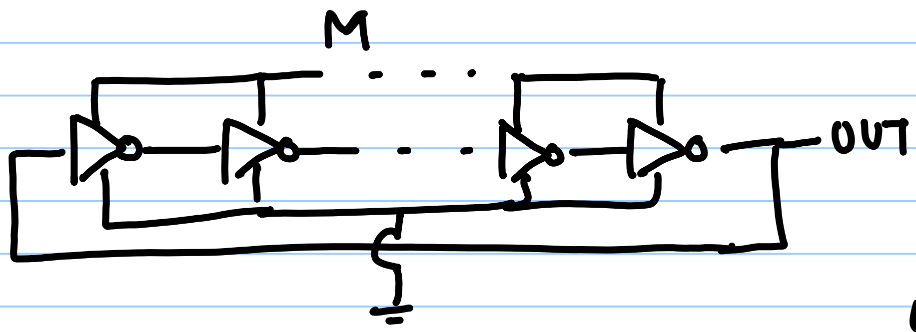
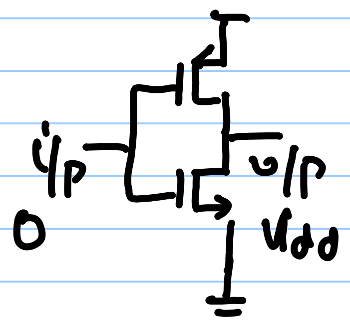
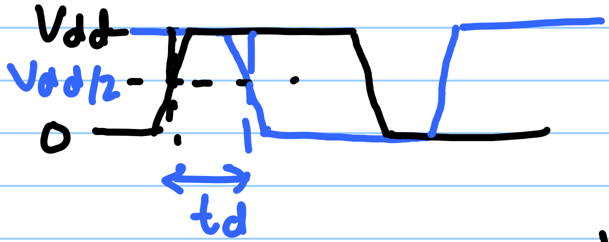
Note Title 13-10-2017

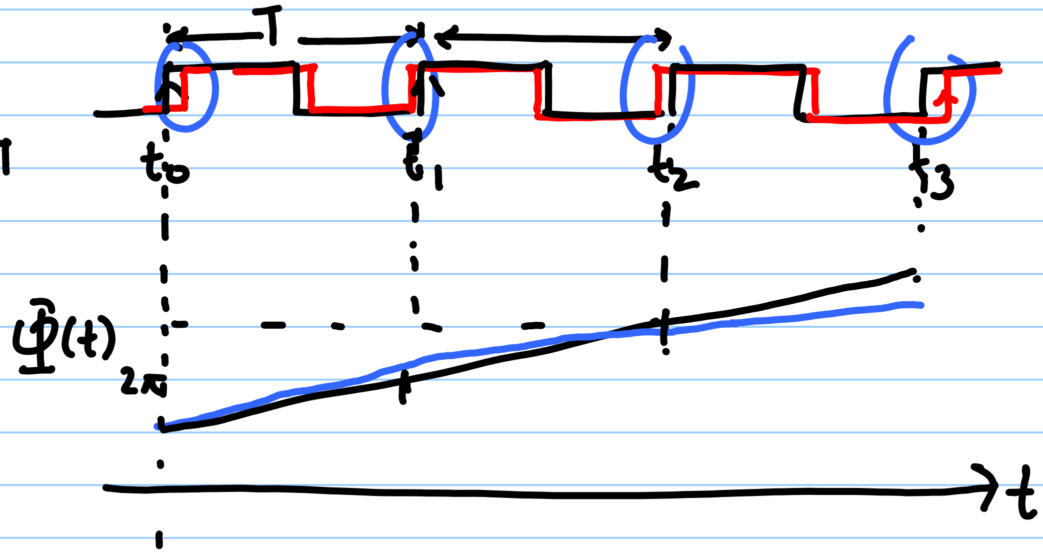
Lecture # 38

Phase noise in ring oscillators.

