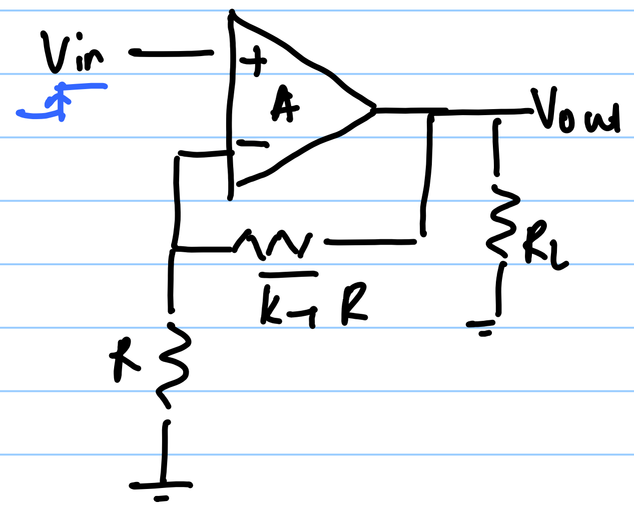
Note Title 15-02-2022

# Lecture # 13




$A(s) = \dfrac{A_0}{1+s/\omega_p}$ ✓

if $V_{in} = a_{in}\sin(\omega_{in} \cdot t)$

$\underline{\omega_{in} \ll \omega_p} \Rightarrow A(s) \approx A_0$

- Static error $\propto 1/A_0$
- Dynamic error $\propto \underline{1/A_0}$

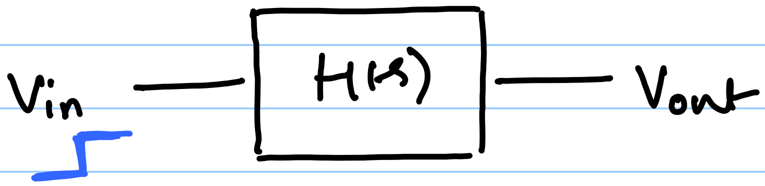
$H(s) = \dfrac{A_{0C}}{1+s/\omega_{p1}}$

$\omega_{p1} = p_1\left(1+\dfrac{A_0}{K}\right)$, $A(s) = \dfrac{A_0}{1+s/p_1}$

$\epsilon_{static} = \dfrac{K}{1+A_0/K}$

$$\epsilon_{dyn} = \frac{\lim_{t\to\infty} V_{out}(t) - \lim_{t\to t_d} V_{out}}{\lim_{t\to\infty} V_{out}(t)}$$

