Lecture #1

Note Title 31-07-2018

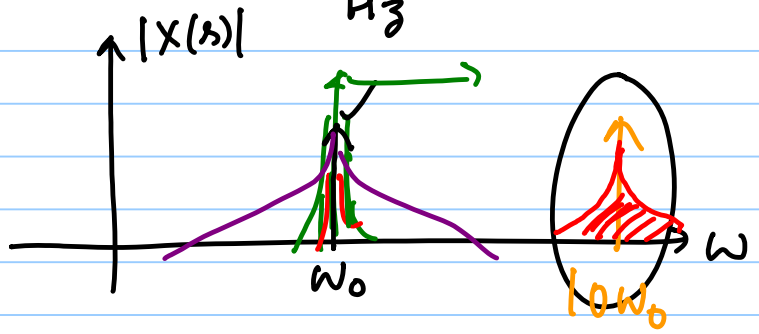
Phase-Locked Loops

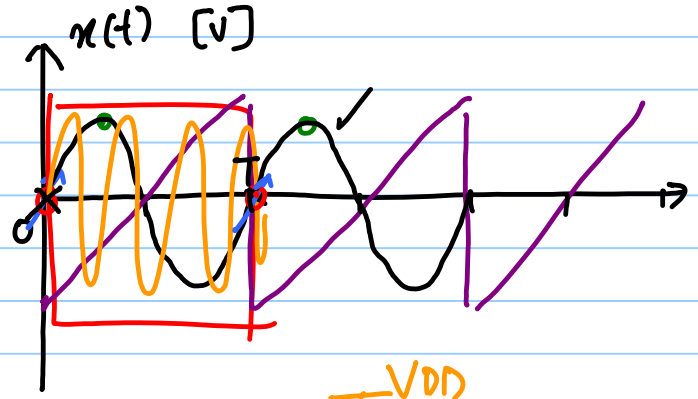
Synchronous — clock

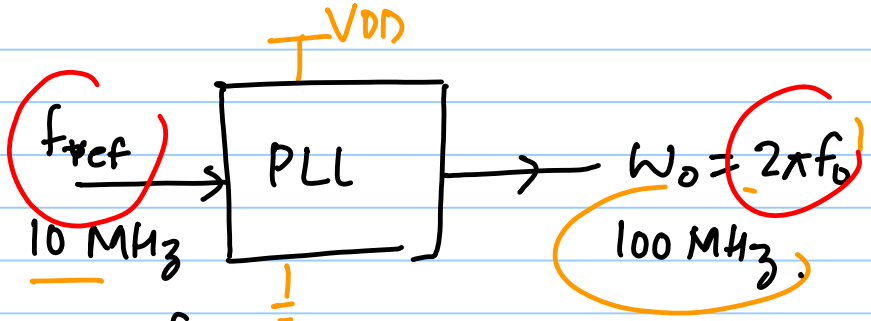
$x(t) = \sin(\omega_0 t)$ ✓

$\omega_0 = 2\pi f_0 = \frac{2\pi}{T}$

(rad/s) | 1/s | Hz | [s]







$f_0 = N f_{ref}$ (Integer-N PLL)

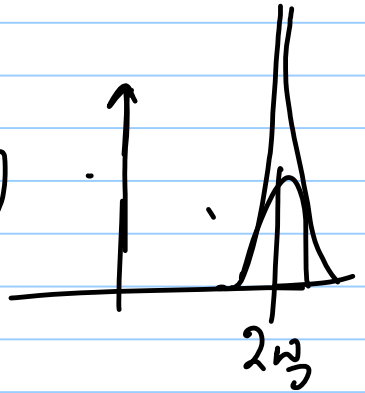
N integer

$f_0 = (N+\alpha) f_{ref}$, $0 < \alpha < 1$

(Fractional-N PLL)

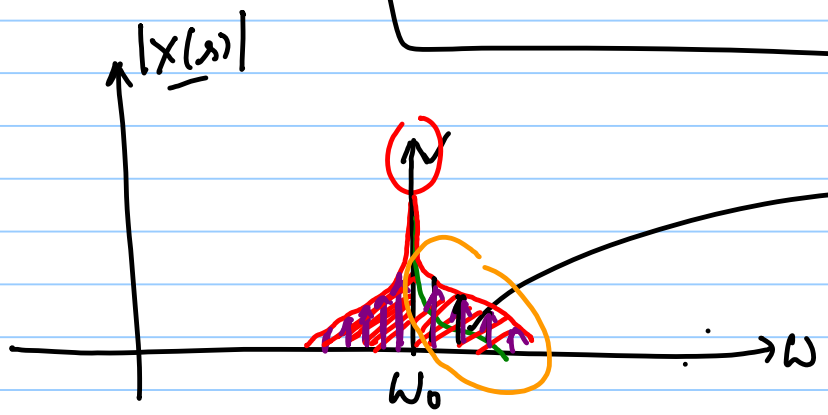
$x(t) = \sin(\omega_0 t)$ $\quad \omega_0 = 2\pi \times 10\,MHz$.

