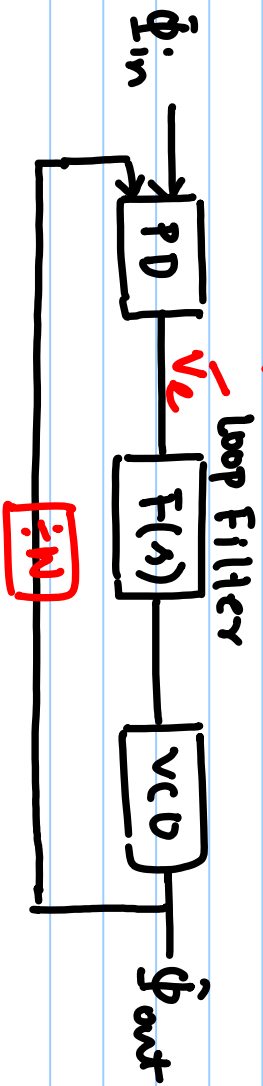


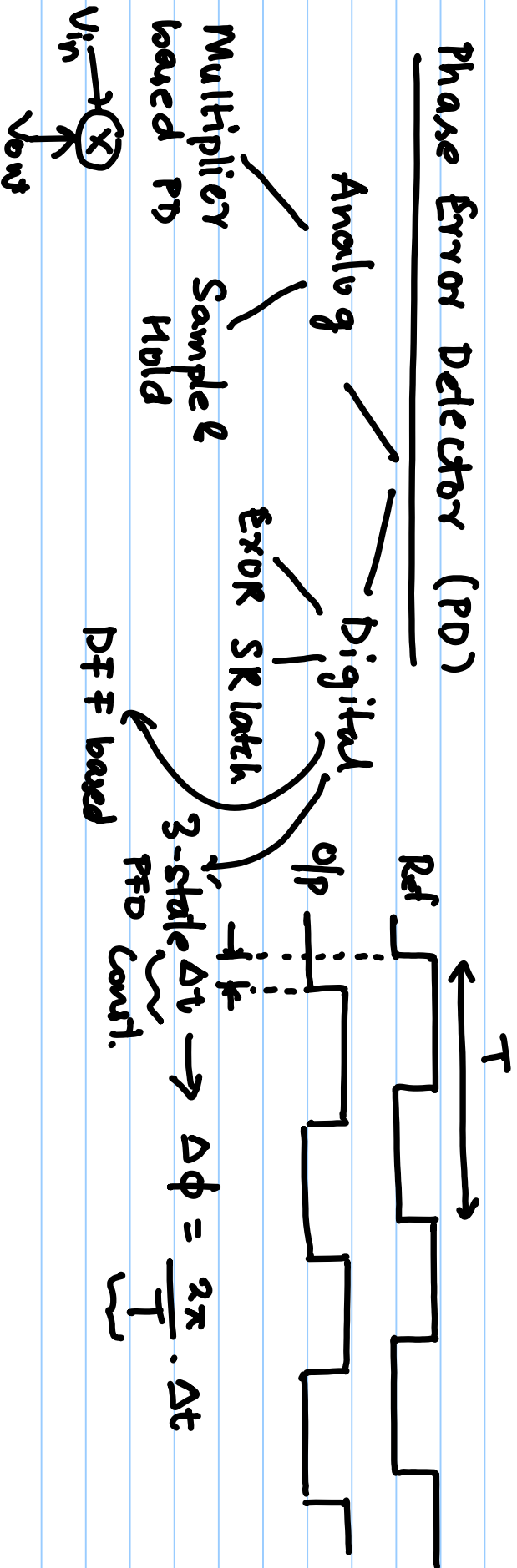
# Lecture #12



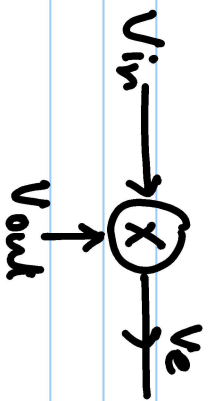
\* Input/output signals are voltage/current

