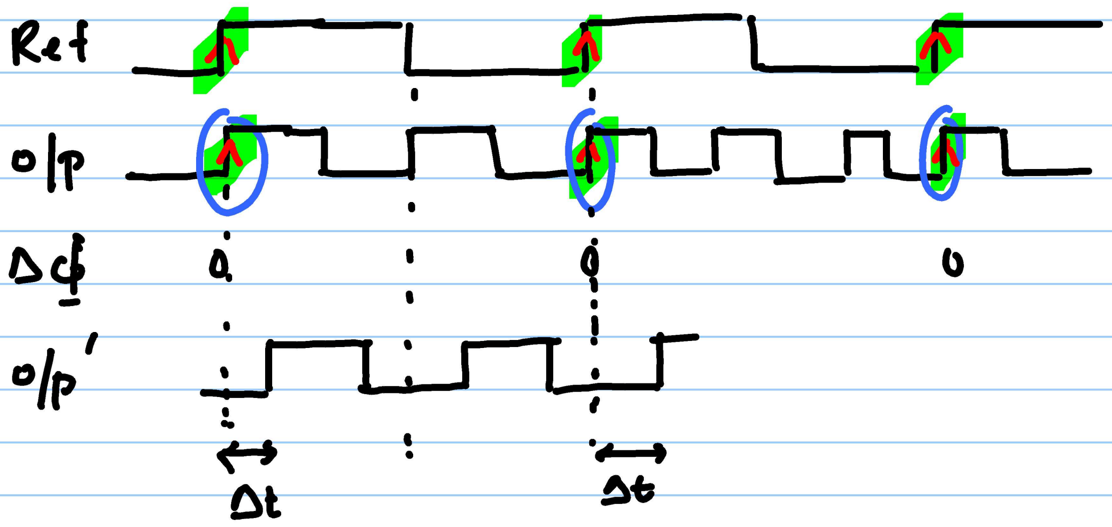
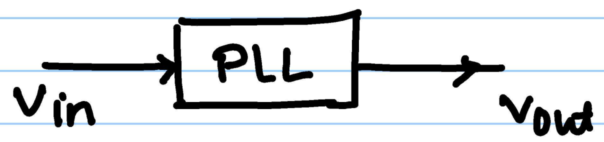
Note Title 02-08-2017

Lecture #3



Basics of PLL



$V_{in} = A_{in} \sin(\omega_{in} t + \Phi_{in}(0))$ ✓

$V_{out} = A_{out} \sin(\omega_{out} t + \Phi_{out}(0))$ ✓

$\Phi_{in} = \omega_{in} t + \Phi_{in}(0)$

$\Phi_{out}(t) = \omega_{out} t + \Phi_{out}(0)$

$$f_{in} = \frac{1}{2\pi}\frac{d\hat{\Phi}_{in}}{dt}, \quad f_{out} = \frac{1}{2\pi}\frac{d\hat{\Phi}_{out}}{dt}$$

PLL: $\frac{f_{out}}{f_{in}} = N$ (constant) frequency locking.

$$\frac{d(\hat{\Phi}_{in} - \hat{\Phi}_{out})}{dt} = 0 \quad \text{(phase locked)}$$

PLL doesn't enforce any relationship b/w A_{in} & A_{out}

$$V_{in} = A_{in} \sin(\omega_{in} t + \hat{\Phi}_{in}(0))$$

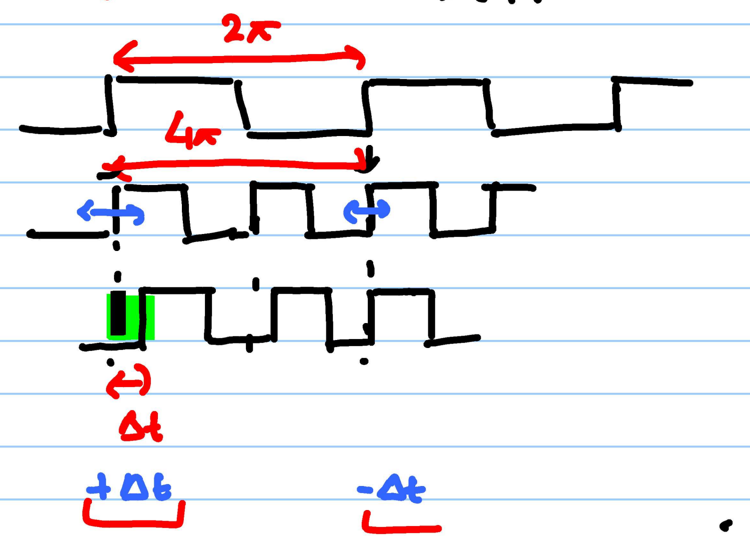
$$V_{out} = A_{out} \sin((\omega_{in} + \Delta\omega)t + \hat{\Phi}_{out}(0))$$

