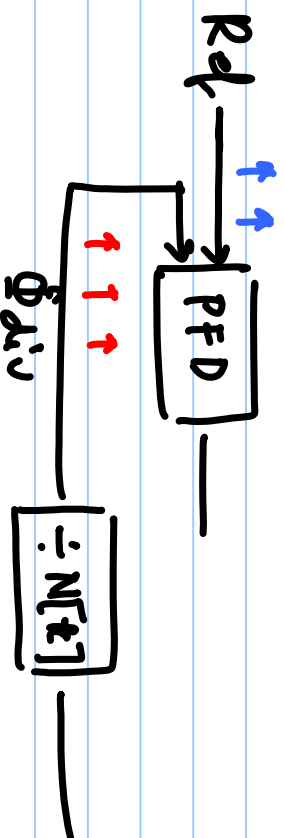


Lecture # 49

Fractional-N PLL

1. t_{rk} : time instant of rising edge on ref.

$t_{rd} + \Delta t_{rk}$: time instant of rising edge on divided clode.



$t_{kH} - t_k = T$ (ref. time period)

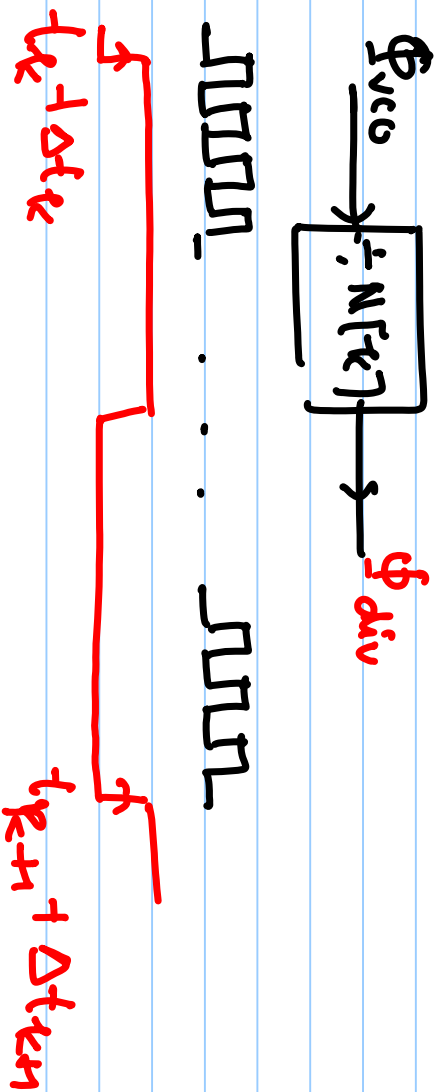
2. $\Phi_{VCO} = 2\pi K_{VCO} \int V_{ctrl} \cdot dt = 2\pi f_{nom} t + \Phi_{offset}(t)$

$f_{nom} = (N + \alpha) f_{ref}$, $0 < \alpha < 1$

3. $t_{k+1} + \Delta t_{k+1}$: time instant for $(k+1)^{th}$ rising edge on Φ_{div}

$t_k + \Delta t_k$: k^{th}

$$\Phi_{vco}(t_{k+1} + \Delta t_{k+1}) - \Phi_{vco}(t_k + \Delta t_k) = 2\pi N[k]$$



$$\begin{aligned} \text{LHS} &= 2\pi f_{nom}(t_{k+1} + \Delta t_{k+1} - t_k - \Delta t_k) + \Phi_{ovd}(t_{k+1} + \Delta t_{k+1}) - \Phi_{ovd}(t_k + \Delta t_k) \\ &= 2\pi f_{nom}(t_{k+1} - t_k) + 2\pi f_{nom}(\Delta t_{k+1} - \Delta t_k) \\ &\quad + \underbrace{\Phi_{ovd}(t_{k+1} + \Delta t_{k+1}) - \Phi_{ovd}(t_k + \Delta t_k)} \end{aligned}$$

