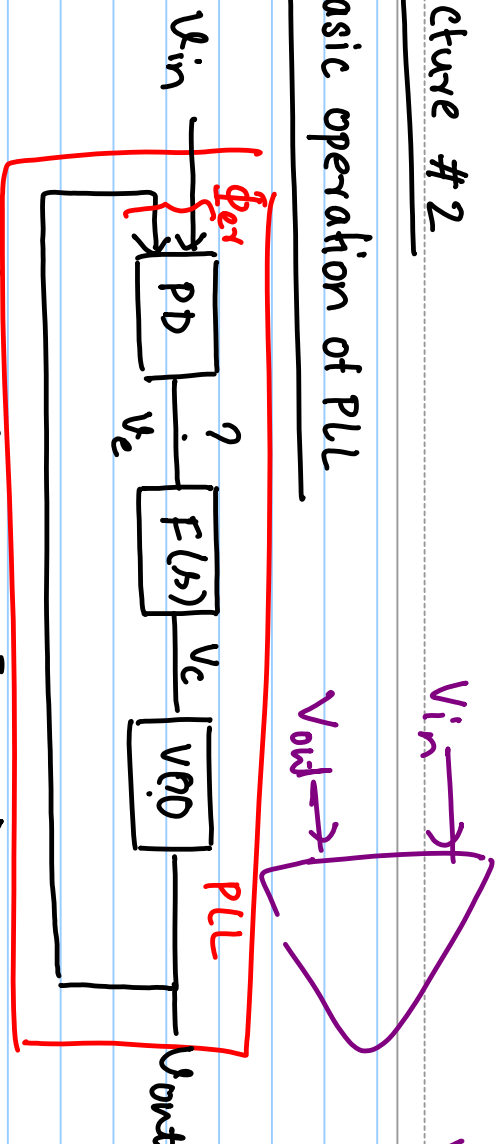


Lecture #2

Basic operation of PLL



~~Vin - Vout~~

$\Phi_{in} - \Phi_{out}$

$$\Phi_{er} \rightarrow \text{PD} \xrightarrow{V_c} K_{pd} = \frac{dV_c}{d\Phi_{er}} \quad [V/rad]$$

$$V_{in} = A_{in} \sin(\omega_{in} t + \Phi_{ino}(t)) \rightarrow \Phi_{in}(t) = \omega_{in} t + \Phi_{ino}(t)$$

$$V_{out} = A_{out} \sin(\omega_{out} t + \Phi_{outo}(t)) \rightarrow \Phi_{out}(t) = \omega_{out} t + \Phi_{outo}(t)$$

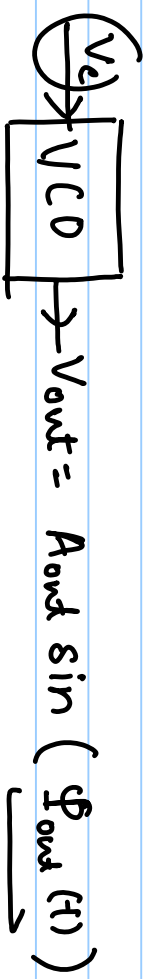
$$\omega_{out} = \omega_{in}$$

$$\Phi_{er}(t) = \Phi_{in}(t) - \Phi_{out}(t)$$

$$= (\omega_{in} - \omega_{out})t + (\Phi_{ino}(t) - \Phi_{outo}(t)) = \Phi_{ino}(t) - \Phi_{outo}(t)$$

$$\frac{d(\Phi_{er}(t))}{dt} = 0 \quad \nabla \quad \Phi_{er}(t) = 0$$

$$\omega_{out} = 2\pi f_{free} + 2\pi K_{vco} \cdot V_c$$



VCO: Voltage controlled Oscillator

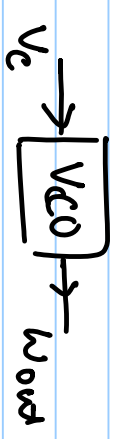
$$\Phi_{osc}(t) = \int 2\pi f_{out} \cdot dt$$

