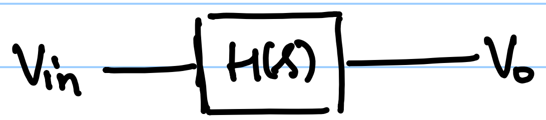
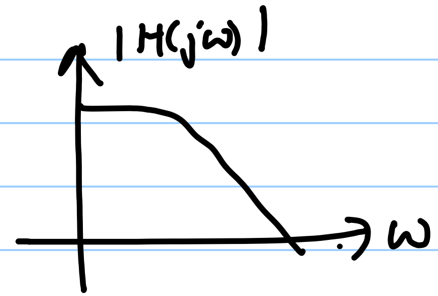
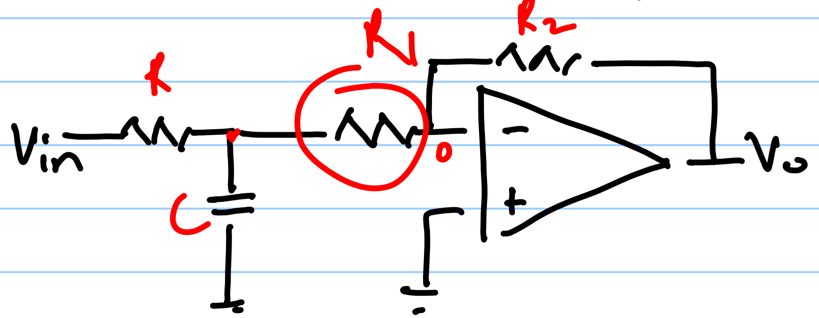
Lecture # 29

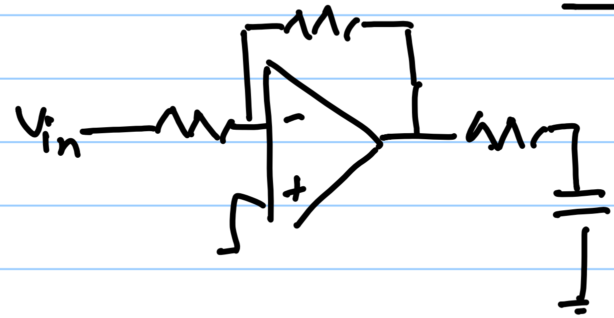
Low Pass Filter



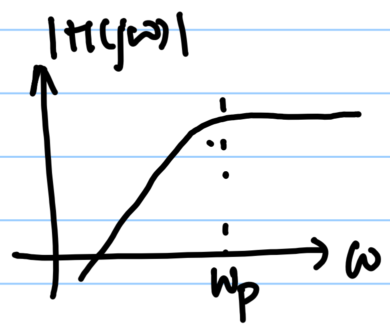
$$\frac{V_o}{V_{in}} = H(s) = \frac{1}{1+s/\omega_p} = \frac{A_o}{1+sRC}$$





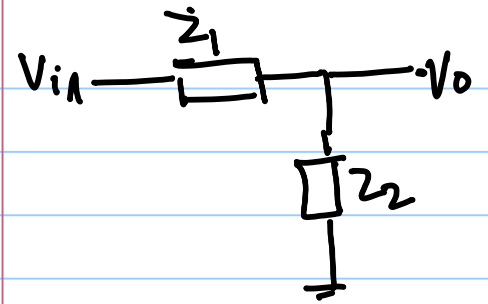


High Pass Filter (HPF)



$$H(s) = \frac{(s/\omega_p)}{1+s/\omega_p} = \frac{s}{s+\omega_p}$$

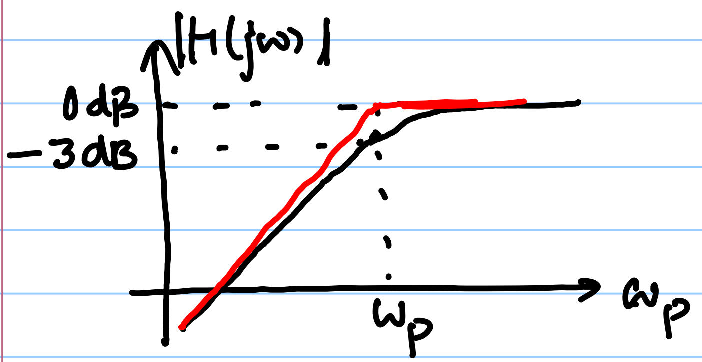
$$\frac{s}{s+\omega_p} \approx \frac{s}{s}$$



