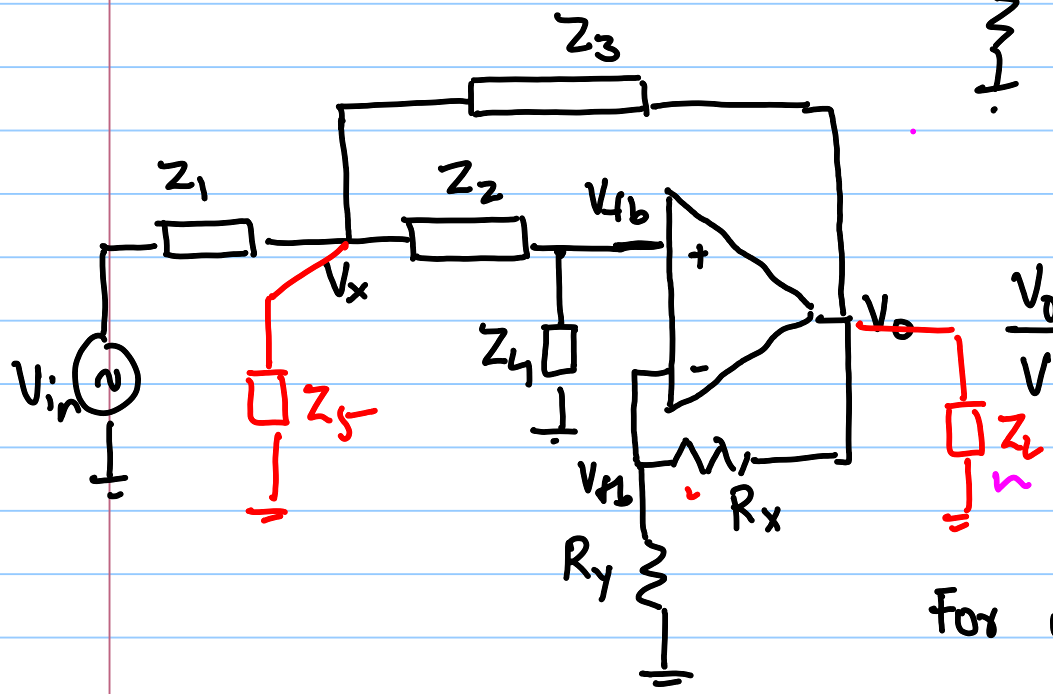
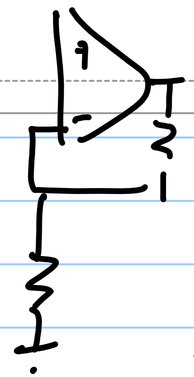
Note Title 28-04-2021

Lecture # 40

Sallen Key filter





$$V_{fb} = \frac{R_y}{R_y + R_x} V_o = \alpha V_o = \frac{1}{K} V_o$$

$$\frac{V_o}{V_i} = \frac{\frac{1}{\alpha} \quad Z_3 Z_4}{Z_3 Z_4 + Z_1 Z_4 \left(1 - \frac{1}{\alpha}\right) + Z_1 Z_3 + Z_1 Z_2 + Z_2 Z_3}$$

For unity feedback.

$$H(s) = \frac{1}{1 + \frac{s}{\omega_0 Q_p} + \frac{s^2}{\omega_0^2}}$$

$$\frac{V_o}{V_i} = \frac{Z_3 Z_4}{Z_3 Z_4 + Z_1 Z_3 + Z_1 Z_2 + Z_2 Z_3}$$

$$\frac{V_o}{V_i} = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_2 Y_4 + Y_3 Y_4 + Y_1 Y_4}$$

Num: $Y_1 Y_2$ no frequency term

Den: $Y_1 Y_2 + Y_2 Y_4 + Y_1 Y_4 + Y_3 Y_4$ (s^2 term, s, s^0)