✓ $$\hat{\Phi}_{out} - \hat{\Phi}_{in} = (\Delta\omega)t + \underbrace{\hat{\Phi}_{out}(0) - \hat{\Phi}_{in}(0)}_{\hat{\Phi}_{err}(0)}$$

$$\hat{\Phi}_{err}(t) = (\Delta\omega)t + \hat{\Phi}_{err}(0)$$

if $\Delta\omega = 0 \Rightarrow \hat{\Phi}_{err}(t) = \hat{\Phi}_{err}(0) = 0$

Ex:



$V_{in} = A\sin(\omega_{in} t)$

$V_{out} = A\sin(\omega_{out} t)$

$\omega_{out} = \omega_{in} \Rightarrow \frac{d(\hat{\Phi}_{out} - \hat{\Phi}_{in})}{dt} = 0$

$\hat{\Phi}_{out} = \hat{\Phi}_{in}$

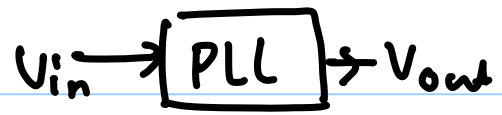
$\omega_{out} = N\omega_{in}$

$\hat{\Phi}_{out} = N\hat{\Phi}_{in}$

$\hat{\Phi}_{err}(t) = (\Delta\omega)t + \hat{\Phi}_{err}(0)$, $\hat{\Phi}_{err}(0) = 0$

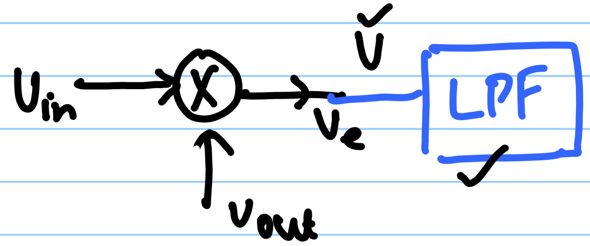
$\hat{\Phi}_{err}(t) = (\Delta\omega)t$

$\frac{\hat{\Phi}_{err}(t)}{t} = \Delta\omega$ freq. error



$$V_{in} = A_{in} \sin(\omega_{in} t + \Phi_{in}(0))$$

$$V_{out} = A_{out} \sin(\omega_{out} t + \Phi_{out}(0))$$



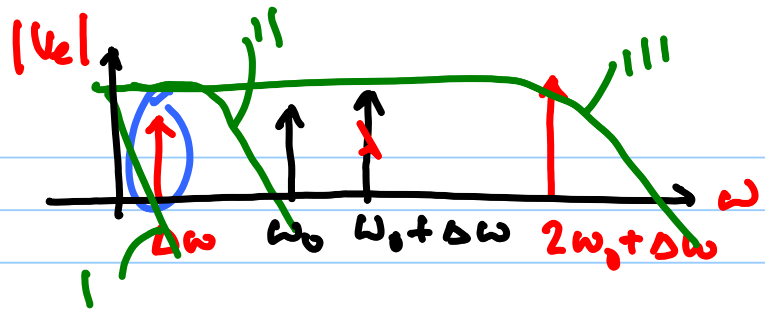
$$V_e = V_{in} V_{out}$$

$$= \frac{1}{2}\left[\cos\left(\overline{\omega_{in} + \omega_{out}}\, t + \Phi_{in}(0) + \Phi_{out}(0)\right) + \cos\left(\overline{\omega_{in} - \omega_{out}}\, t + \Phi_{in}(0) - \Phi_{out}(0)\right)\right]$$