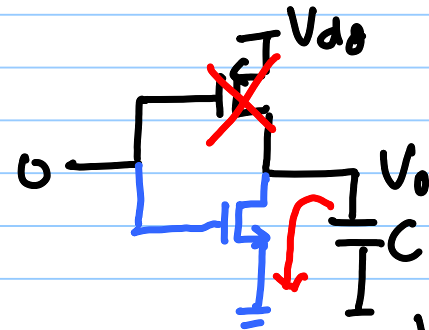








$$T = \frac{1}{2 M t_d}$$

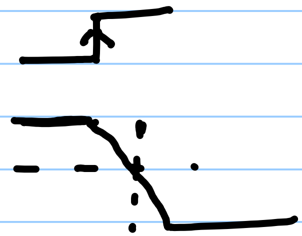


$V_{out} = V_{dd} + V_n$

$\overline{V^2_{out,n}} = \frac{KT}{C}$

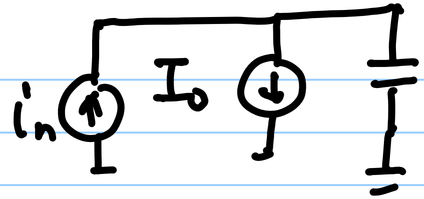
$t=0$

i/p



$V_{dd} + V_n = V_{out}$

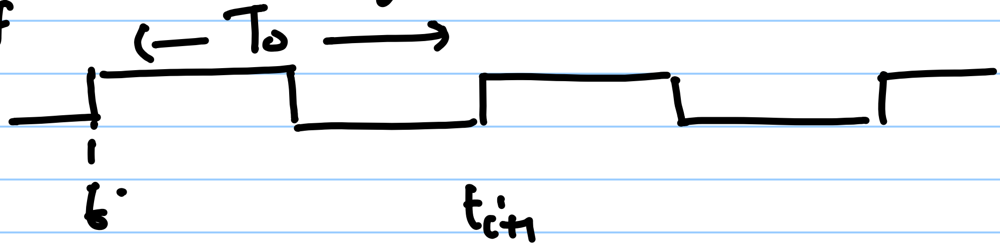
$$I_o = \frac{kW}{2L}\left(V_{dd} - V_{tn}\right)^2$$



$$\frac{1}{C}\int_0^{t_{dN}} (I_o + i_n)\,dt = \int_{V_{dd}+V_n} dV_{out}$$

M inverters

$$\frac{\overline{i_n^2}}{\Delta f} = 4kT\gamma_N g_m$$



✓ $$\tau_i = \frac{1}{2\pi f_0}\left(\Phi(t_{i+1}) - \Phi(t_i)\right), \quad f_0 = \frac{1}{T_0}$$

freq. domain.

✓ $$S_\tau(f) = \frac{1}{(2\pi f_0)^2} S_\Phi(f) \left|\underbrace{1 - e^{-j2\pi f T_0}}\right|^2 = \frac{S_\Phi(f)}{4\pi^2 f_0^2}\left|e^{-j\pi f/f_0}\left(e^{j\pi f/f_0} - e^{-j\pi f/f_0}\right)\right|^2$$

$$= \frac{S_\Phi(f)}{4\pi^2 f_0^2}\, 4\sin^2(\pi f/f_0)$$

✓ $$\sigma_\tau^2 = \int_0^\infty S_\tau(f)\,df$$

$$\sigma_x^2 = E\left[(x - \bar{x})^2\right]$$ ✓

$$\Phi(kT_0) = 2\pi f_0 (kT_0) + \sum_{i=1}^{K} \frac{2\pi}{T_0} . \Delta t_i$$

$$\Phi(\overline{k+1}\, T_0) = 2\pi f_0 (\overline{k+1}\, T_0) + \sum_{i=1}^{k+1} \frac{2\pi}{T_0} \Delta t_i$$

$$\Phi(\overline{k+1}\, T_0) - \Phi(kT_0) = 2\pi f_0 T_0 + \frac{2\pi}{T_0} . \Delta t_{k+1}$$

$$= 2\pi f_0 \underbrace{\left(T_0 + \Delta t_{k+1} \right)}$$

$$\frac{x(t)}{x^2(t)} = \int S_x(f)\, df$$

$$S_\tau(f) = \frac{S_\Phi(f)}{\cancel{4}\pi^2 f_0^2} \quad \cancel{4} \sin^2(\pi f / f_0)$$