$$\begin{aligned}
 &= 2\pi f_{\text{nom}} \cdot T + \Phi. \\
 &\quad \nearrow 1/f_{\text{ref}} \\
 &= 2\pi N_{\text{nom}} + \quad = \\
 &= 2\pi N[f_c]
 \end{aligned}$$

$$2\pi f_{\text{nom}} (\Delta t_{k+1} - \Delta t_k) = 2\pi (N[f_c] - N_{\text{nom}}) - (\Phi_{\text{out}}(t_{k+1} + \Delta t_{k+1}) - \Phi_{\text{out}}(t_k + \Delta t_k))$$

Sum k samples.

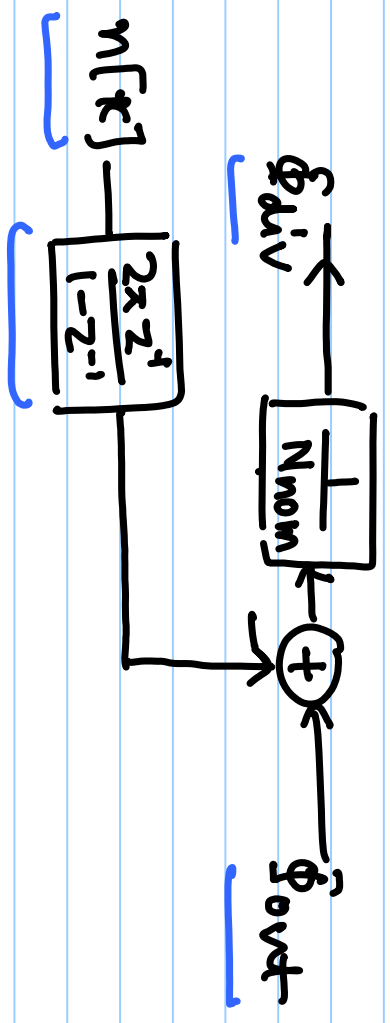
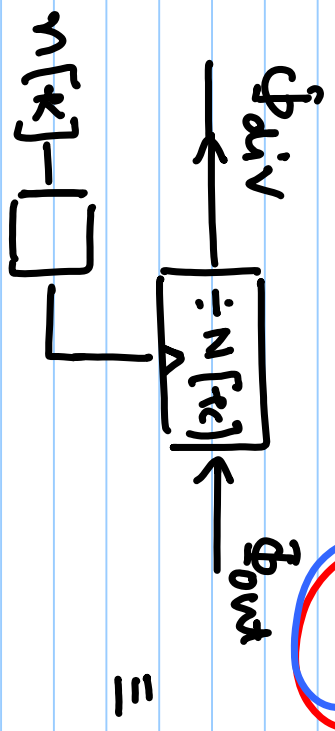
$$2\pi f_{\text{nom}} (\Delta t_{k+1} - \Delta t_0) = 2\pi \sum_{i=1}^k n[i] - (\Phi_{\text{out}}(t_{k+1} + \Delta t_{k+1}) - \Phi_{\text{out}}(t_0 + \Delta t_0))$$

