Note Title 03-08-2017

Lecture #4

$v_{out}(t) = A \sin\left(2\pi \int (f_0 + K_{vco} \cdot V_{ctrl})\, dt\right)$

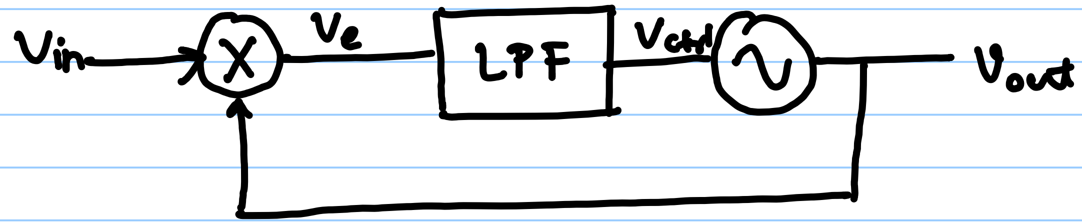
V_{ctrl}, V_{out} : Non-linear

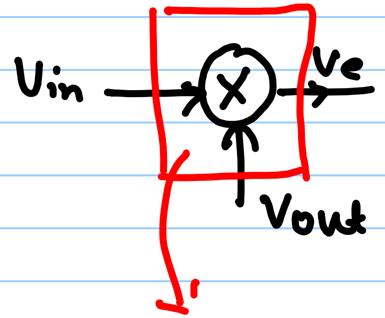
f_{out}, V_{ctrl} : $f_{out} = f_0 + K_{vco} \cdot V_{ctrl}$

$[K_{vco}] = Hz/V$

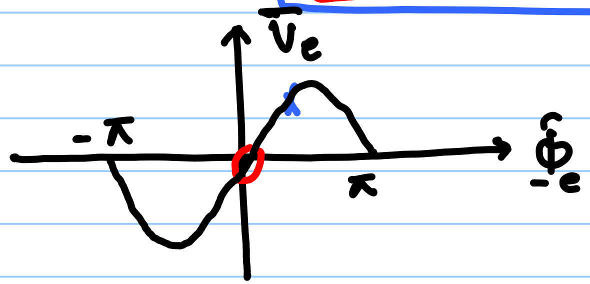
$\omega_{out} = 2\pi \left(f_0 + K_{vco} \cdot V_{ctrl}\right)$

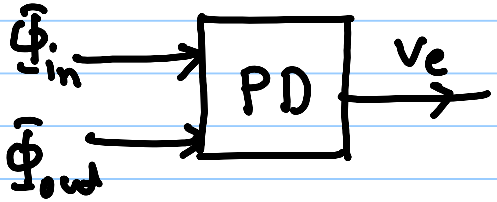
$\Phi_{out} = \int 2\pi \left(f_0 + K_{vco} \cdot V_{ctrl}\right) dt$



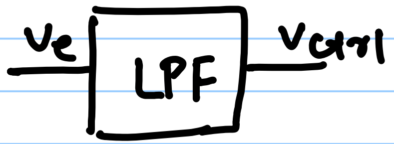


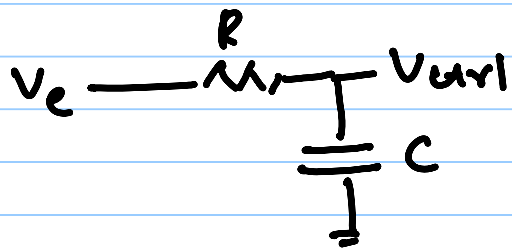
$$\boxed{V_e = \frac{1}{2}\sin(\Delta\omega \cdot t + \hat{\Phi}_e)} + \sin(\overline{\omega_{in}+\omega_{out}}\,t + \hat{\Phi}(0))$$

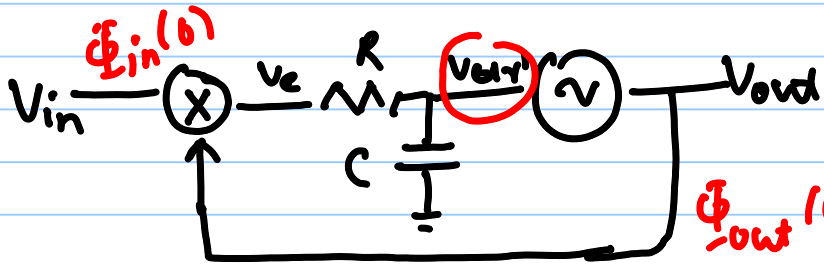




$$K_{pd}\Big|_{\hat{\Phi}_e=0} = \frac{1}{2}\cos(\hat{\Phi}_e)\Big|_{\hat{\Phi}_0=0} = \frac{1}{2}$$