$$= 2\pi \int (f_{free} + K_{VCO} V_c) dt$$

$$= 2\pi f_{free} t + 2\pi K_{VCO} \cdot \int V_c \cdot dt$$

$$\Phi_{osc}(t) = 2\pi K_{VCO} \int V_c \cdot dt$$

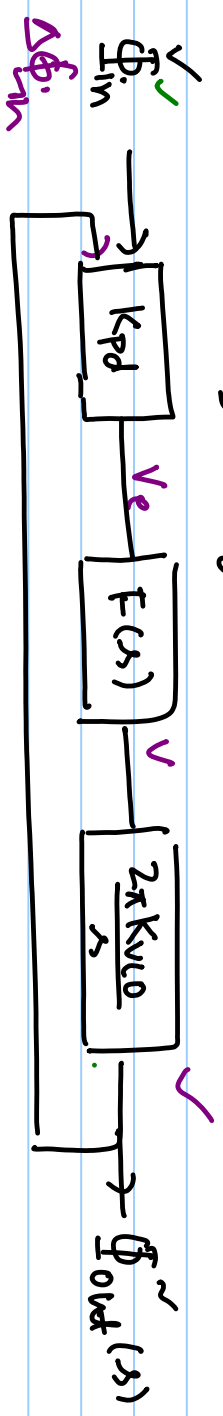
$$[K_{VCO}] = Hz/V$$

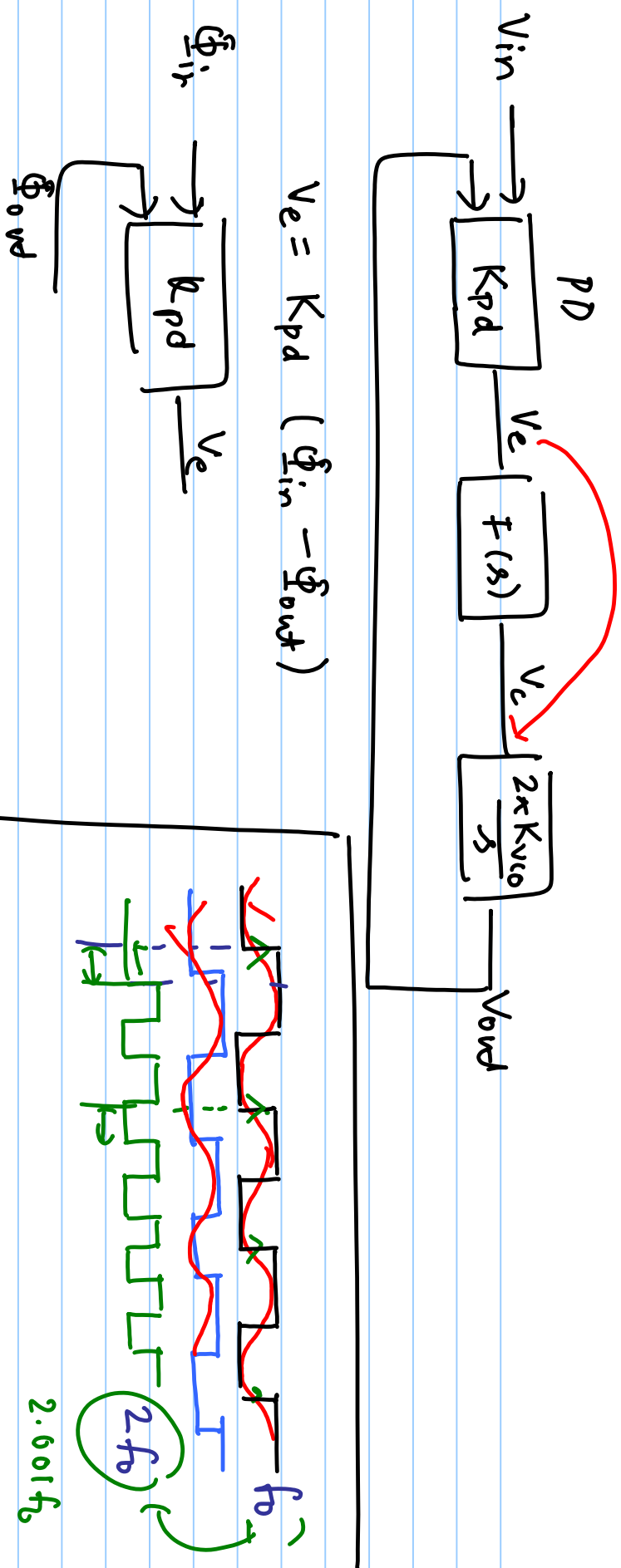


$$\omega_{osc} = 2\pi K_{VCO} \cdot V_c$$

$$\omega_{osc}(s) = 2\pi K_{VCO} \cdot V_c(s)$$

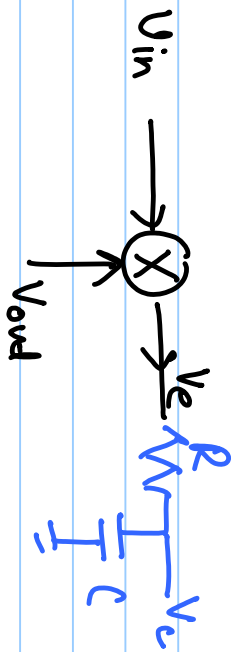
$$\Phi_{osc}(s) = 2\pi K_{VCO} \cdot \frac{V_c(s)}{s}$$





$$L_G(s) = K_{pd} \times F(s) \times \frac{2\pi K_{vco}}{s} = K_{pd} \times \frac{1}{1+sRC} \times \frac{2\pi K_{vco}}{s}$$

$$\frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{L_G}{1+L_G}$$



$$\Delta \omega = 0.1 \omega_{in}$$

$$F(s) = \frac{1}{1 + sRC}$$

