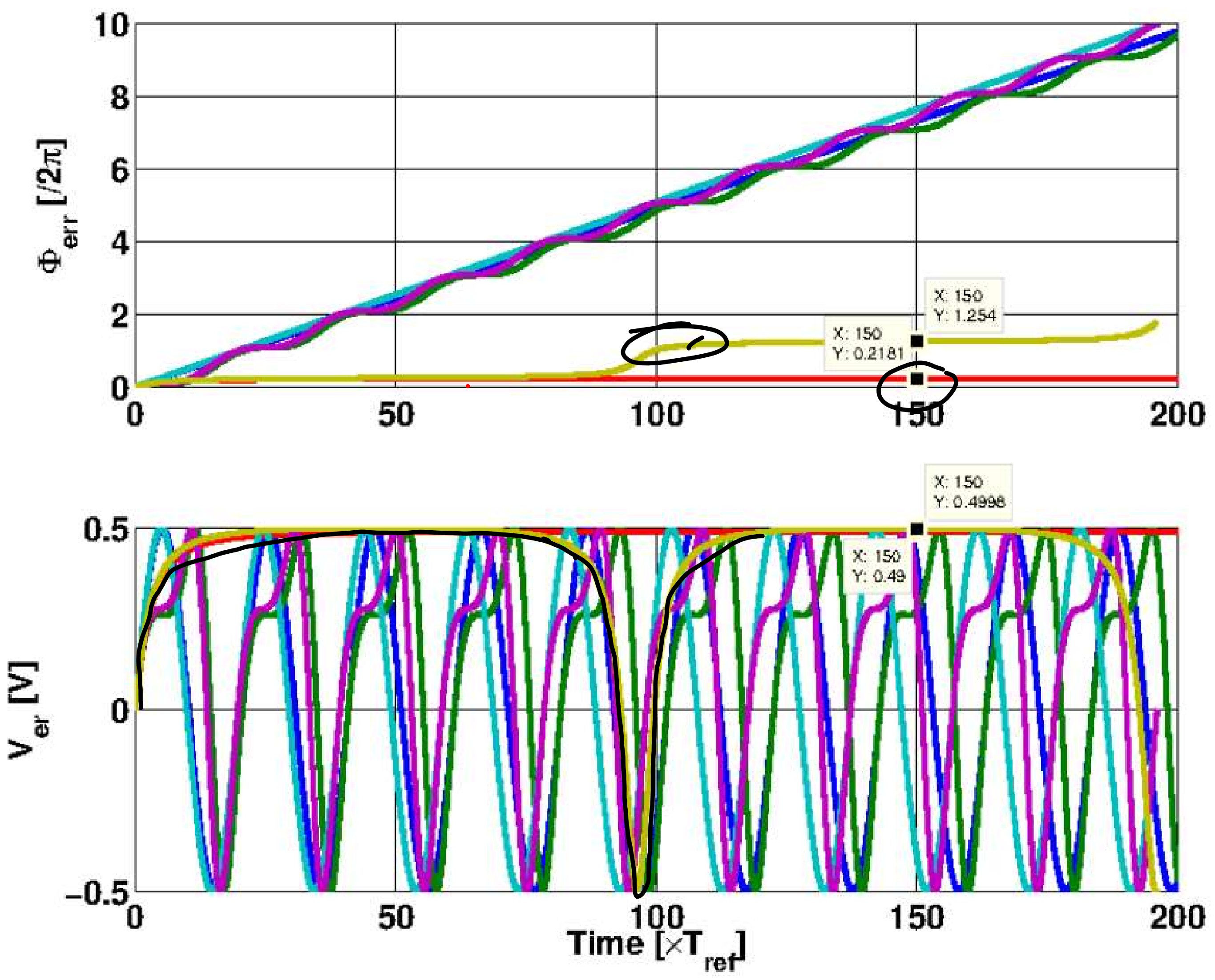
$\Delta f \leq K_{vco} . K_{pd}$



$$\omega_{out} = \omega_{free} + 2\pi K_{vco} \cdot K_{pd} \cdot \sin(\phi_e)$$

$$= \omega_{in}$$

$$\frac{\omega_{in}' - \omega_{free}}{2\pi K_{vco} \cdot K_{pd}} = \sin(\phi_e)$$



$\Delta F_1 = 49\,MHz$

$\Delta F_2 = 51\,MHz$.