$x(t) \times x(t) = \sin^2(\omega_0 t) = \frac{1}{2}[1 - \cos(2\omega_0 t)]$



$\omega_0 \longrightarrow 2\omega_0 \longrightarrow 4\omega_0 \longrightarrow 8\omega_0$

$8\omega_0$, $10\omega_0$, $6\omega_0$



$x_{ideal}(t) = \sin(\omega_0 t)$

$x(t) = \sin(\omega_0 t + \Phi_n(t))$

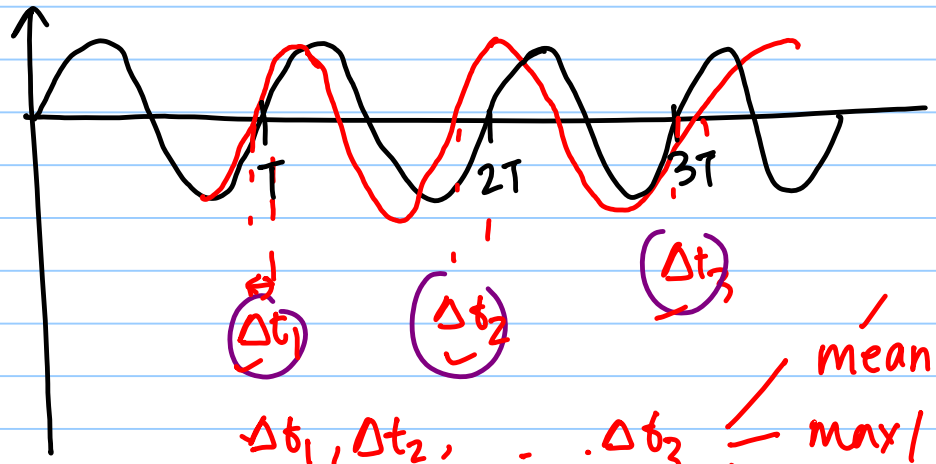
$\Phi(t) = \omega_0(t) + \Phi_n(t)$

$= \omega_0 t$

$x(t) = \sin(\omega_0 t + \phi_n(t))$ — Ideal: after every $T = \frac{2\pi}{\omega_0}$

$\phi(t) = \omega_0 t + \phi_n(t)$ — Real: $\omega_0 t + \phi_n(t) = 2n\pi$

$$t = \frac{2n\pi + \phi_n(t)}{\omega_0}$$



$\Delta t_1, \Delta t_2, \ldots \Delta t_3$

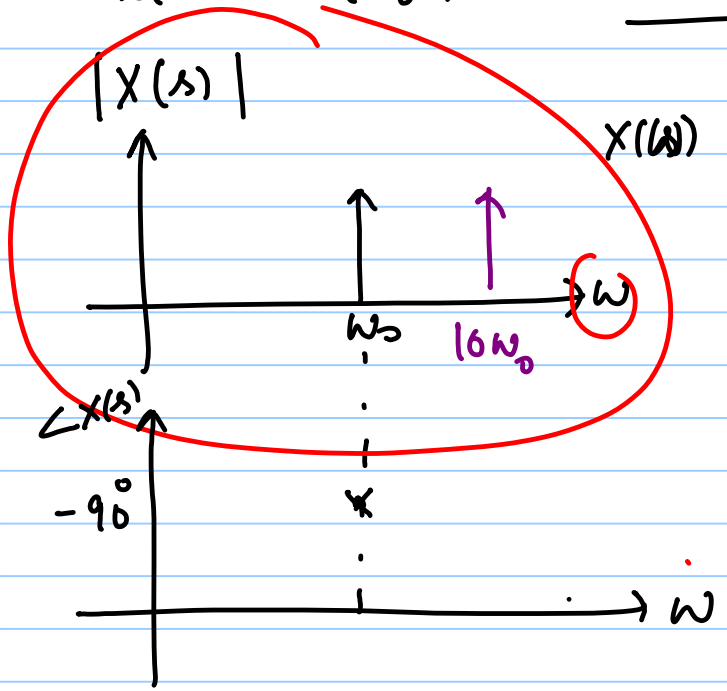
- mean — avg. time period.
- max / min — peak-to-peak jitter
- rms — Jitter

$\Delta t(t) =$

$\Delta t(t_1) =$, $\Delta t(t_2) =$

$$x(t) = \sin(\omega_0 t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$X(j\omega) = \frac{1}{2j} 2\pi \left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$$



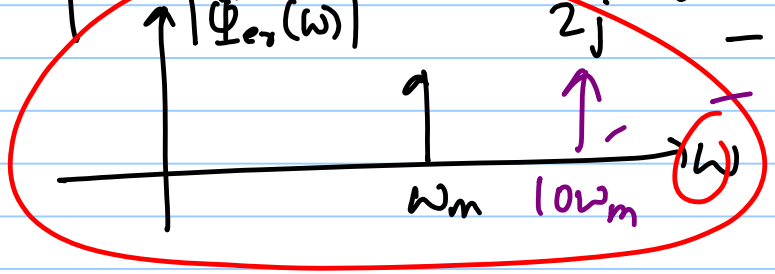
$$x(t) = \sin(\omega t + \Phi_n(t)) \stackrel{?}{=} \sin(\omega t + a_m \sin(\omega_m t))$$

$$\Phi(t) = \omega_0 t + \Phi_n(t)$$

$$\Phi(t) - \omega_0 t = \Phi_{er}(t) = \Phi_n(t) = a_m \sin(\omega_m t)$$

$$\Phi_{er}(j\omega) = \frac{a_m}{2j}\left(e^{j\omega_m t} - e^{-j\omega_m t}\right)$$

$$\Phi_{er}(\omega) = \frac{a_m}{2j}\left[2\pi\,\delta(\omega - \omega_m) - 2\pi\,\delta(\omega + \omega_m)\right]$$



$$x(t) = \sin\left(\omega_0 t + a_m \sin(\omega_m t)\right)$$

$$= \sin(\omega_0 t) \cdot \cos(a_m \sin(\omega_m t)) + a_m \cos(\omega_0 t) \sin(\sin(\omega_m t))$$

$$\sin(a_m \sin(\omega_m t))$$

$$\Phi(t) = \omega_0 t + a_m \sin(\omega_m t)$$

$$\Phi(t) - \omega_0 t = a_m \sin(\omega_m t) = \phi_{er}(t)$$