$\times A_{in} A_{out}$

$\omega_{in} = \omega_0 + \Delta\omega, \quad \omega_{out} = \omega_0$

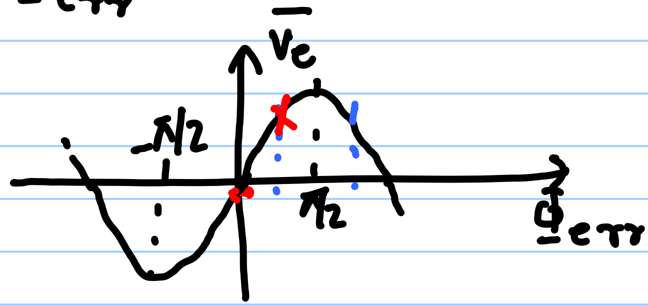
$$\Rightarrow V_e = \frac{1}{2}\left[\cos\left(\overline{2\omega_0 + \Delta\omega}\, t + \Phi_{in}(0) + \Phi_{out}(0)\right) + \cos\left(\Delta\omega . t + \Phi_{err}(0)\right)\right]$$



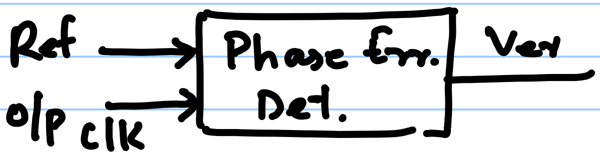
if $\Delta\omega = 0$ $\Rightarrow$ $V_2 = \frac{1}{2}\left[\cos\left(2\omega_0 t + \phi_{in}(0) + \phi_{out}(0)\right) + \cos\left(\phi_{err}(0)\right)\right]$

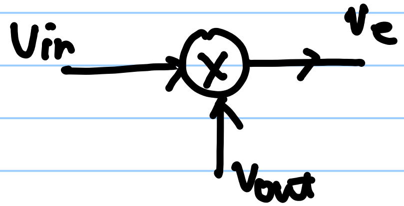
$\bar{V_e} = \frac{1}{2}\cos\left(\phi_{err}(0)\right)$

$\bar{V_e} = \frac{1}{2}\sin\left(\phi_{err}\right)$



Range $= \pm\pi/2$

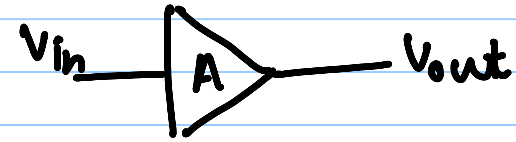




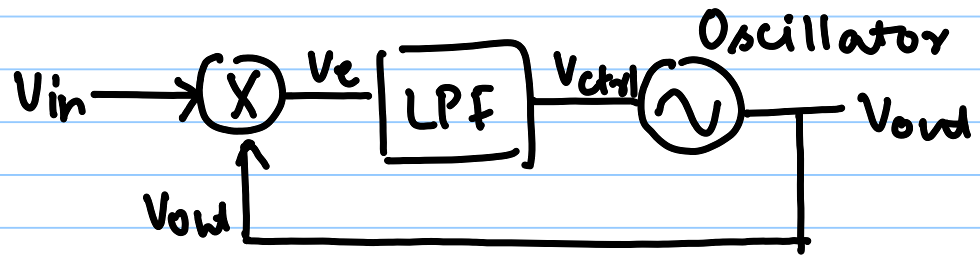
$K_{pd} = \frac{d\bar{V_e}}{d\phi_e} = \frac{1}{2}\cos\left(\phi_e\right)$ Ain Aout

$$K_{pd}\big|_{\phi_e=0} = \frac{1}{2} A_{in} A_{out}$$

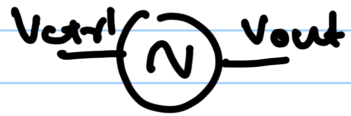
$$K_{pd}\big|_{\phi_e=\pi/2} = \frac{1}{2\sqrt{2}} A_{in} A_{out}$$



$V_{out} = A V_{in}$



$f_{out} = K_{vco} \cdot V_{in}$



$$V_{out} = A_{out} \sin\left(2\pi \int (f_0 + K_{vco} \cdot V_{ctrl})\, dt\right)$$

A_{out} : Amp.

f_0 : free-running freq. of osc.

K_{vco} : freq. gain [Hz/V]