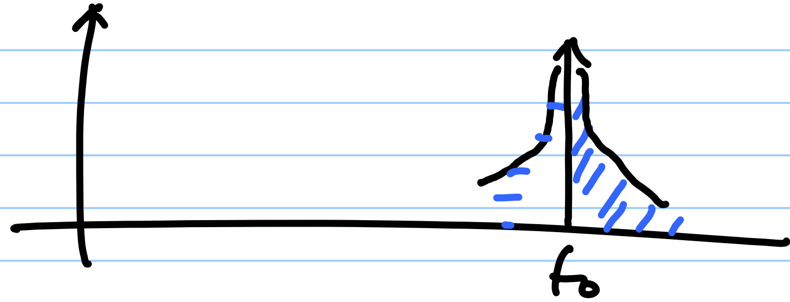
$$\boxed{\sigma_\tau^2 = \int S_\Phi(f) \frac{\sin^2(\pi f / f_0)}{(\pi f_0)^2} df}$$

Phase Noise & Jitter in CMOS Ring Osc. by Abidi

$$V_{out} = \underbrace{A \sin(\omega_0 t + \phi(t))}$$

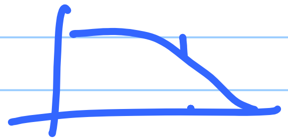
$$\sin(\omega_0 t + \underbrace{\int K_{vco} a_m \sin(\omega_m t)\, dt}_{\Phi(t)})$$

SSB (Single sideband) Phase Noise.



$$\mathcal{L}(f) = \frac{S_\phi(f)}{2} = \left(\frac{S_w}{f^2}\right)$$

-90 dBc/Hz



f : w.r.t carrier freq.

S_w : oscillator constant.

$$\sigma_\tau^2 = \int \frac{2S_w}{f^2} \frac{\sin^2(\pi f/f_0)}{\pi^2 f_0^2}\, df = \frac{2S_w}{f_0^4} \int_0^\infty \frac{\sin^2(\pi f/f_0)}{(\pi f/f_0)^2}\, df$$

$x = \pi f/f_0 \quad \Rightarrow$

$$\sigma_\tau^2 = \frac{2S_w}{f_0^4} \frac{f_0}{\pi} \int_0^\infty \frac{\sin^2 x}{x^2}\, dx$$

$$\sigma_\tau^2 = \frac{S_w}{f_0^3} \frac{2}{\pi} \times \frac{\pi}{2} = \frac{S_w}{f_0^3}$$

$$\boxed{\mathcal{L}(f) = \sigma_\tau^2 \frac{f_0^3}{f^2}}$$

M: no. of inverters in ring osc.

$$f_0 = \frac{1}{M(t_{dN} + t_{dP})}$$

$$= \frac{2}{M C V_{dd}} \left(\frac{1}{I_N} + \frac{1}{I_P} \right)^{-1}$$

$$= \frac{2}{M C V_{dd}} \times \frac{I}{2}$$

$$f_0 = \frac{I/C}{M V_{dd}}$$

$$T_0 = M(\check{t}_{dN} + \check{t}_{dP})$$

$$\sigma_T^2 = M(\sigma_{t_{dN}}^2 + \sigma_{t_{dP}}^2)$$

$$\frac{1}{C}\int_0^{t_{dN}} (I_N + i_n)\, dt = \frac{V_{dd}}{2}$$

$$\langle t_{dN} \rangle = \frac{C V_{dd}}{2 I_N},$$

$$\sigma_{t_{dN}}^2 = \left\langle \left(\frac{1}{I} \int_0^{t_{dn}} i_n\, dt \right)^2 \right\rangle$$

$$\int_0^{t_{dN}} i_n\, dt = \frac{C V_{DD}}{2} - \int_0^{t_{dN}} I_N\, dt$$

$$\boxed{\check{t}_{dn} = \frac{C V_{DD}}{2 I_N} - \frac{1}{I_N} \int_0^{t_{dN}} i_n dt}$$

$$\sigma_{t_{dN}}^2 = \frac{4kT\gamma_N t_{dN}}{I_N (V_{DD} - V_{tn})}$$

$$\sigma_T^2 = \frac{M\, 4kT\, t_{dN}}{I\,(V_{DD} - V_t)} (\gamma_N + \gamma_p)$$

$$+ \left(\qquad \right)$$

$V_{tn} = V_{tp}$

$I_N = I_p = I$

$$t_{dN} = t_{dp} = \frac{C V_{DD}}{2I}$$

$$\overline{v_n^2} = \frac{kT}{C}$$