\* PLL enforces a relationship b/w i/p & o/p phase & frequency

## Phase Error Detector (PD)



# Multiplier-based PD



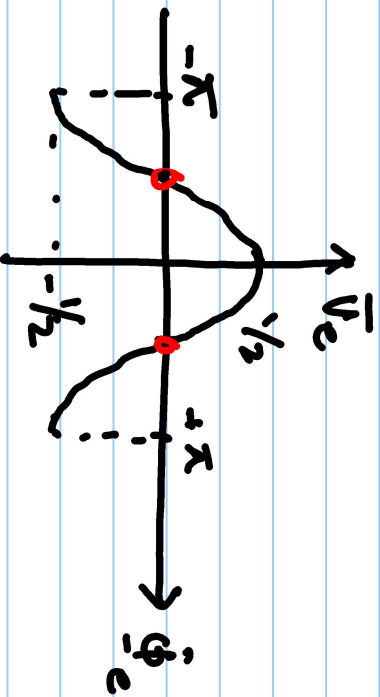
$$V_{in} = a_{in} \cdot \sin(\omega_{in} t)$$

$$V_{out} = a_{out} \cdot \sin(\omega_{out} t - \phi_e)$$

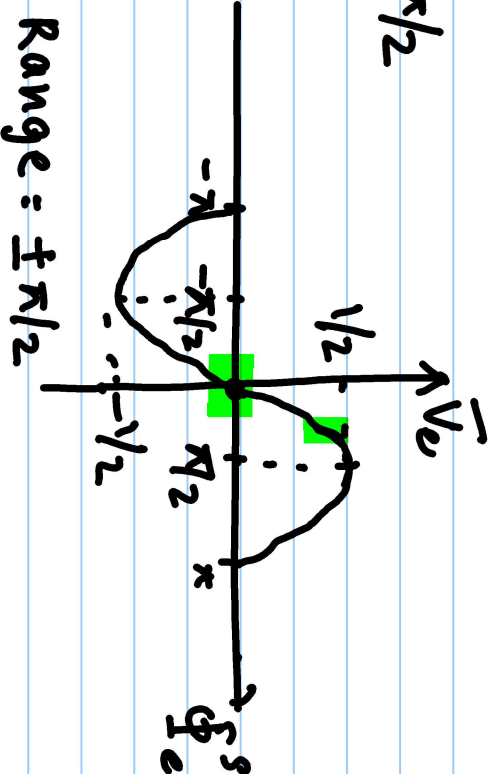
$$\omega_{in} = \omega_{out}, \quad a_{in} = a_{out} = 1$$

$$V_e = \frac{-1}{2} [\cos(2\omega_{in} t - \phi_e) - \cos(\phi_e)]$$

$$\bar{V}_e = \frac{1}{2} \cos(\phi_e)$$



$$\phi_e^s = \phi_e + \pi/2$$



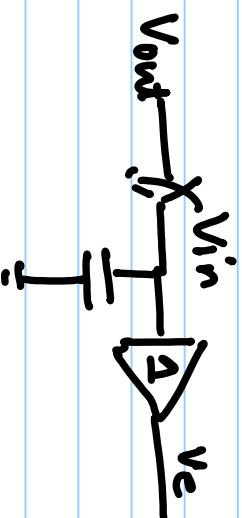
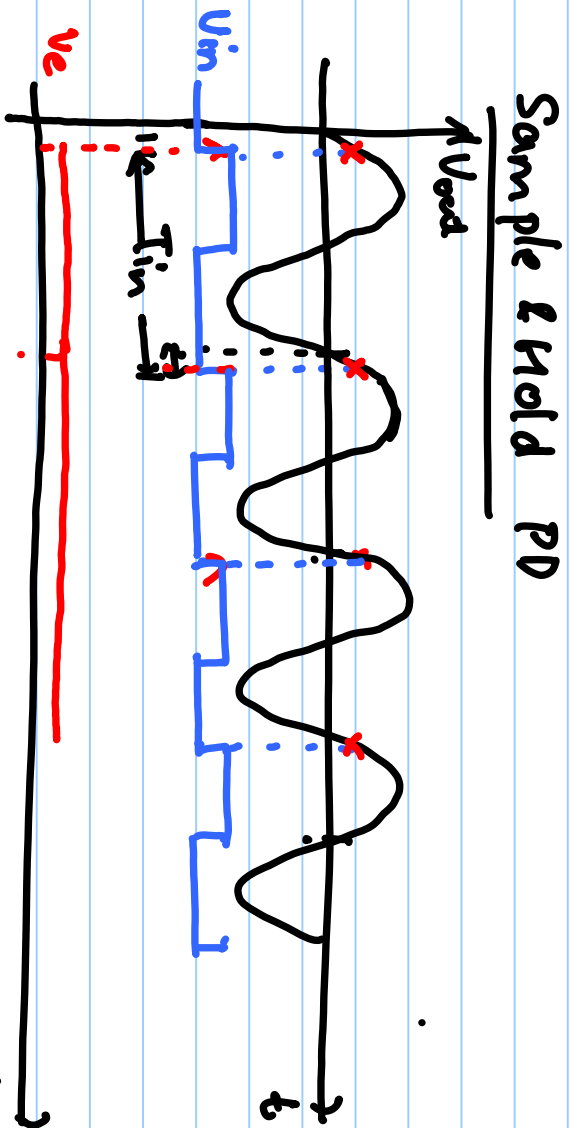
$$\left. \begin{aligned} &= -2 \sin A \cdot \sin B \\ &= \cos(A+B) - \cos(A-B) \end{aligned} \right\}$$