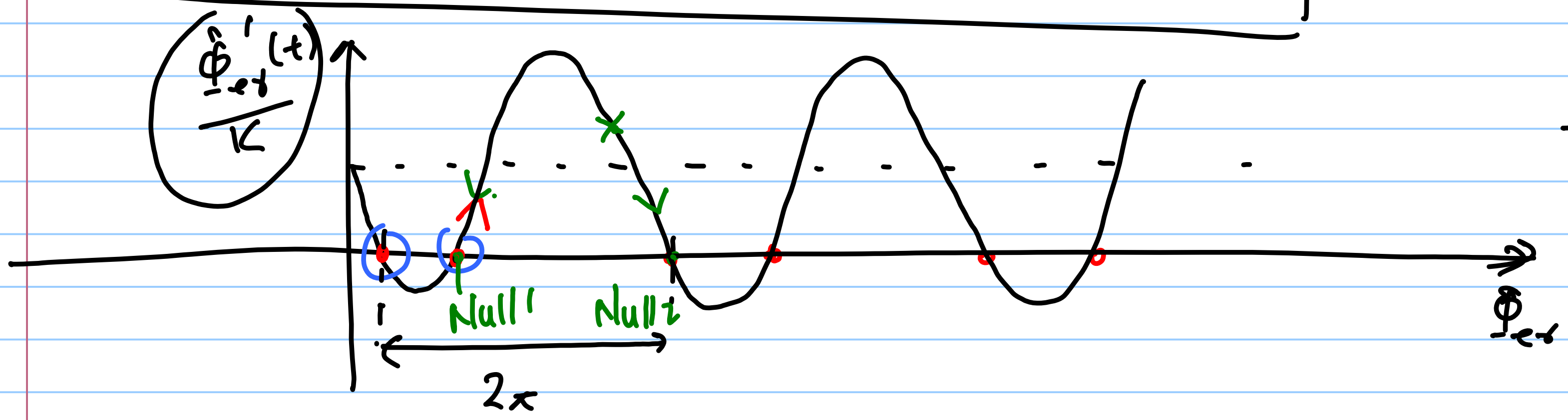
$$\Phi_{in}(t) = \omega_{in} \cdot t + \Phi_{in}(0)$$

$$\Phi_{out}(t) = \omega_{out}\, t + \Phi_{out}(0)$$

$$\Phi_{er}(t) = \overline{\omega_{in} - \omega_{out}}\; t + [\Phi_{in}(0) - \Phi_{out}(0)]$$

$$\Phi_{er}(t) = \Delta\omega \cdot t - \int 2\pi K_{vco} \cdot K_{pd} \sin(\Phi_{er})\, dt + \Phi_{er}(0)$$

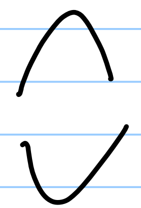
$$\boxed{\frac{d\Phi_{er}(t)}{dt} = \Delta\omega - 2\pi K_{vco} \cdot K_{pd} \sin(\Phi_{er}(\infty))} = \Phi'_{er}(t)$$

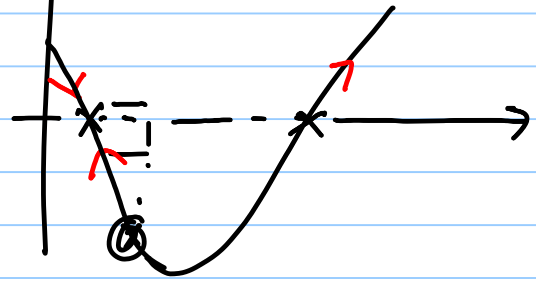


$$\frac{\Phi'_{er}(t)}{K} = \underbrace{\frac{\Delta\omega}{K}}_{=0} - \underbrace{\sin(\Phi_{er})}$$

