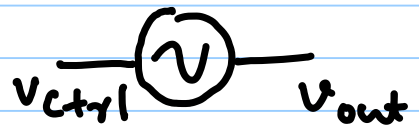
Note Title 07-08-2017

Lecture # 5



$$V_{out} = A_{out} \sin\left(\int 2\pi\left(K_{vco}\cdot V_{ctrl} + f_{free}\right) dt\right)$$

f_{free} : free running freq. of osc

K_{vco} : freq. gain

$K_{vco}\cdot V_{ctrl}$ = Δf controlled by V_{ctrl}

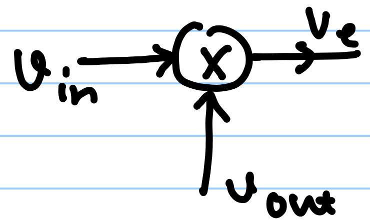
$$f_{total} = f_{free} + \underbrace{K_{vco}\cdot V_{ctrl}}$$

$$\Phi_{out}(t) = 2\pi\int\left(f_{free} + K_{vco}\cdot V_{ctrl}\right) dt$$

$$\frac{\Phi_{out}(s)}{V_{ctrl}(s)} = \frac{2\pi K_{vco}}{s}$$

$[K_{vco}] = Hz/V \longrightarrow 2\pi K_{vco}$

$[K_{vco}] = \frac{rad/s}{V} \longrightarrow K_{vco}$



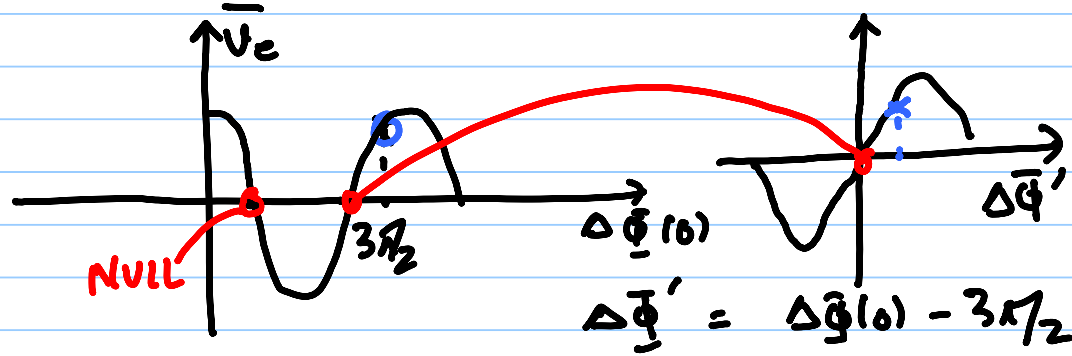
$v_{in} = 1 \cdot \sin(\omega_{in} t)$

$v_{out} = 1 \cdot \sin(\omega_{in} t + \Delta\phi(0))$

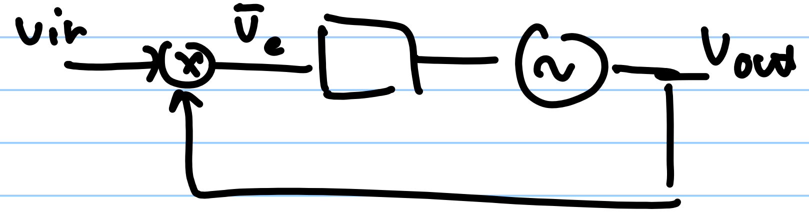
$v_e = v_{in} \times v_{out}$

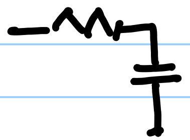
$= \frac{-1}{2}\left[\cos(2\omega_{in} t + \Delta\phi(0)) - \cos(\Delta\phi(0))\right]$

$\bar{v}_e = \frac{1}{2}\cos(\Delta\phi(0))$



$\Delta\phi' = \Delta\phi(0) - 3\pi/2$

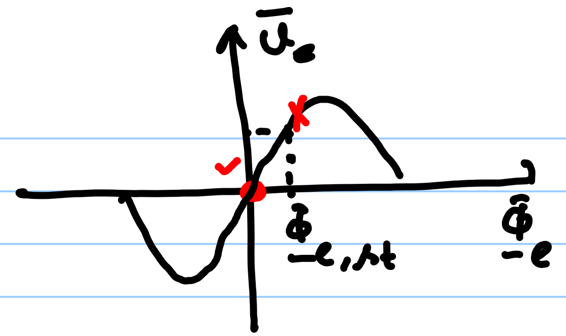




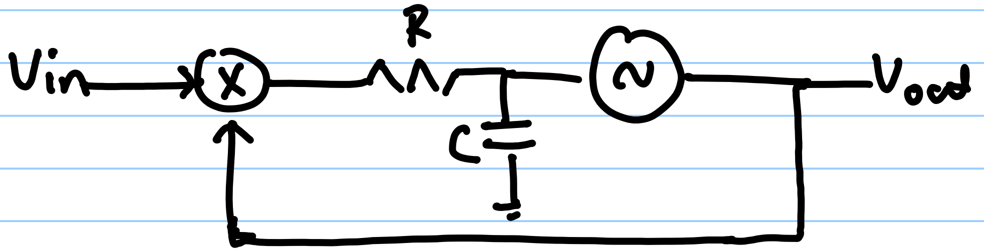
* V_{in} & V_{out} w/ $\omega_{in} = \omega_{out}$
locks to phase offset $\pi/2$, $\bar{v}_e = 0$

at $t=0$, $\omega_{in} - \omega_{out} = \Delta\omega$

$$\sin(\Phi_e) = \frac{\Delta\omega}{\underbrace{2\pi K_{VCO} \cdot K_{pd}}_{K}} \leq 1$$