$Y_1 = G_1$, $Y_2 = G_2$

Den: $G_1 . G_2 + G_1 . Y_4 + G_2 Y_4 + Y_3 Y_4$

$Y_3 = sC_3$, $Y_4 = sC_4$

$$H(s) = \frac{G_1 G_2}{G_1 G_2 + sG_1 C_4 + sG_2 C_4 + s^2 C_3 C_4}$$

$$\frac{V_o}{V_i} = \frac{1}{1 + sC_4\left(\frac{1}{G_2} + \frac{1}{G_1}\right) + \frac{s^2 C_3 C_4}{G_1 G_2}}$$

$$\frac{V_o}{V_i} = \frac{1}{1 + sC_4(R_1 + R_2) + s^2 C_3 C_4 R_1 R_2}$$

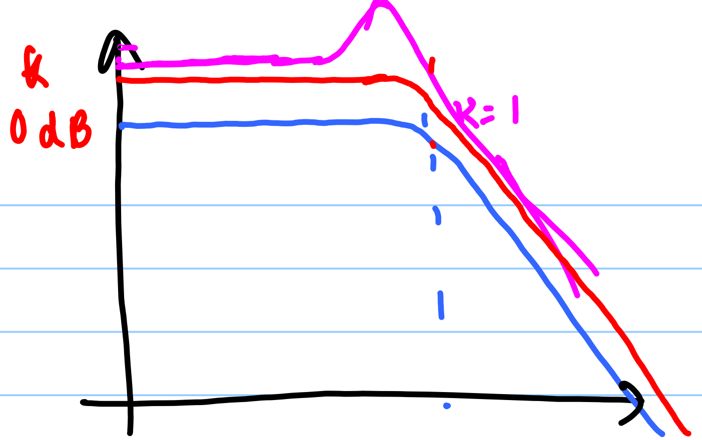
$$\frac{V_o}{V_i} = \frac{\frac{1}{\alpha} \quad Z_3 Z_4}{Z_3 Z_4 + Z_1 Z_4 \left(1 - \frac{1}{\alpha}\right) + Z_1 Z_3 + Z_1 Z_2 + Z_2 Z_3}$$

$$= \frac{K \quad Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 (1-K) + Z_2 Z_3 + Z_3 Z_4}$$

$$= \frac{K \times 1/sC_3 sC_4}{R_1 R_2 + \frac{R_1}{sC_3} + \frac{R_1}{sC_4}(1-K) + \frac{R_2}{sC_3} + \frac{1}{s^2 C_3 C_4}}$$

$$= \frac{K}{1 + sC_4 R_1 + sC_3 R_1 (1-K) + sC_4 R_2 + s^2 C_3 C_4 R_1 R_2}$$

$$\frac{V_o}{V_i} = \frac{K}{1 + s\left[\underbrace{R_1 C_3 (1-K) + R_1 C_4}_{} + R_2 C_4\right] + s^2 \underbrace{C_3 C_4 R_1 R_2}_{}}$$