$$\phi_e^s \rightarrow \bar{V}_e$$

$$\bar{V}_e = K_{pd} \cdot \phi_e^s \quad K_{pd} = \frac{d\bar{V}_e}{d\phi_e^s} = \frac{1}{2} \cos(\phi_e^s)$$

$\bar{V}_e$  is a non-linear function of  $\phi_e^s$ .

### Sample & Hold PD



$$\omega_{in} = \omega_{out}$$

$$V_e = \sin(\omega_{in} \cdot \Delta t)$$

$$V_{out} = \sin(\omega_{out} t)$$

$$V_e = \sin(\omega_{out} (n \cdot T_{in} + \Delta t))$$

$$= \sin(n \cdot \omega_{out} \cdot T_{in} + \omega_{out} \cdot \Delta t)$$

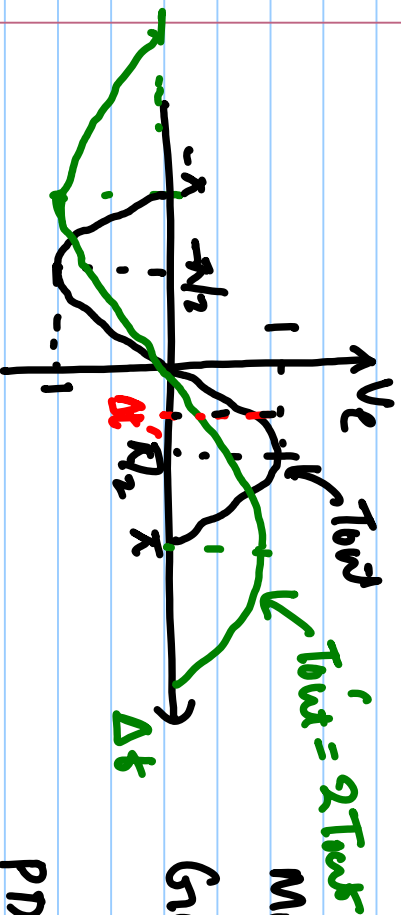
if  $\omega_{out} = \omega_{in} \Rightarrow \omega_{out} \cdot T_{in} = 2\pi \Rightarrow V_e = \sin(\omega_{out} \cdot \Delta t) = \sin(\psi_e)$

if  $\omega_{out} = N\omega_{in} \Rightarrow \omega_{out} \cdot T_{in} = 2N\pi$

$$\omega_{out} \cdot \frac{2\pi}{\omega_{in}} = 2N\pi$$

$$V_e = \sin(2N\pi + \omega_{out} \cdot \Delta t)$$

$$= \sin(\psi_e) = \sin\left(\frac{2\pi}{T_{out}} \cdot \Delta t\right)$$



Montonous Range =  $-\pi/2 \leq \psi_e \leq \pi/2$

Gain,  $K_{pd} = \frac{dV_e}{d\psi_e} = \cos(\psi_e)$

PD is non-linear.

Case:  $\omega_{out} = N\omega_{in}$ ;  $\psi_e$  vs  $\omega_{out}$  or  $\omega_{in}$ ?

$\phi_c$  range  $-\pi/2$  to  $+\pi/2 \Rightarrow -\frac{T_{out}}{4}$  to  $+\frac{T_{out}}{4}$

$\phi_c$  range  $-\pi/2$  to  $+\pi/2$  w.r.t ref.  $T_{in} \Rightarrow -\frac{T_{in}}{4}$  to  $+\frac{T_{in}}{4}$

$V_{in}(f_{in})$   $V_{out}(f_{out})$

$F_{in} \xrightarrow{F_{out}}$  Normalizing phase error w.r.t  $f_{in}$ .  
if Normalizing " w.r.t  $f_{out}$ .

$$V_{out} = \sin(\omega_{out} t)$$

$$\omega_{out} = N \omega_{in}$$

$$V_e = V_{out} \times \underbrace{\sum \delta(t - nT_{in})}$$

$$V_e(f) = V_{out}(f) * \frac{1}{T} \sum \delta(f - n f_{in})$$