$t=0$ $\hat{\Phi}_{in}(0) \neq \hat{\Phi}_{out}(0)$

$\hat{\Phi}_{out}(0)$; $t=0$ $t \to \infty$ $\hat{\Phi}_{in}(t) = \hat{\Phi}_{out}(t)$

$$V_{in} = 1 \cdot \sin(2\pi \, 100\,\text{MHz} \cdot t)$$

$$V_{out} = 1 \cdot \underset{\sin}{\cos}\left(2\pi \, \underbrace{100\,\text{MHz}} \cdot t + \Delta\underline{\phi}(0) + 2\pi \int K_{VCO} \cdot V_{ctrl} \, dt\right)$$

$$V_e = V_{in} \times V_{out} = \frac{1}{2}\left[\sin\left(2\pi \cdot 200\,\text{MHz} \cdot t + \overset{\Delta\underline{\phi}(0)}{\Delta\underline{\phi}(0)}\right) + \underbrace{\sin(\Delta\underline{\phi}(0))}\right]$$

$$V_{ctrl} = \frac{1}{2}\sin(\Delta\underline{\phi}(0))$$

$$V_{out} = \cos\left(2\pi \cdot 100\,\text{MHz} + \underbrace{\Delta\underline{\phi}(0) + 2\pi \int K_{VCO} \cdot \frac{1}{2}\sin(\Delta\underline{\phi}(0)) \, dt}\right)$$

$$\Delta\underline{\phi}(0) + 2\pi \times K_{pd} \times K_{VCO} \int \sin(\Delta\underline{\phi}(0)) \, dt$$

Sol: $\Delta\underline{\phi}(0) = 0 \implies V_e = 0 \implies V_{ctrl} = 0$

V_{out} locks to V_{in} w/ phase offset $\pi/2$

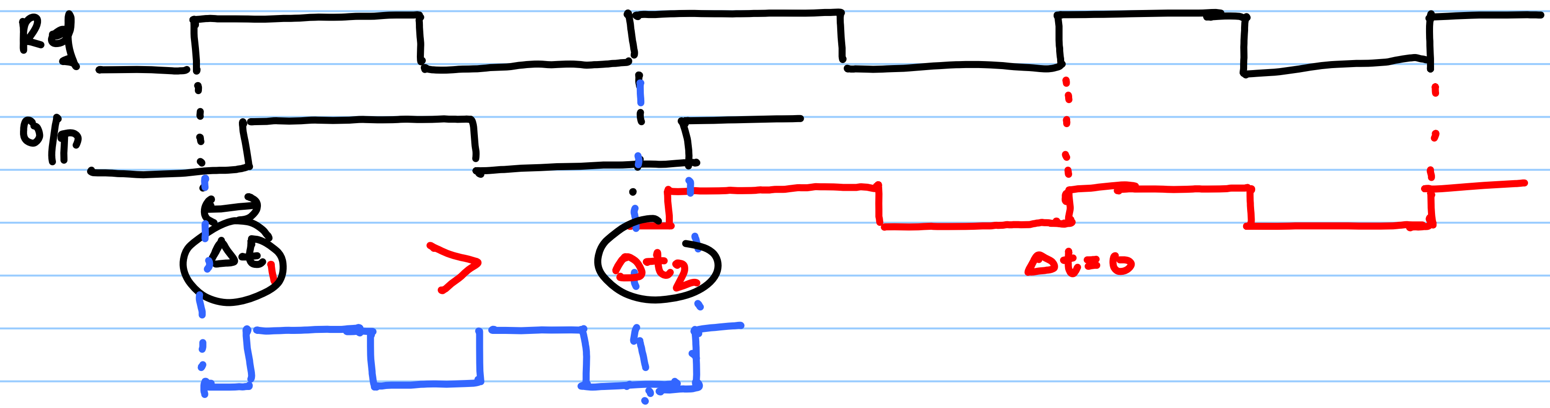
* V_{in}, V_{out} in above PLL locks to zero steady state phase error provided ref. & free runing clock freq. are same.

$$V_{in} = \sin(\omega_0 t)$$

$$V_{out} = \sin(\omega_0 t + \underbrace{\Delta\phi(0)})$$

at $t=0$

$$t=0 \begin{cases} W_{in} = 100\text{ MHz}, & \phi_{in}(0) = 0 \\ W_{out} = 100\text{ MHz}, & \phi_{out}(0) = \pi/3 \end{cases}$$

