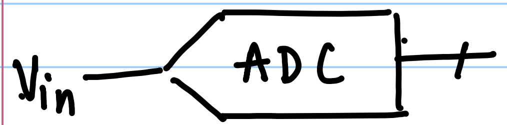
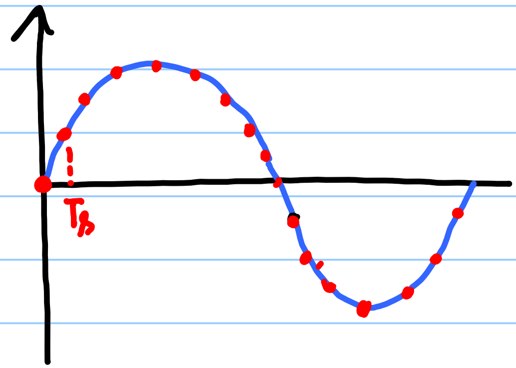
Note Title 15-03-2022

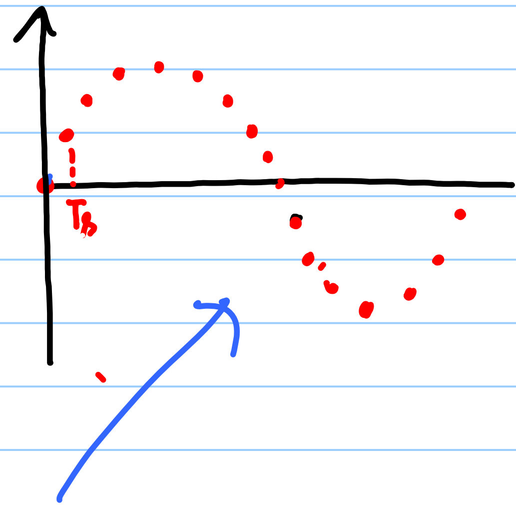
Lecture #28

Analog Filters



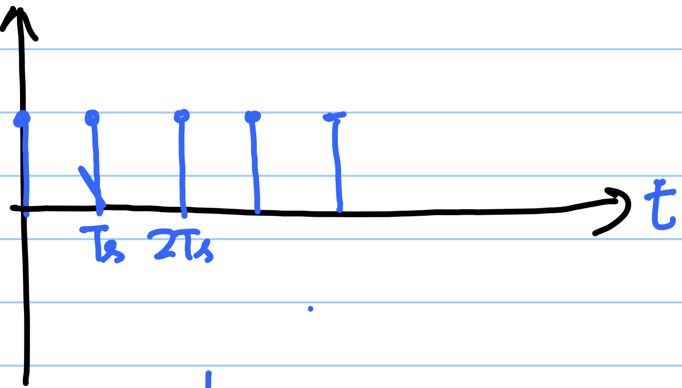
Sampling
+ Digitization.





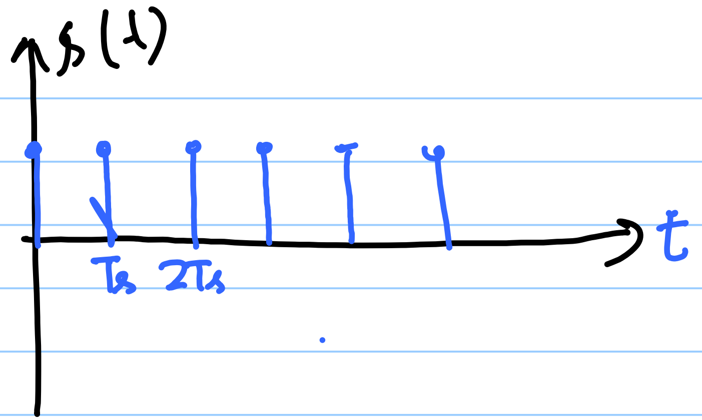
Sampling freq. $f_s = \frac{1}{T_s}$

$\times$



$\sum \delta(t - kT_s)$, Here $\delta(t - kT_s) = 1$ for $t = kT_s$

$\delta(t - kT_s) = 0$



$$s(t) = \sum_{k=0}^{\infty} \delta(t - kT_s)$$

$$= \sum a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \checkmark$$

$$\omega_0 = \frac{2\pi}{T_s}$$

$$\int_0^{T_s} s(t) \cdot \cos(m\omega_0 t)\, dt = \sum \int_0^{T_s} a_n \cos(n\omega_0 t) \cdot \cos(m\omega_0 t)\, dt + -$$