$= \underline{e^{-\omega_{p1} t_d}}$

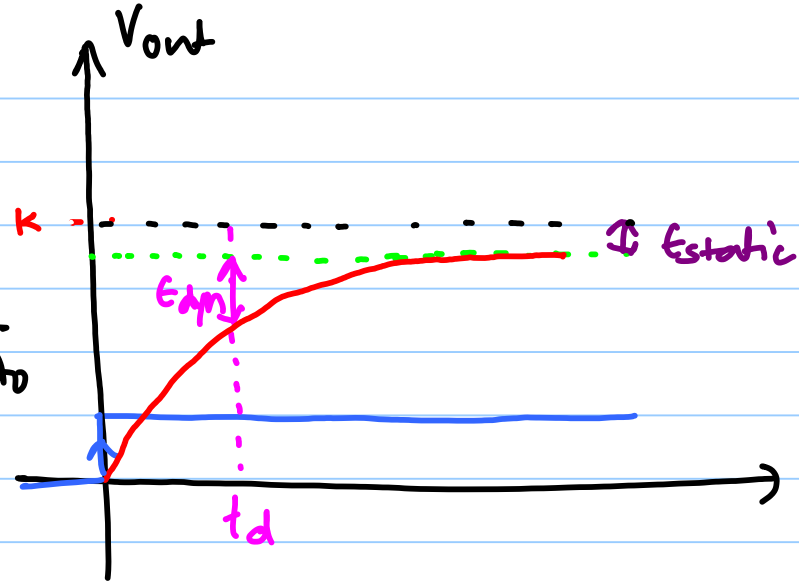


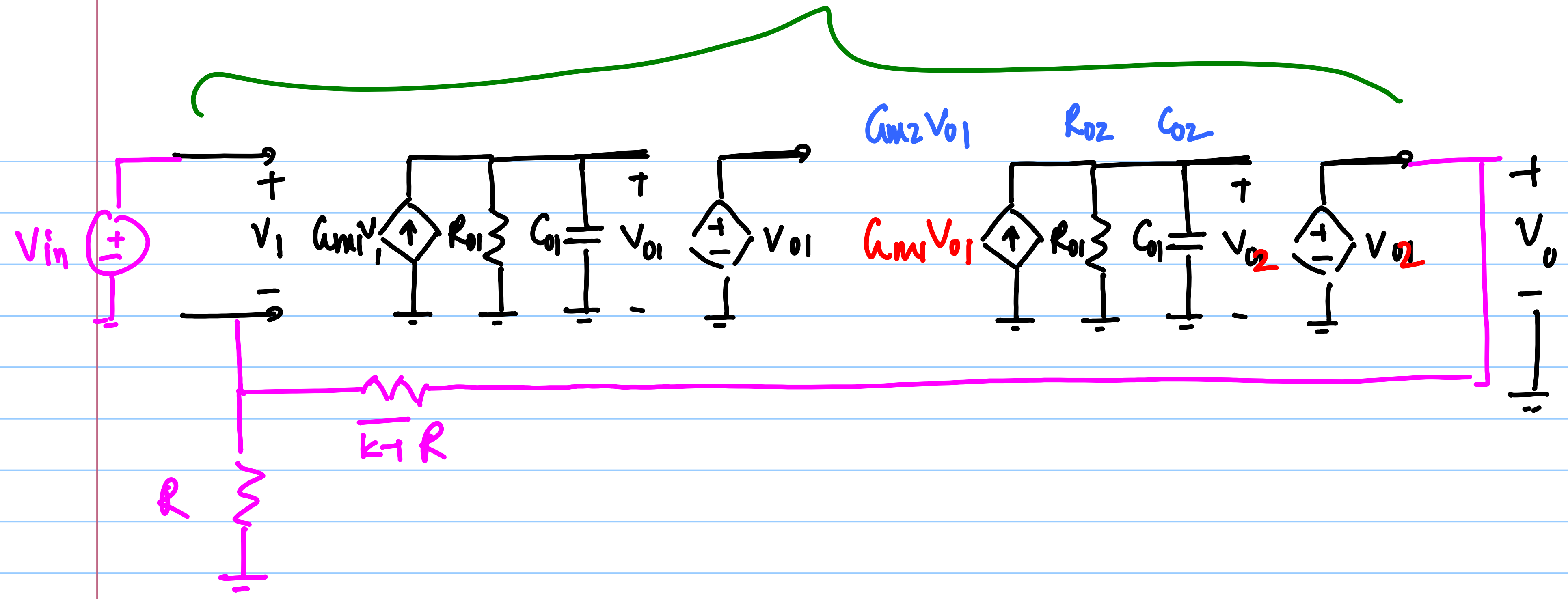

$$H(s) = \frac{A_{DC}}{1 + s/\omega_{p1}}, \quad A_{DC} = \frac{K}{1 + K/A_0}$$

where $\omega_{p1} = p_1\left(1 + \frac{A_0}{K}\right)$

$$V_{out}(t) = A_{DC}\left(1 - e^{-\omega_{p1} \cdot t}\right)$$

$$A = \frac{A_0}{1 + s/p_1}$$






$$A(s) = \frac{G_{m1} \cdot R_{o1}}{1 + sC_{o1}R_{o1}} \times \frac{G_{m2} \cdot R_{o2}}{1 + sC_{o2} \cdot R_{o2}} = \frac{A_0}{(1 + s/p_1)(1 + s/p_2)}$$

$$p_1 = \frac{1}{R_{o1} C_{o1}}, \quad p_2 = \frac{1}{R_{o2} C_{o2}}, \quad A_0 = G_{m1} \cdot R_{o1} \cdot G_{m2} \cdot R_{o2}$$