* PLL locks before Φ_e cross 2π

$$\frac{dx(t)}{dt} = 0$$





– Locks before phase error exceeds 2π

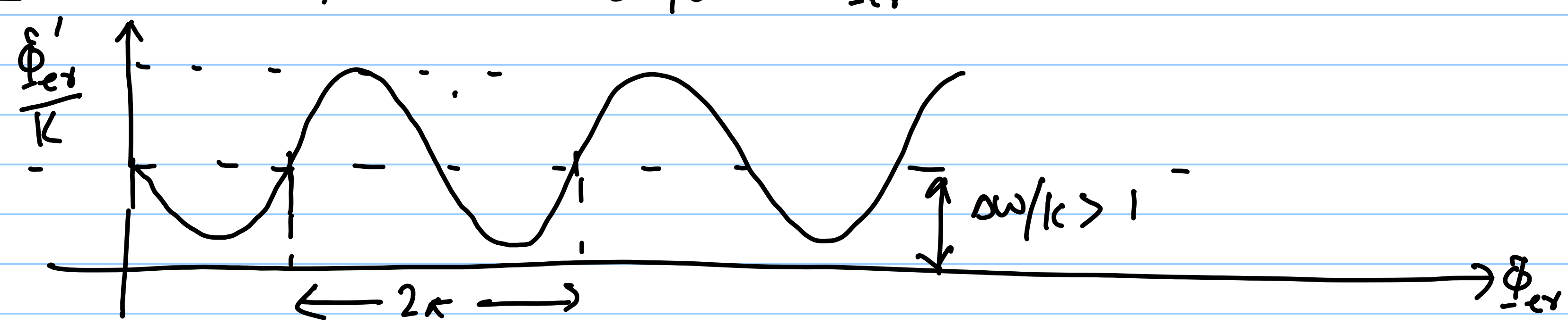
<u>Lock-in-range</u> : Frequency ($\Delta\omega$) error which PLL can lock before phase error exceeds 2π.

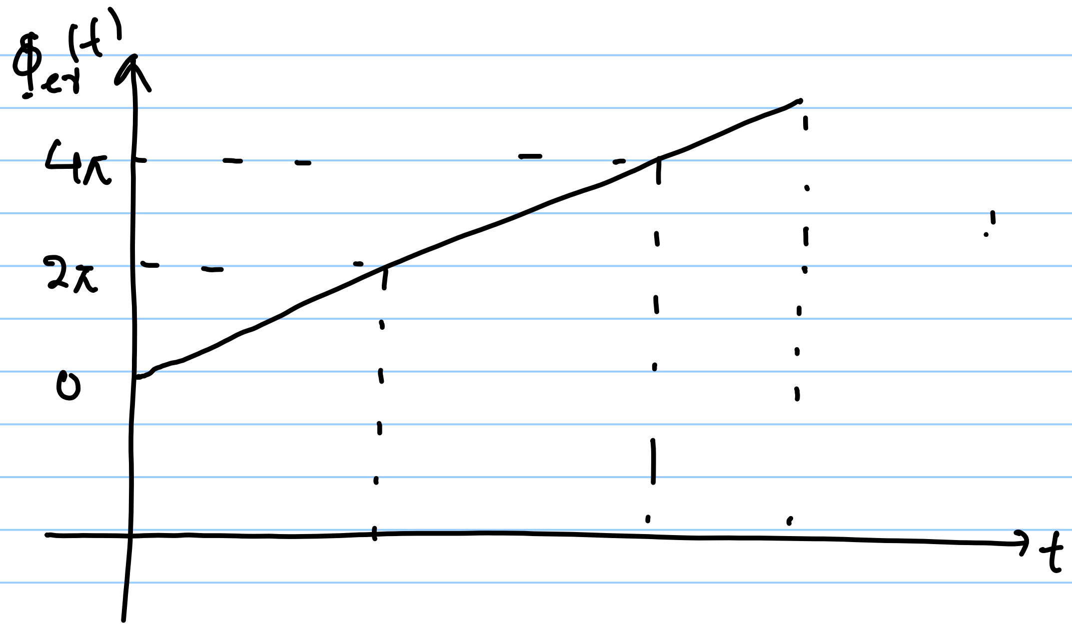
$$\Delta\omega = 2\pi K_{vco} \cdot K_{pd} \leq 1$$

$$\frac{\Delta\omega}{K} > 1$$

$$\Phi_{er}(t) = \Delta\omega(0) \cdot t - 2\pi K_{vco} \cdot K_{pd} \int V_e \cdot dt$$

$$\frac{d\Phi_{er}(t)}{dt} = \Delta\omega(0) - 2\pi K_{vco} K_{pd} \sin(\Phi_{er}(t))$$





$$\Phi_{er}(t) = \Phi_{er}(0) + \frac{d\Phi_{er}(t)}{dt} \cdot t$$

$\frac{\Delta\omega}{K} > 1 \longrightarrow$ Asymmetric phase error & voltage error w/ non-zero DC voltage