$$\frac{V_o}{V_{in}} = \frac{Z_2}{Z_2+Z_1} = \frac{1}{1+\frac{Z_1}{Z_2}} = \frac{s}{s+\omega_p}$$

$$= \frac{1}{1+\frac{\omega_p}{s}}$$

Z_1	R	$1/sC$
Z_2	sL	$\overset{?}{R}$



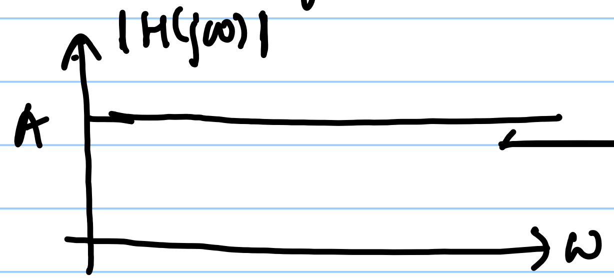
$$H(s) = \frac{s}{s+\omega_p}$$

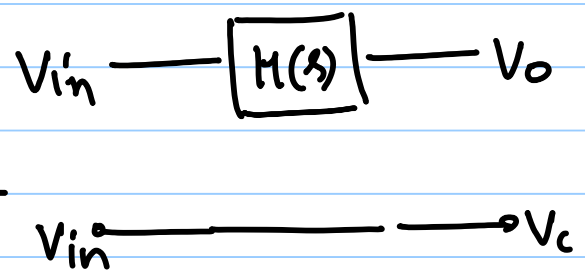
$$|H(j\omega)| = \frac{j\omega}{j\omega+\omega_p}$$