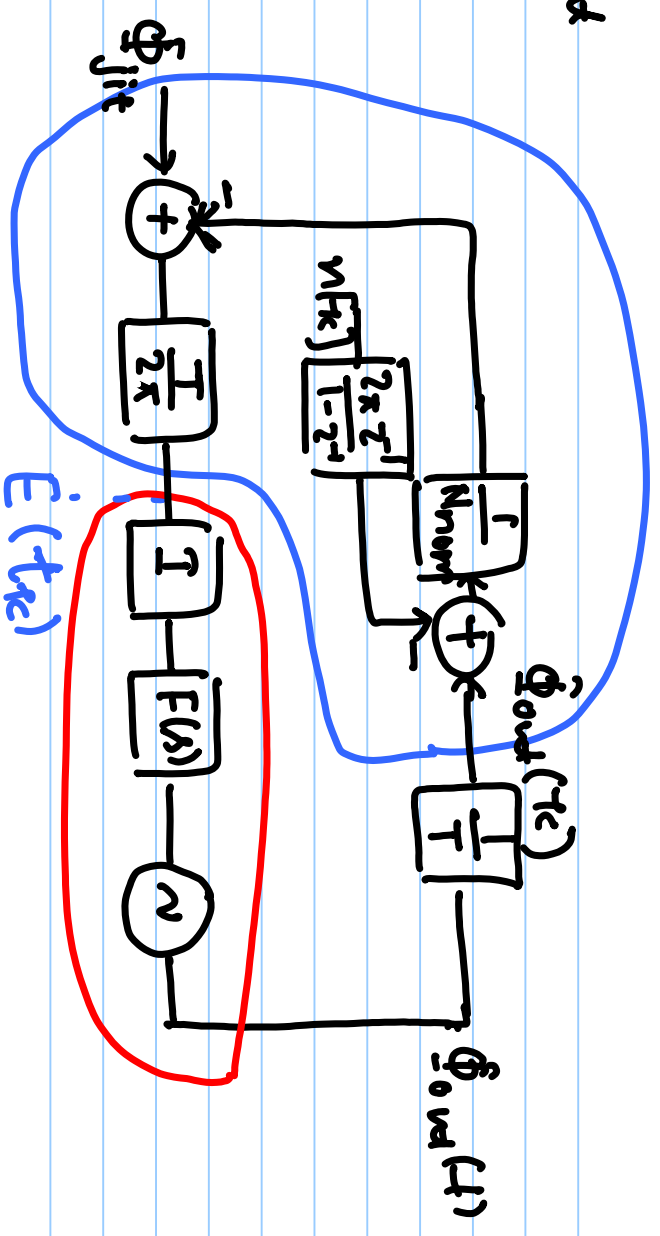
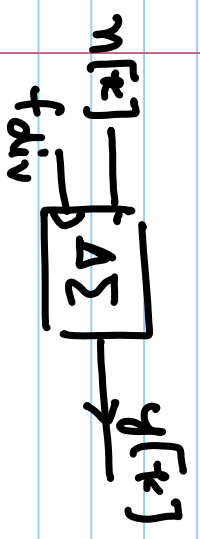
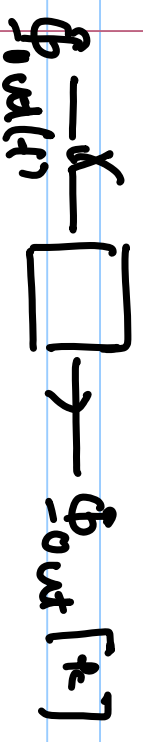
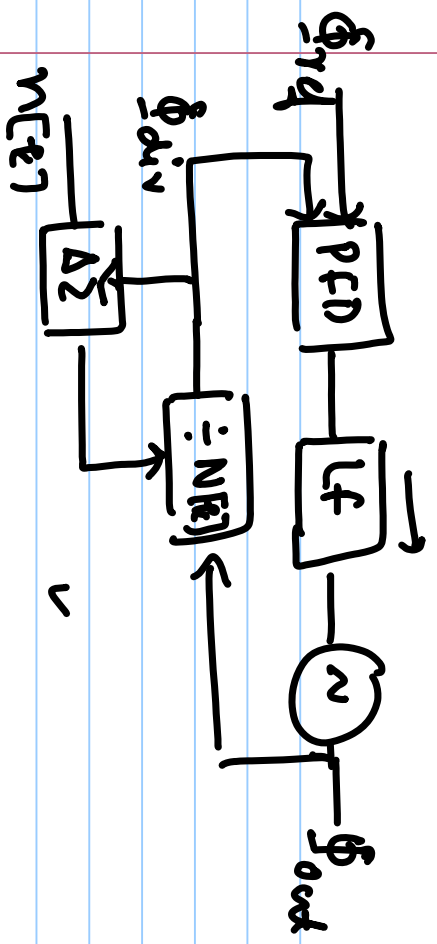
$$\Delta t_0 = 0, \quad \Phi_{\text{out}}(t_0 + \Delta t_0) = 0$$