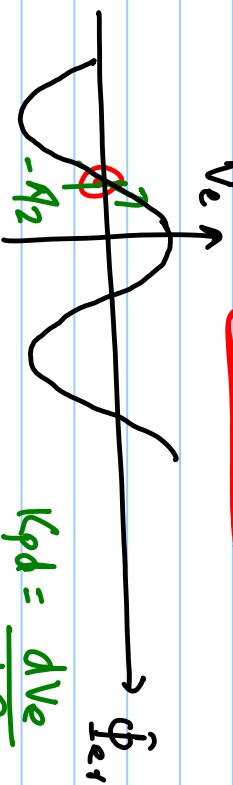
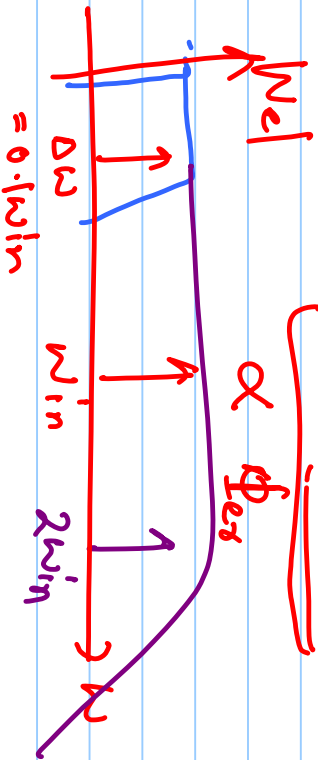
$$V_e = \frac{A_{in} \cdot A_{out}}{2} \left[2 \cdot \sin(\omega_{in} t + \phi_{in0}) \cdot \sin(\omega_{out} t + \phi_{out0}) \right]$$

$$= \frac{A_{in} A_{out}}{2} \left[\cos(\omega_{in} t + \phi_{in0}) + \cos(\omega_{out} t + \phi_{out0}) - \cos(\omega_{in} - \omega_{out} t + \phi_{in0} - \phi_{out0}) \right]$$

$$\approx A_{pd} \cos(\Delta \omega t + \phi_{in0} - \phi_{out0}) - A_{pd} \cos(\omega_{out} + \omega_{in} t + \dots)$$

$\Delta \omega = 0$

$$= A_{pd} \cos(\phi_{in0} - \phi_{out0}) - A_{pd} (\cos(2\omega_0 t + \dots))$$



$$K_{pd} = \frac{dV_e}{d\phi_{err}} = -A_{pd} \sin(\phi_{err}) = A_{pd}$$