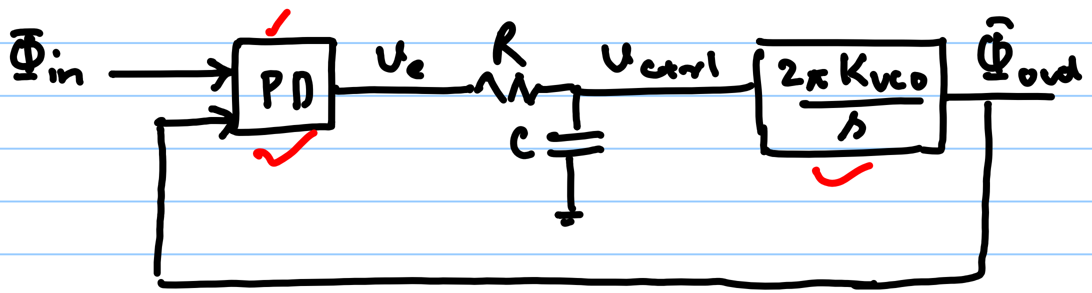


In steady state $\frac{d\Phi_e}{dt} = 0$ where $\Phi_e = \Phi_{in} - \hat{\Phi}_{out}$



In case $\frac{\Delta\omega}{K} > 1$

PLL willn't lock in freq./phase



$$K_{pd} = \frac{d\bar{v}_e}{d\Phi_e} = \frac{1}{2}\cos(\Phi_e) \checkmark = \frac{1}{2}$$

$v_{in} = \sin(\omega_{in} t)$

$V_{out} = \cos(\omega_{in} t)$

$[K_{pd}] = V/rad$

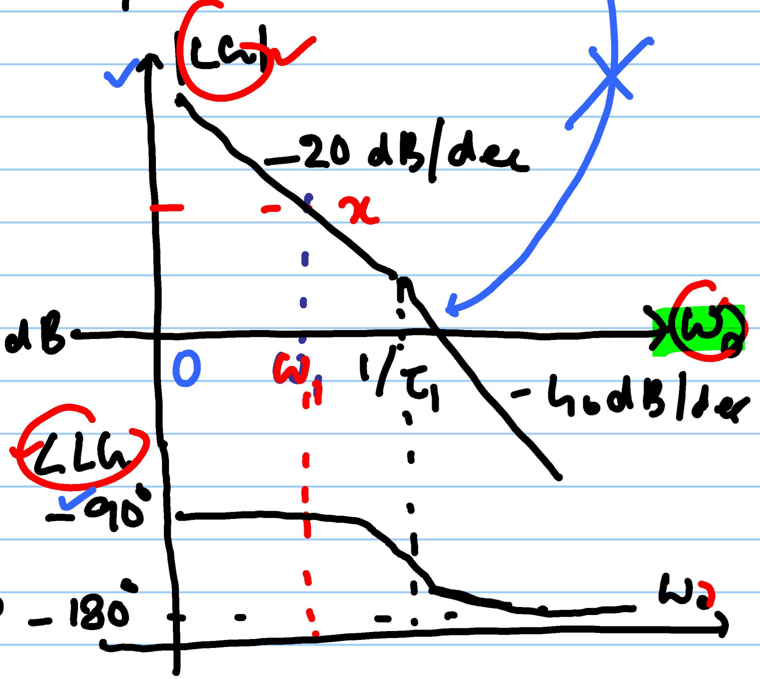
$[F(s)] = V/V$ $\qquad F(s) = \frac{1/sC}{R + 1/sC} = \frac{1}{1+sRC} = \frac{1}{1+s\tau_1}$

$[K_{vco}] = Hz/V$

$V_{out} = A_{out} \sin(\omega_{out} t + \Delta\phi)$

$LG(s) = K_{pd} F(s) \frac{2\pi K_{VCO}}{s} = \frac{\Phi_{out}(s)}{\Phi_{in}(s)} \Big|_{open\ loop}$

$LG(s) = \frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{1}{2} \frac{1}{1+s\tau_1} \frac{2\pi K_{vco}}{s}$



Φ_{in}, Φ_{out} : $V_{in}, V_{out}, V_e, V_{ctrl}$ signal

ω_0 :