$x_1 + x_2 \le 0$

$x_1 = a + jb$

$x_2 = a - jb$

$x_1 + x_2 \le 0$

$-\frac{b}{a} \le 0$

$b \geq 0$

$R_1 C_3 (1-K) + R_1 C_4 + R_2 C_4 \geq 0$

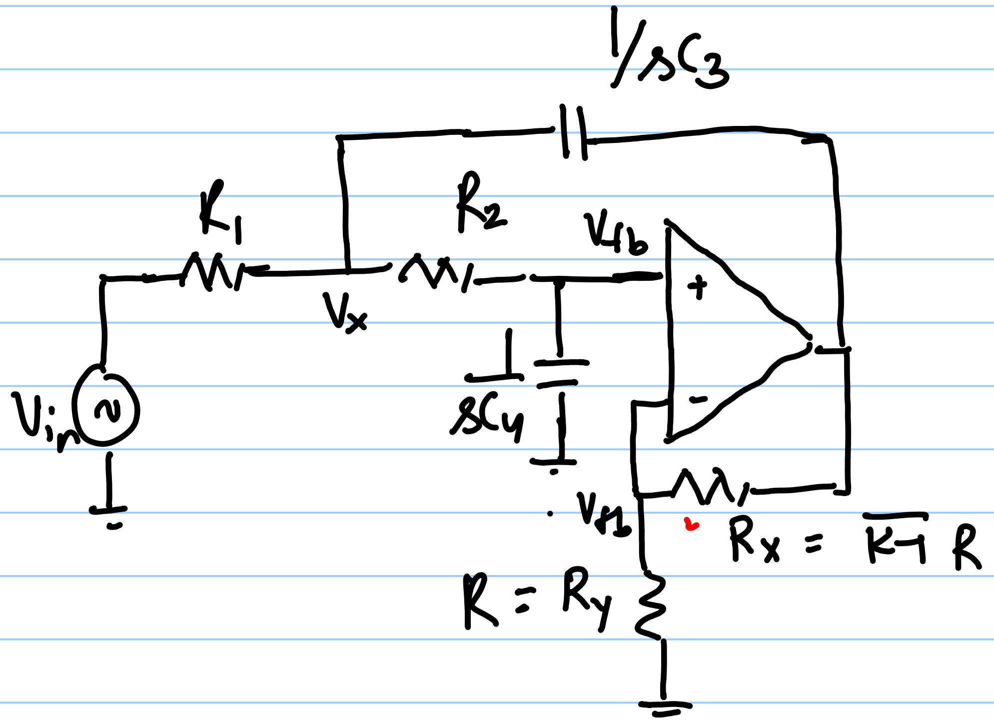
$1 - k + \frac{C_4}{C_3} + \frac{R_2}{R_1} \frac{C_4}{C_3} \geq 0$