$$\frac{V_o}{V_{in}} = \frac{K}{1 + K/A(s)} = \frac{K}{1 + \frac{K}{A_0}(1 + s/p_1)(1 + s/p_2)}$$

$$= \frac{K}{\left(1 + K/A_0\right) + \frac{K}{A_0} s\left(\frac{1}{p_1} + \frac{1}{p_2}\right) + \left(\frac{K}{A_0} \frac{s^2}{p_1 p_2}\right)}$$

$$\frac{V_o}{V_{in}} = \frac{K \cdot \frac{A_0 p_1 p_2}{K}}{\frac{A_0}{K} p_1 p_2 \left(1+\frac{K}{A_0}\right) + \frac{\cancel{K}}{\cancel{A_0}} \cdot \frac{\cancel{A_0} p_1 p_2}{\cancel{K}} s\left(\frac{1}{p_1}+\frac{1}{p_2}\right) + s^2}$$

$$= \frac{A_0 p_1 p_2}{p_1 p_2\left(\frac{A_0}{K}+1\right) + s(p_1+p_2) + s^2}$$

$$= \frac{A_{DC}}{\omega_n^2 + 2\zeta\omega_n s + s^2}$$

$\omega_n$ : natural frequency. $\qquad \omega_n = \sqrt{\left(\frac{A_0}{K}+1\right) p_1 p_2}$

$\zeta$ : damping coefficient.

$$\zeta = \frac{p_1+p_2}{2 \cdot \omega_n} = \frac{p_1+p_2}{2\sqrt{\left(\frac{A_0}{K}+1\right) p_1 p_2}} = \underbrace{\frac{1}{2\sqrt{1+\frac{A_0}{K}}} \left(\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}}\right)}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{A_{DC}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

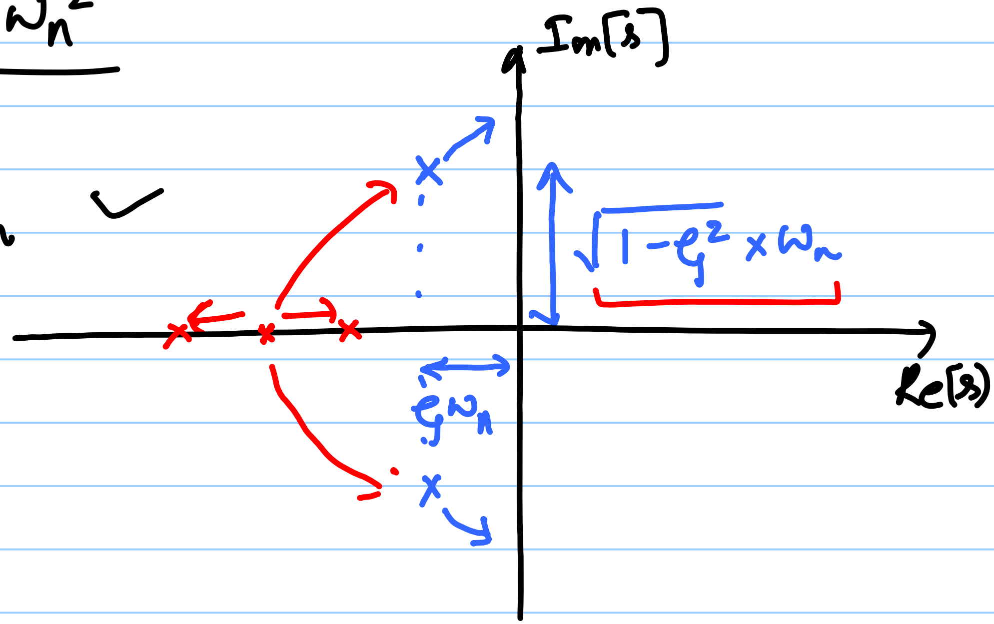
$$D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \left[-\zeta \pm \sqrt{\zeta^2 - 1}\right]\omega_n \quad \checkmark$$