$$1 = a_m \int_0^{T_s} \cos^2(m\omega_0 t)\, dt$$

$$1 = \frac{a_m}{2} \int_0^{T_s} (1 + \cos(2m\omega_0 t))\, dt$$

$$1 = \frac{a_m}{2} \cdot T_s \Rightarrow a_m = \frac{2}{T_s}$$

$$b_n = \frac{2}{T_s} \int_0^{T_s} s(t) \cdot \sin(m\omega_0 t)\, dt = 0$$

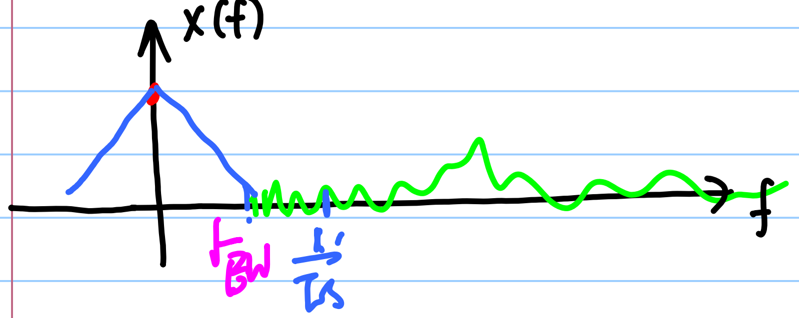
$$s(t) = \sum_{n=0}^{\infty} \frac{2}{T_s} \cos(n\omega_0 t)$$

$$= \sum \frac{2}{T_s} \frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2}$$

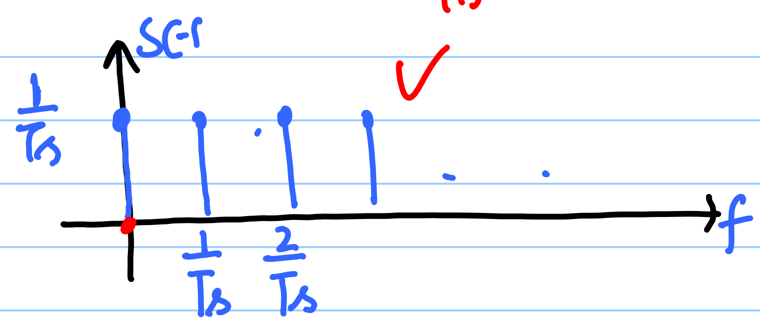
$$s(t) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} e^{jk\omega_0 t}$$

$$S(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$

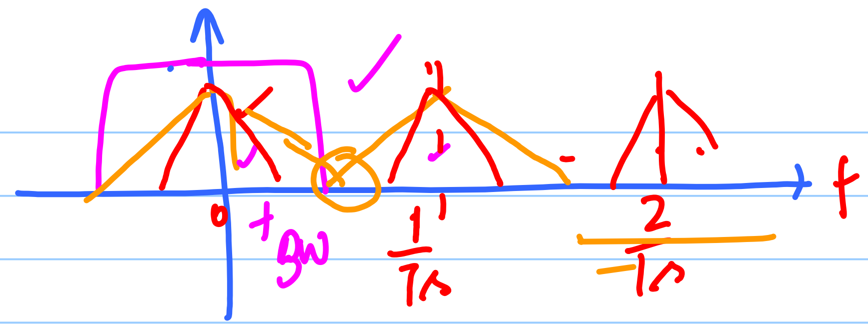
$$x(t) \times s(t) \xrightarrow{\mathcal{F}} X(f) * S(f) = \frac{1}{T_s} \sum X(f - nf_s)$$



