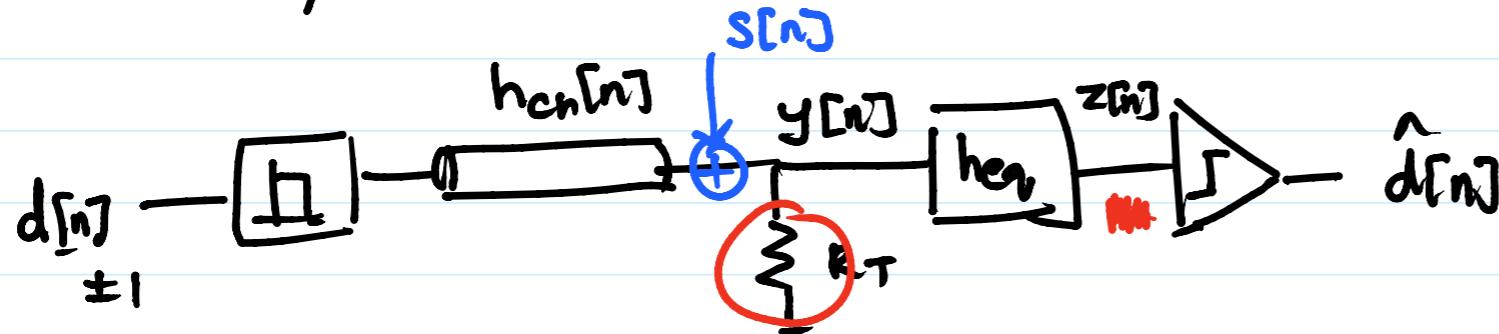


Lecture - 42

Equalization on Rx side

1. FFE
2. Passive high pass filter based eq.
3. CTLE, CTLE + TIA



$$h_{eq}[n] = \delta(n) - \alpha \delta(n-1)$$

$$s[n] * h_{eq}[n] = s[n] - \alpha s[n-1]$$

$$\left. \begin{array}{l} \text{Eq: } h_{ch}[z] = 1 + \alpha z^{-1} \\ h_{eq}[z] = 1 - \alpha z^{-1} \end{array} \right\} \quad \begin{array}{l} h_{ch}[z] h_{eq}[n] = 1 - \alpha^2 z^{-2} \\ s[n] \sim N(0, \sigma) \end{array}$$

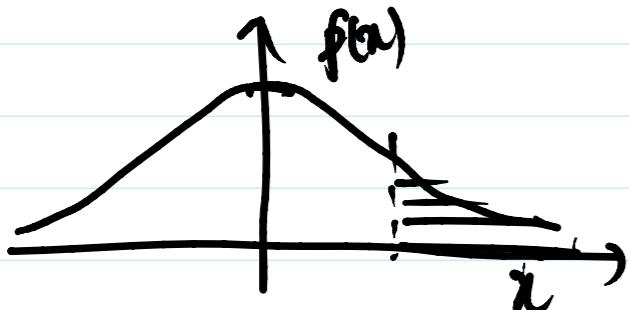
$$z[n] = h_{ch}[n] * h_{eq}[n]$$

$$= \delta(n) - \alpha^2 \delta(n-2) + (\underbrace{\delta(n) - \alpha \delta(n-1)}_{n'[n]})$$

$$\text{i/p of eq. } (y[n] + s[n]) * h_{eq}[n] \quad n'[n] \sim N(0, \sigma \sqrt{1+\alpha^2})$$

$$\text{i/p of comp, } z[n] = d[n] - \alpha^2 d[n-2] + n'[n]$$

$$P(\hat{d}[n] \neq d[n]) = P(z[n] < 0) \times P(d[n]=1) + P(z[n] > 0) \times P(d[n]=-1)$$