$$\frac{s}{\omega_n} = -\zeta \pm \sqrt{\zeta^2 - 1}$$

Real part of pole $\gtreqless$




**Case 1:** if $\zeta > 1$, $s = \omega_n\left[-\zeta \pm \sqrt{\zeta^2 - 1}\right]$

**Case 2:** if $\zeta < 1$, $s = \omega_n\left[-\zeta \pm j\sqrt{1 - \zeta^2}\right]$

$$V_{in}(s) \longrightarrow \boxed{H(s)} \longrightarrow V_o(s)$$

$$V_o(s) = V_{in}(s) \cdot H(s)$$

$$= \frac{1}{s} \times \frac{A_{DC}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s}\left(\frac{K}{1+K/A_0}\right)\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

if $\zeta > 1$

$$A_{DC} = A_0 p_1 \cdot p_2$$

$$\omega_n^2 = \left(\frac{A_0}{K} + 1\right) p_1 \cdot p_2$$

$$A_{DC} = A_0 \cdot \frac{\left(1+\frac{A_0}{K}\right) p_1 \cdot p_2}{\left(1+\frac{A_0}{K}\right)}.$$

$$= \frac{\omega_n^2}{\left(\frac{1}{A_0} + \frac{1}{K}\right)}$$

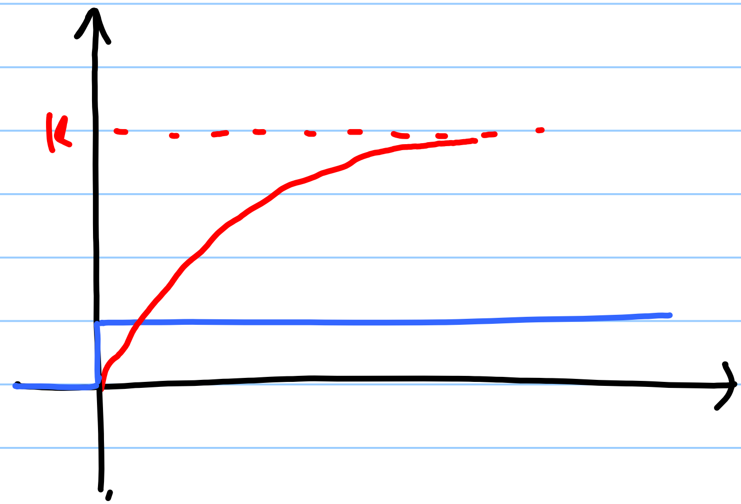
$$= \frac{K}{(1+K/A_0)}\omega_n^2$$

for $\zeta > 1$

$$V_o(t) = A_{DC}\,u(t) - \frac{A_{DC}\cdot e^{-\zeta\omega_n t}}{2\sqrt{1-\frac{1}{\zeta^2}}}\left[\left\{e^{\zeta\omega_n\sqrt{1-\frac{1}{\zeta^2}}\,t} - e^{-\zeta\omega_n\sqrt{1-\frac{1}{\zeta^2}}\cdot t}\right\}\right.$$

$$\left. + \sqrt{1-\frac{1}{\zeta^2}}\left\{e^{\zeta\omega_n\sqrt{1-\frac{1}{\zeta^2}}\,t} + e^{-\zeta\omega_n\sqrt{1-\frac{1}{\zeta^2}}\cdot t}\right\}\right]u(t)$$

$$\lim_{t\to\infty} V_o(t) = A_{DC}\cdot u(t)$$



$\zeta < 1$

$$V_o(t) = A_{DC} \cdot u(t) \left[ 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \underbrace{\left\{ \zeta \sin\left(\omega_n \sqrt{1-\zeta^2}\, t\right) + \sqrt{1-\zeta^2} \cos\left(\omega_n \sqrt{1-\zeta^2}\, t\right) \right\}} \right]$$

$$\zeta \ll 1 \Rightarrow \sqrt{1-\zeta^2} \approx 1$$

$$V_o(t) \approx A_{DC}\, u(t) \left[ 1 - \frac{e^{-\zeta \omega_n t}}{1} \left\{ \zeta \sin(\omega_n t) + 1 \cdot \cos(\omega_n t) \right\} \right]$$

$$\approx A_{DC} \cdot u(t) \left[ 1 - e^{-\zeta \omega_n t} \cos(\omega_n t) \right]$$