$$\Rightarrow 2\pi f_{\text{nom}} (\Delta t_{k+1}) = 2\pi \sum_{i=1}^k n[i] - \Phi_{\text{out}}(t_{k+1} + \Delta t_{k+1})$$

$$\Delta t_{rk} = \frac{T}{2\pi} \frac{1}{N_{nom}} \left[2\pi \sum_{i=1}^k n[i] - \Phi_{out}(rc) \right]$$

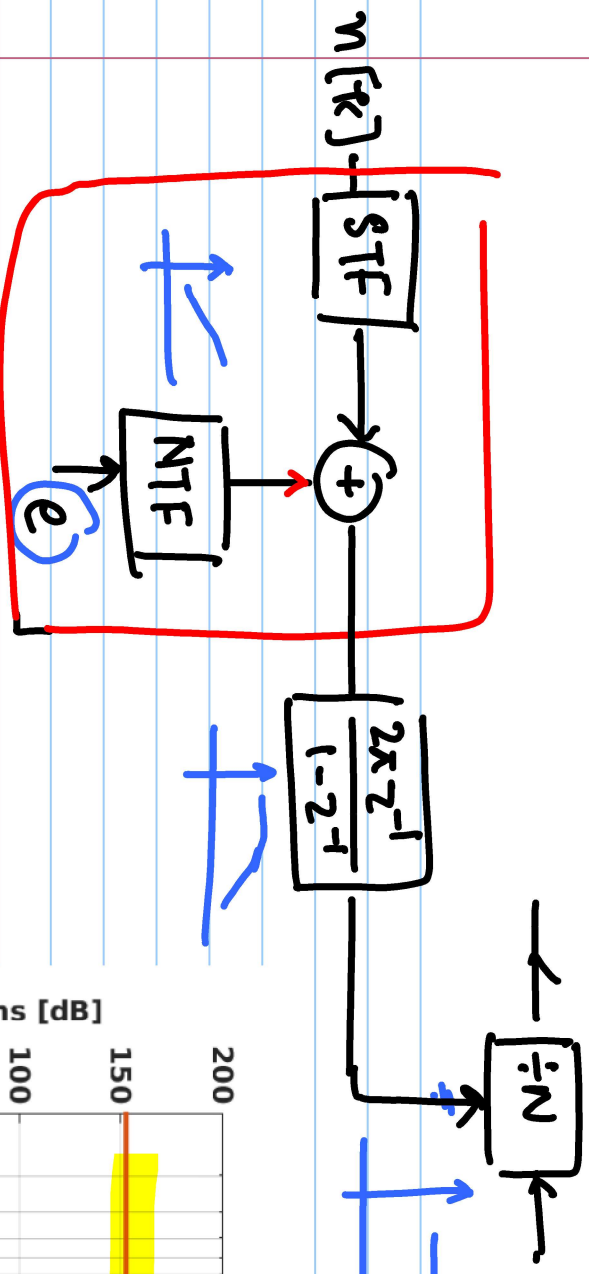
$$\Phi_{div}(rc) = - \frac{1}{N_{nom}} \left[2\pi \sum_{i=1}^k n[i] - \Phi_{out}(rc) \right]$$





$$Y(z) = STF(z) N(z) + NTF(z) E(z)$$

$$NTF(z) = (1 - z^{-1})^m$$



$f_{ref} = 40 \text{ MHz}$
 $N = 64$
 $f_{out} = 2.56 \text{ GHz}$

$$\begin{aligned}
 |NTF_{DSM}| &= (1-z^{-1})^2 \\
 &= \frac{2z^2}{1-z^{-1}} \times \frac{L_n}{1+L_n}
 \end{aligned}$$