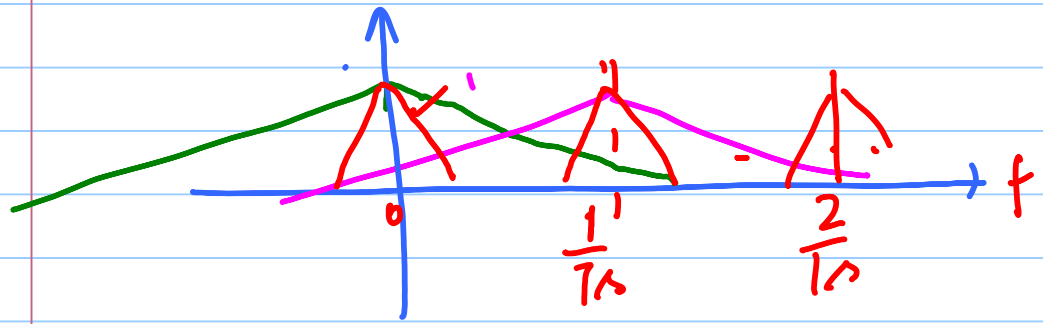
*

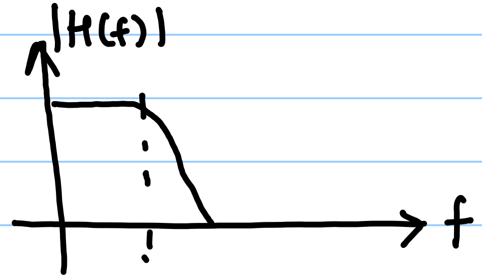


#1



$$f_{BW} \leq \frac{f_s}{2}$$



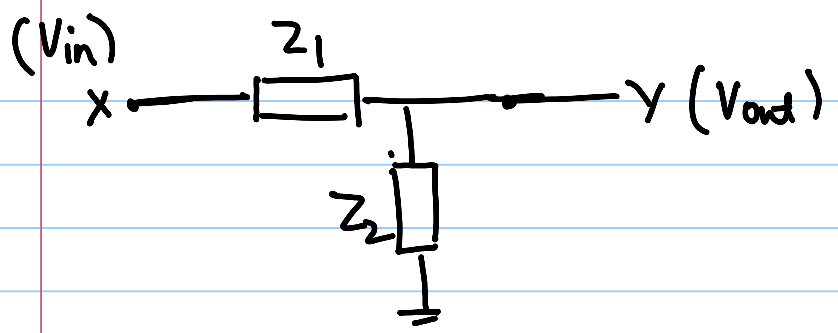


$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(f)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}, \quad \frac{Y(s)}{X(s)} = H(s)$$

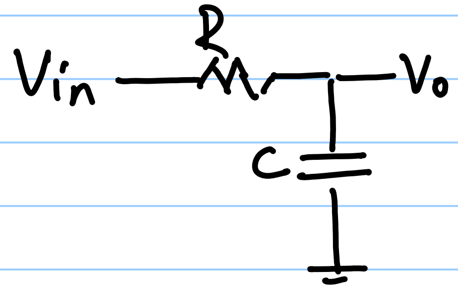
$$H(s) = \frac{1}{1 + s/\omega_p} = \frac{A_0}{1 + s/\omega_p}$$

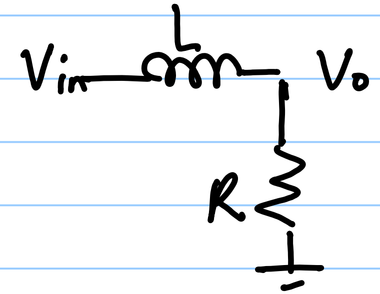
$$|H(j\omega)| = |H(j2\pi f)|$$



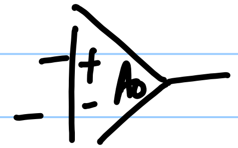
$$\frac{V_{out}}{V_{in}} = \frac{Y}{X} = \frac{Z_2}{Z_2+Z_1} = \frac{1}{1+\frac{Z_1}{Z_2}}$$ ✓