$$P(x > x) = Q(x)$$

$$x \sim N(0, 1)$$

$$\begin{aligned}
&= \frac{1}{2} \left[P(z[n] < 0) + P(z[n] > 0) \right] \\
&\quad d[n] = 1 \qquad \qquad \qquad d[n] = -1 \\
&= \frac{1}{2} \left[\frac{1}{2} P(-1 - \alpha^2 + s'[n] < 0) + \frac{1}{2} P(1 + \alpha^2 + s'[n] < 0) \right. \\
&\quad \qquad \qquad \qquad d[n-2] = 1 \qquad \qquad \qquad d[n-2] = -1 \\
&\quad \left. + \frac{1}{2} P(-1 - \alpha^2 + s'[n] > 0) + \frac{1}{2} P(1 + \alpha^2 + s'[n] > 0) \right] \\
&= \frac{1}{4} \left[P(s'[n] < -1 + \alpha^2) + P(s'[n] < -(1 + \alpha^2)) \right. \\
&\quad \qquad \qquad \qquad \text{underlined} \\
&\quad \left. + P(s'[n] > 1 + \alpha^2) + P(s'[n] > -1 - \alpha^2) \right] \\
&= \frac{1}{2} \left[P(s'[n] > 1 - \alpha^2) + P(s'[n] > 1 + \alpha^2) \right] \\
&= \frac{1}{2} \left[Q\left(\frac{1 - \alpha^2}{\sigma_n \sqrt{1 + \alpha^2}}\right) + Q\left(\frac{1 + \alpha^2}{\sigma_n \sqrt{1 + \alpha^2}}\right) \right]
\end{aligned}$$

\Rightarrow Linear eq. is amplifying the noise.

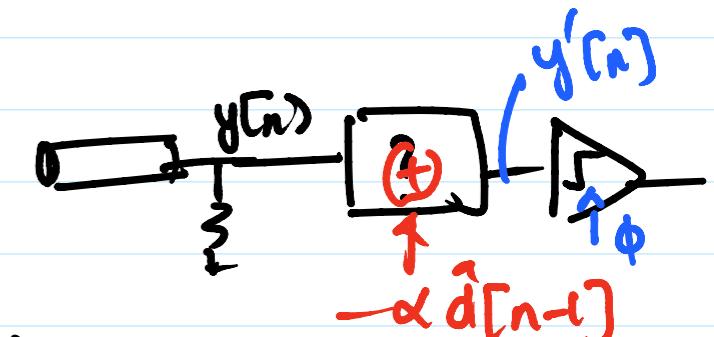
W/P to the eq. $y[n] = d[n] + \alpha d[n-1] + s[n]$

$- \alpha \hat{d}[n-1]$

$$y[n] = d[n] + \alpha(d[n-1] - \hat{d}[n-1]) + s[n]$$

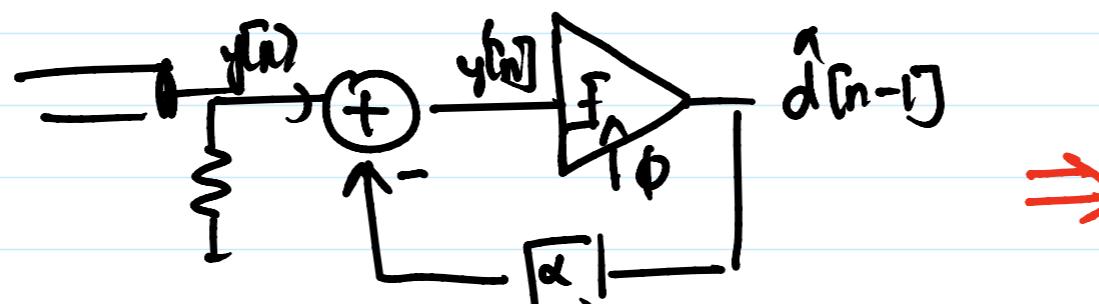
If $\hat{d}[n-1] = d[n-1] \Rightarrow y[n] = d[n] + s[n]$

If $y'[n]$ is i/p to comp. $\Rightarrow P(\hat{d}[n] \neq d[n]) = \frac{1}{2} P(1 + s[n] < 0)$