$\Phi_{out} = \omega_{out} t + \Delta\Phi$

$$\frac{\Phi_{out}}{\Phi_{in}} = \frac{LG}{1+LG} = \frac{\kappa K_{VCO}}{\kappa K_{VCO} + s(1+s\tau_1)}$$

$$\Phi_{er} = \Phi_{in} - \Phi_{out} = \Phi_{in} - \frac{LG}{1+LG}\Phi_{in} = \frac{1}{1+LG}\Phi_{in}$$

$$t=0, \quad \Phi_{in}(t) = \Delta\Phi(0)\, u(t)$$

$$\lim_{t\to\infty} \Phi_{er}(t) = \lim_{s\to 0} s \cdot \frac{1}{1+LG}\Phi_{in}(s)$$

$$= \lim_{s\to 0} \cancel{s} \cdot \frac{\Delta\Phi_{in}(0)}{\cancel{s}} \frac{1}{1+\frac{\kappa K_{VCO}}{s(1+s\tau_1)}}$$

$$= 0$$

$t=0$; $\quad f_{in}(t) = \Delta f(0)\, u(t) \Rightarrow \hat{\Phi}_{in}(t) = 2\pi t .\ \Delta f(0) . u(t)$

$$\hat{\Phi}_{in}(s) = \frac{2\pi \Delta f(0)}{s^2}$$

$$\lim_{t\to\infty} \Phi_{er}(t) = \lim_{s\to 0} \cancel{s} . \frac{2\pi\Delta f(0)}{s^2} \frac{1}{1+\frac{\pi K_{vco}}{s(1+s\tau_1)}}$$

$$= \lim_{s\to 0} \frac{2\pi \Delta f(0)}{s + \frac{\pi K_{vco}}{\cancel{1+s\tau_1}}}$$

$$= \frac{2\cancel{\pi}\Delta f(0)}{\cancel{\pi} K_{vco}} = \frac{\Delta f(0)}{(K_{vco} . \underbrace{K_{pd}}_{=1/2})}$$

$\frac{1}{s^3}$

$\frac{1}{s}$

∞

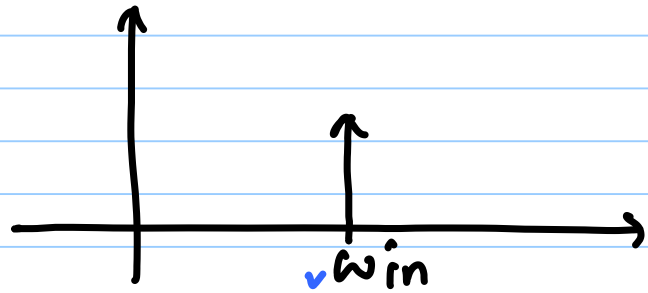
at $t=0$,

$$f_{in}(t) = t \cdot \Delta f(0)\, u(t)$$

$$\Phi_{in}(t) = \frac{2\pi t^2}{2} \Delta f(0)\, u(t)$$

$$\Phi_{in}(s) = \frac{2\pi}{2} \frac{\Delta f(0)}{s^3}$$

$$\frac{\Delta\omega \uparrow}{\underbrace{K\uparrow}} = \underbrace{\sin(\Phi_e)} \leq 1$$



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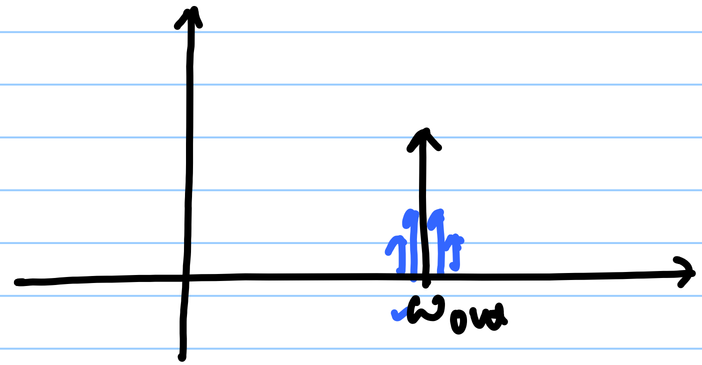

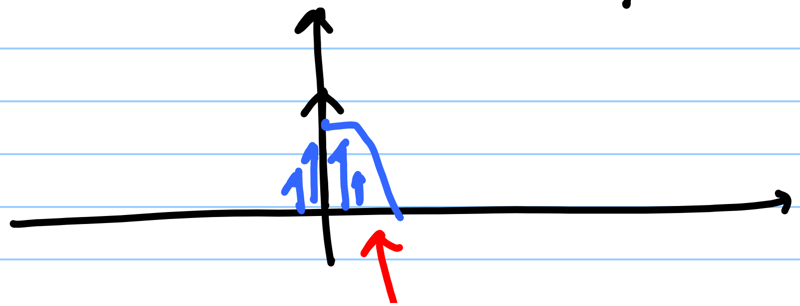

$$v_{out} = A_{out} \sin\left(\int (\omega_0 + K_{VCO} \cdot \cancel{v_{ctrl}})\, dt\right)$$

$$a_c \sin(\omega_1 t)$$

$(\omega_0 + \omega_1)$, $(\omega_0 - \omega_1)$