$k \le 1 + \left(\frac{R_2}{R_1} + 1\right) \frac{C_4}{C_3}$

$K \le 3$

$C_4 = C_3$

$R_1 = R_2$



$$\frac{1}{1+\frac{s}{\omega_0 Q_P}+\frac{s^2}{\omega_0^2}}$$

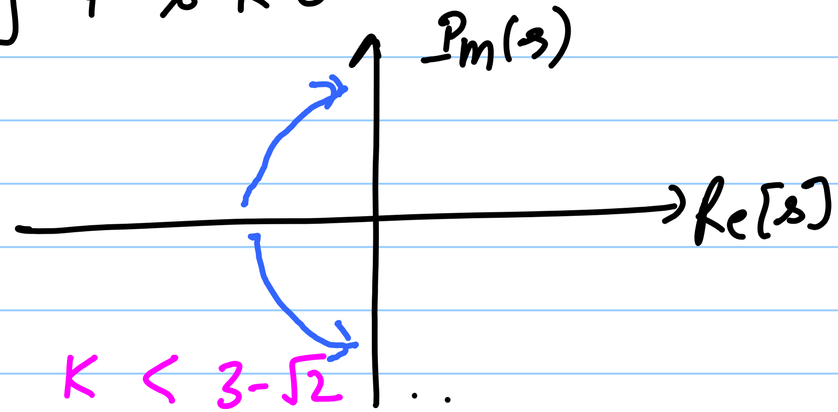
$$\omega_0 = \frac{1}{RC}$$

$$\omega_0 Q_p = \frac{1}{RC(3-k)} = \frac{Q_p}{RC}$$

$$Q_p = \frac{1}{3-K} \checkmark$$

$$\frac{V_o}{V_i} = \frac{K}{1+s[RC(1-K)+RC+RC]+s^2R^2C^2}$$

$$= \frac{K}{1+sRC(3-k)+s^2R^2C^2}$$

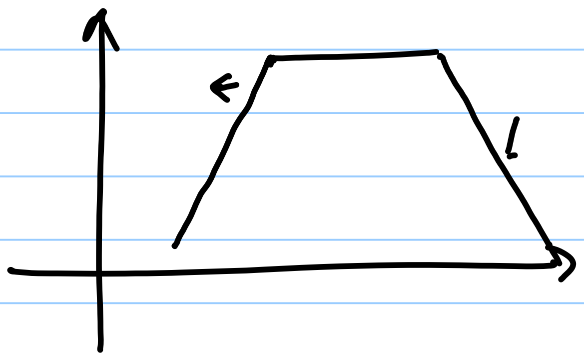


$$Q_p < \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{3-K} < \frac{1}{\sqrt{2}} \Rightarrow K < 3-\sqrt{2}$$

$K=1$

$$\frac{V_0}{V_i} = \frac{Z_3 Z_4 Z_5}{Z_1 Z_3 Z_4 + Z_1 Z_3 Z_5 + Z_1 Z_2 Z_3 + Z_1 Z_2 Z_5 + Z_3 Z_4 Z_5 + Z_2 Z_3 Z_5}$$

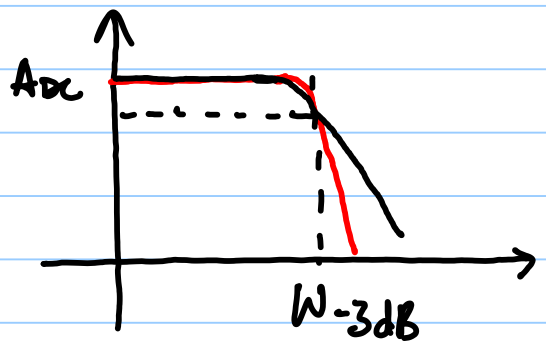
$$= \frac{Y_1 Y_2}{Y_2 Y_5 + Y_2 Y_4 + Y_4 Y_5 + Y_3 Y_4 + Y_1 Y_2 + Y_1 Y_4} = \frac{(s)p.}{as^2 + bs + c}$$



$Y_1 = G_1$, $Y_2 = sC_2$

$Y_3 = sC_3$, $Y_4 = G_4$, $Y_5 = sC_5$

Butterworth filter



$$H(s) = \frac{1}{D(s)}$$

$$|H(j\omega)|^2 = H(j\omega)\, H(j\omega)^*$$

$$= H(j\omega)\, H(-j\omega)$$

$$= \text{Even fn. of } \omega.$$

$$H(s) = \frac{1}{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \dots}$$

$$= \frac{1}{\sum_{k=0}^{\infty} \alpha_{2k} (s^2)^k + \sum_{p=0}^{\infty} \alpha_{2p+1} s^{2p+1}}$$

$$|H(j\omega)|^2 = \frac{1}{\left| \sum_{k=0}^{\infty} \alpha_{2k} (-\omega^2)^k + \sum_{p=0}^{\infty} \alpha_{2p+1} (\pm j)^{2p+1} \omega^{2p+1} \right|^2}$$

$$= \frac{1}{\left| \underbrace{\sum_{k=0}^{\infty} \alpha_{2k} (-1)^k \omega^{2k}} + j \underbrace{\sum_{p=0}^{\infty} \alpha_{2p+1} (-1)^p \omega^{2p+1}} \right|^2}$$

$$= \frac{1}{\left\{ \sum \alpha_{2k} (-1)^k \omega^{2k} \right\}^2 + \left\{ \sum_{p=0}^{\infty} \alpha_{2p+1} (-1)^p \omega^{2p+1} \right\}^2}$$

$$(\omega^{2p+1})^2$$

$$\omega^{2p+1} \cdot \omega^{2q+1} = \omega^{2(p+q)+2}$$

$$|H(j\omega)|^2 = \frac{1}{\underbrace{1 + K_1 \omega^2 + K_2 (\omega^2)^2 + \cdots + K_{n-1} (\omega^2)^{n-1} + K_n (\omega^2)^n}_{|D(j\omega)|^2}}$$

"maximally flat"