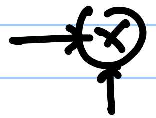


Not using $\otimes$ as PD for square wave

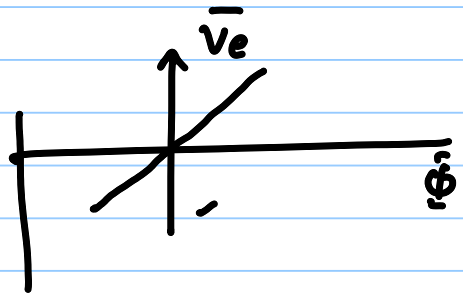
$\underline{v_{in}(t),\ v_{out}(t)}$

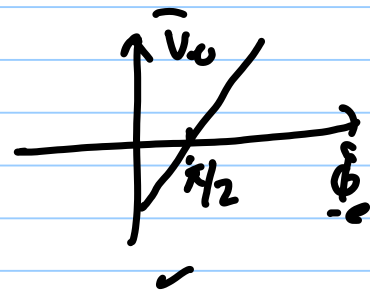
$\lim_{t\to\infty} \underline{\phi}_e(t) = 0 \quad \Rightarrow v_e = \sin(\underline{\phi}_e)$

$\sin(\omega_1 t) \cdot \sin(\omega_1 t + \underline{\phi}) = \frac{1}{2}\left(\underset{\omega}{1} + \cos(2\omega_1 t)\right)$



$\left.\begin{array}{l}\sin(\omega_1 t)\\ \cos(\omega_1 t)\end{array}\right\} \quad v_e = 0$





$$t=0 \begin{cases} v_{in} = \sin(2\pi \cdot \underbrace{100}_{\omega_0} \text{ MHz } t) \\ v_{out} = \cos(2\pi\ 99 \text{ MHz } t + \Delta\underline{\phi}(0)) \end{cases} \qquad \begin{array}{l} \omega_0 = 100 \text{ MHz} \\ \Delta\omega = 1 \text{ MHz} \end{array}$$

$$v_e = \frac{1}{2}\Big[\sin(2\pi \cdot 2\omega_0 \cdot t - 2\pi \cdot \Delta\omega \cdot t + \Delta\underline{\phi}(0)) + \sin(2\pi \cdot \Delta\omega \cdot t - \Delta\underline{\phi}(0))\Big]$$

$$V_{ctrl} = \frac{1}{2} \sin\left(2\pi \cdot \Delta\omega \cdot t - \Delta\underline{\Phi}(0)\right)$$

$$V_{out} = \cos\left(2\pi \cdot \omega_0 \cdot t - 2\pi \cdot \Delta\omega \cdot t + 2\pi \int K_{vco} \cdot V_{ctrl} \cdot dt + \Delta\underline{\Phi}(0)\right)$$

$$\underline{\hat{\Phi}}_{in}(t) = 2\pi \cdot 100\ \text{MHz} \cdot t$$

$$\underline{\hat{\Phi}}_{out}(t) = 2\pi(\omega_0 - \Delta\omega)t + 2\pi \int K_{vco} \cdot V_{ctrl} \cdot dt + \Delta\underline{\hat{\Phi}}(0)$$

$$\underline{\hat{\Phi}}_e(t) = \underline{\hat{\Phi}}_{in} - \underline{\hat{\Phi}}_{out}$$

$$\underline{\hat{\Phi}}_e(t) = 2\pi \cdot \Delta\omega \cdot t - 2\pi \int K_{vco} \cdot V_{ctrl} \cdot dt - \Delta\underline{\hat{\Phi}}(0)$$

$$\underline{\hat{\Phi}}_e(t) = 2\pi \cdot \Delta\omega \cdot t - 2\pi K_{vco} \int K_{pd} \underbrace{\sin\left(2\pi \cdot \Delta\omega \cdot t - \Delta\underline{\hat{\Phi}}(0)\right)}_{\sin(\underline{\hat{\Phi}}_e(t \to \infty))} dt - \Delta\underline{\hat{\Phi}}(0)$$

$$2\pi K_{vco} \cdot K_{pd} \int \sin(2\pi \cdot \Delta\omega \cdot t - \Delta\phi(0))\, dt = 2\pi \cdot \Delta\omega \cdot t - \Delta\phi(0) - \hat{\phi}_e(t)$$

$$\frac{d\hat{\phi}_e(t)}{dt} = 2\pi \cdot \Delta\omega - 2\pi K_{vco} K_{pd} \sin(2\pi \cdot \Delta\omega \cdot t - \Delta\phi(0))$$

$$2\pi \cdot \Delta\omega - 2\pi K_{vco} \cdot K_{pd} \sin(\phi_e(t \to \infty)) = 0$$

$$\Rightarrow \boxed{\sin(\hat{\phi}_e(t \to \infty)) = \frac{\Delta\omega}{K_{vco} K_{pd}}} \leq 1$$

$$\Rightarrow \Delta\omega \leq K_{vco} K_{pd}$$