$$\frac{Z_1}{Z_2} = \frac{s}{\omega_p}$$



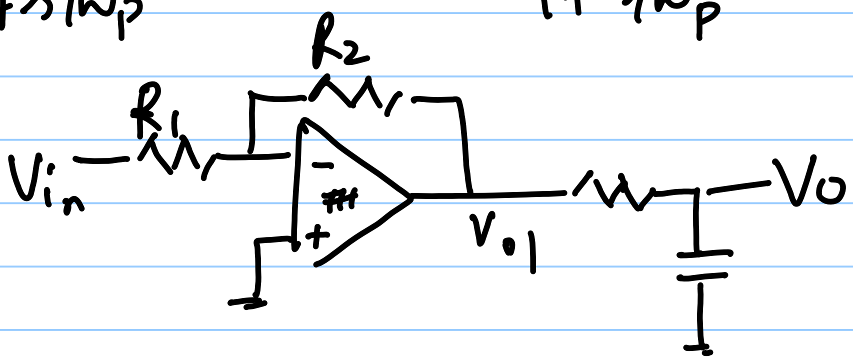


Z_1	R	sL
Z_2	$\frac{1}{sC}$	R
ω_p	$\frac{1}{RC}$	$\frac{R}{L}$



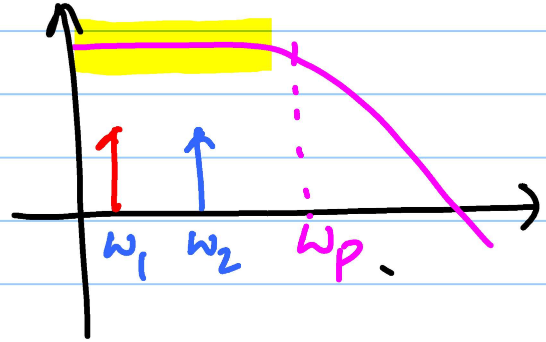
$$\frac{A_0}{1+s/\omega_p}$$

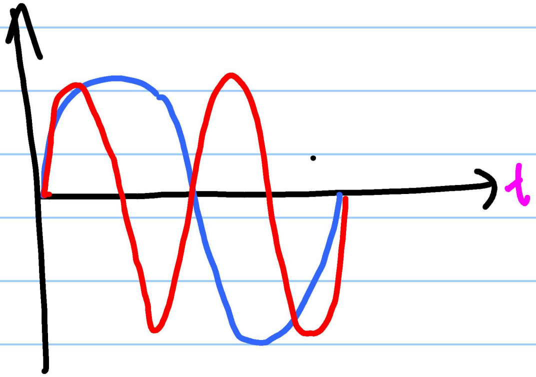
$$\frac{A_0}{1+s/\omega_p} = A_0 \times \frac{1}{1+s/\omega_p}$$

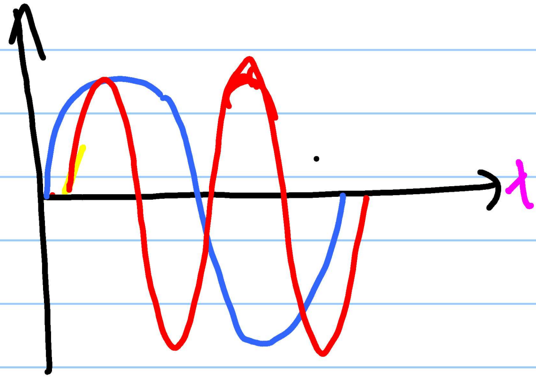


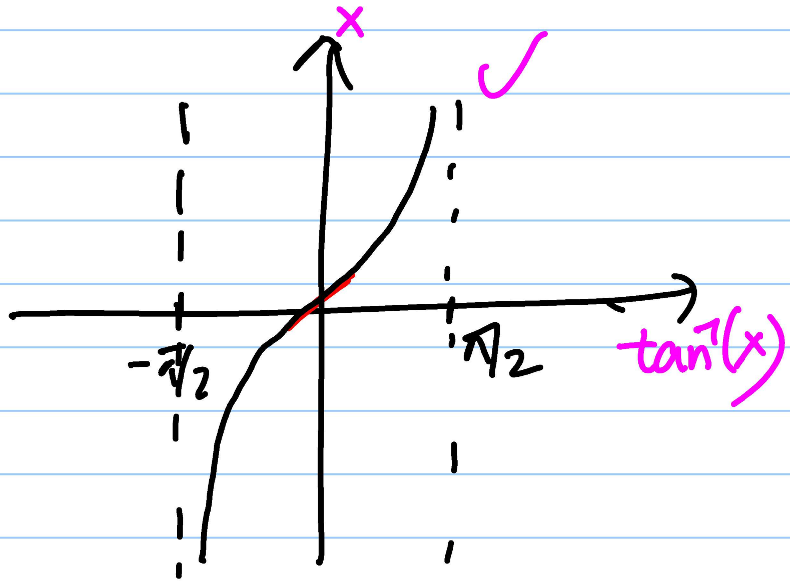
$$H(s) = \frac{A_0}{1 + s/\omega_p}$$

$$\measuredangle H = \underline{-\tan^{-1}\left(\frac{\omega}{\omega_p}\right)}$$









$$\phi_1 = -\tan^{-1}\left(\frac{\omega_1}{\omega_p}\right)$$

$$\phi_2 = -\tan^{-1}\left(\frac{\omega_2}{\omega_p}\right)$$

Δt_1 for ω_1 Δt_2 for ω_2

$$\sin(\omega_1(t - \Delta t_1)) = \sin(\omega_1 t - \underbrace{\omega_1 \cdot \Delta t_1}_{\phi_1})$$

$$\sin(\omega_2(t - \Delta t_2)) = \sin(\omega_2 t - \underbrace{\omega_2 \cdot \Delta t_2}_{\phi_2})$$

$$\frac{d\phi}{d\omega} = 0 \ \cancel{(\text{constant})}$$

$$\sin(\omega_1 t + \phi_1) = \sin\left(\omega_1 \left(t + \frac{\phi_1}{\omega_1}\right)\right)$$

$$= \sin(\omega_1 (t + \Delta t_1))$$

$\phi_1 = \omega_1 \cdot \Delta t$

$\phi_2 = \omega_2 \cdot \Delta t$

$\phi_3 = \omega_3 \cdot \Delta t$