$$20\log_{10}(|H(j\omega)|) = 20\log_{10}\left(\frac{1}{\sqrt{2}}\right)$$

$$\measuredangle H(j\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$

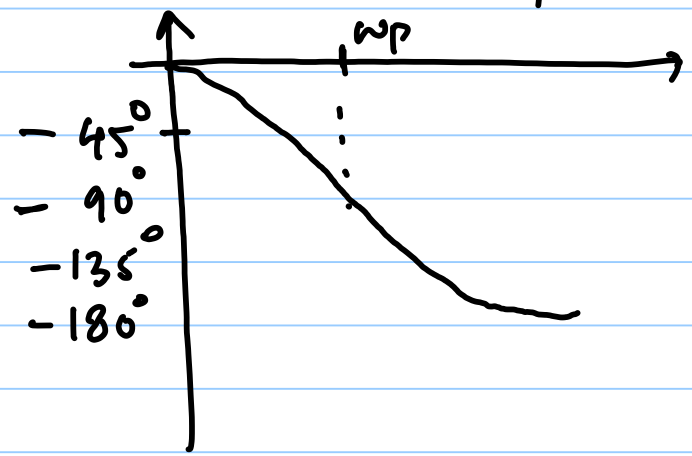
All-pass transfer func

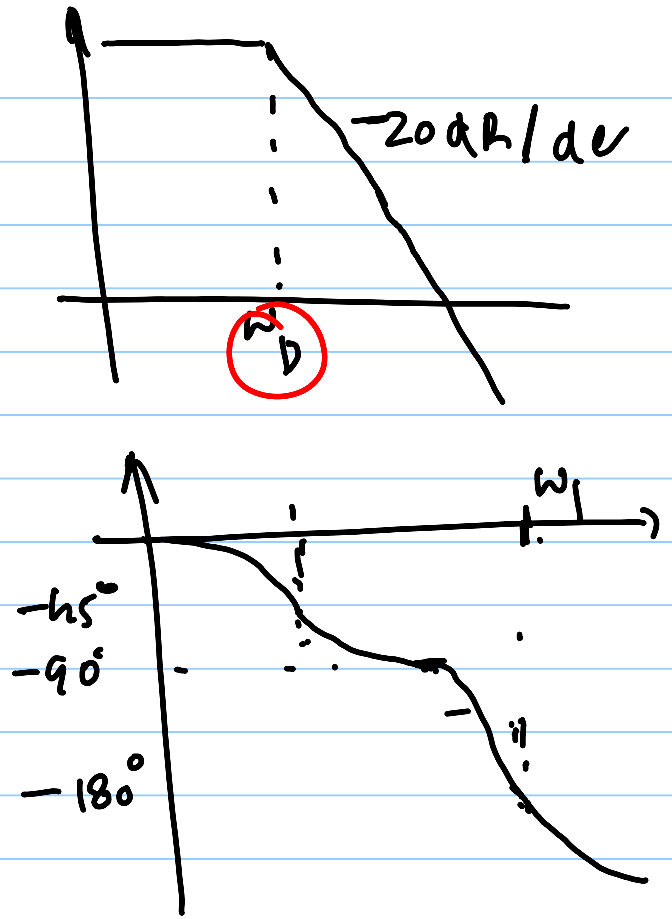




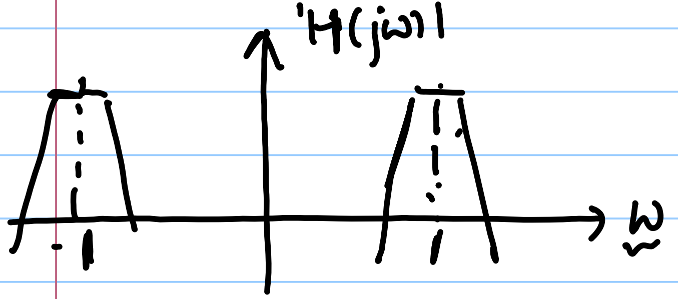
$$H(s) = \frac{1 + s/\omega_z}{1 + s/\omega_p} \quad \text{where} \quad \omega_p = \omega_z \qquad \measuredangle H(j\omega) = 0$$

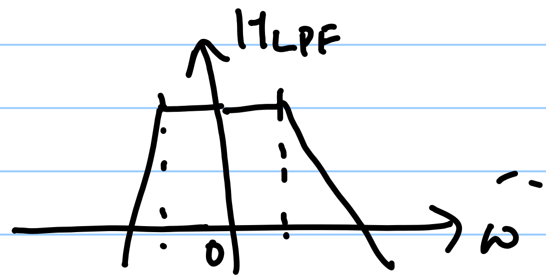
$$H(s) = \frac{1 - s/\omega_z}{1 + s/\omega_p} \quad \text{where} \quad \omega_p = \omega_z \qquad \measuredangle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega}{\omega_p}\right) = -2\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$




-20dB/dec
ω_D
ω_1
-45°
-90°
-180°

Band Pass Filter





$$H_{LPF} = \frac{1}{s+1} \xrightarrow{s=j\omega}$$

$|H(j\omega)|$

$$\begin{cases} \omega = 0 \\ \omega = \pm\infty \end{cases}$$

$s = j\omega = j \cdot 0$

$s = j\omega = \pm j\infty$

$$\left.\begin{aligned} \omega &= \pm 1 \\ \omega &= \pm\infty,\ 0 \end{aligned}\right\}$$

$$s + \frac{1}{s} = s' \xleftarrow{s'} s = j\omega$$

$$s + \frac{1}{s} = j\omega + \frac{1}{j\omega} = \frac{1-\omega^2}{j\omega}$$

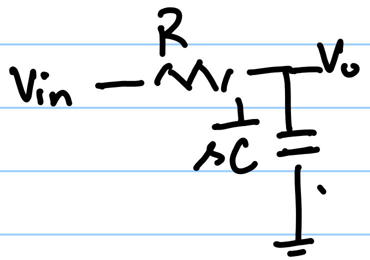
$$H(s) = \frac{1}{\frac{s}{\omega_p}+1} \xrightarrow{s \to s+\frac{1}{s}} H(s) = \frac{1}{\frac{1}{\omega_p}\left(s+\frac{1}{s}\right)+1}$$

$$= \frac{s\omega_p}{s^2+s\omega_p+1}$$

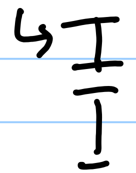
$$s \to Q\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right) \checkmark$$

$$\frac{1}{1+s/\omega_p} \to \frac{1}{\frac{1}{\omega_p}Q\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)+1} = \frac{s\omega_p\omega_0}{Qs^2+s\omega_0\omega_p+Q\omega_0^2}$$

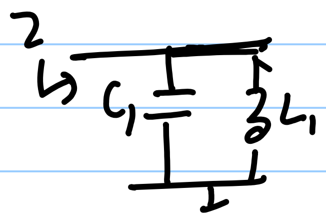
$$= \frac{s\omega_p\omega_0/Q}{s^2+\frac{s\omega_0\omega_p}{Q}+\omega_0^2}$$



$$\longrightarrow \frac{1}{CQ\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)}$$



$$Y = sC = QC\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right) = \frac{QC}{\omega_0}\cdot s + \frac{QC\omega_0}{s}$$



$$Z = \frac{\frac{1}{sC_1} \times sL_1}{\frac{1}{sC_1} + sL_1} = \frac{1}{\frac{1}{sL_1} + sC_1} \checkmark$$

$$C_1 = \frac{QC}{\omega_0} \checkmark, \quad L_1 = \frac{1}{QC\omega_0} \checkmark$$