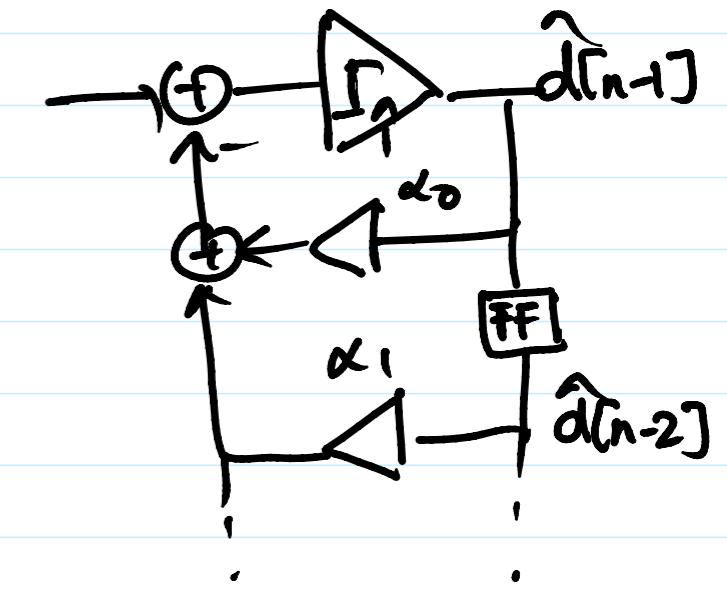


$$\begin{aligned}
 \text{if } y[n] \text{ is i/p to comp.} \Rightarrow P(\hat{d}[n] \neq d[n]) &= \frac{1}{2} P(i + s[n] < 0) \\
 &\quad + \frac{1}{2} P(-1 + s[n] > 0) \\
 &= P(s[n] > 1) \\
 &= Q\left(\frac{1}{\sigma_n}\right)
 \end{aligned}$$

- * No noise amplification
- * Previous bit is known correctly.
- * Can't cancel pre-cursor ISI



$$h_{ch}[n] = \delta(n) + \alpha_0 \delta(n-1) + \alpha_1 \delta(n-2) + \alpha_2 \delta(n-3)$$



$$\begin{aligned}
 \underbrace{P(\hat{d}[n] \neq d[n])}_{\text{red}} &= P(\hat{d}[n-1] = d[n-1]) P(\hat{d}[n] \neq d[n] | \hat{d}[n-1] = d[n-1]) \\
 &\quad + P(\hat{d}[n-1] \neq d[n-1]) P(\hat{d}[n] \neq d[n] | \hat{d}[n-1] \neq d[n-1])
 \end{aligned}$$

$$P(\hat{d}[n-1] \neq d[n-1]) = P(\hat{d}[n] \neq d[n]) = p$$

$$p = (1-p) P_{DFE \mid \text{correct } d[n-1]} + p \times P_{DFE \mid \text{incorrect } d[n-1]}$$

$$P = \frac{P_{DFE \mid \text{correct } d[n-1]}}{\text{underline}}$$

$$1 + P_{DFE} | \text{correct } d[n-1] - P_{DFE} | \text{incorrect } d[n-1]$$

$$P_{DFE} | \text{correct } d[n-1] = Q\left(\frac{1}{\sigma_n}\right)$$

$$y[n] = d[n] + s[n] = \frac{1}{2} \left[Q\left(\frac{1-2\alpha}{\sigma_n}\right) + Q\left(\frac{1+2\alpha}{\sigma_n}\right) \right] \Big|_{\alpha=1/2} = \frac{1}{2} [Q(0) + Q(\infty)]$$

$P_{DFE} | \text{incorrect } d[n-1]$

$$\begin{aligned} y[n] &= d[n] + \alpha(d[n-1] - \hat{d}[n-1]) + s[n] \\ &\approx d[n] + 2\alpha d[n-1] \\ &\quad - 1 + 2\alpha + s[n] \end{aligned}$$

$$y[n] = d[n] + \alpha(d[n-1] - \hat{d}[n-1])$$

$$\begin{array}{ccc} d[n-1] & d[n] & y[n] \\ 1 & 1 & \\ -1 & 1 & \end{array}$$

