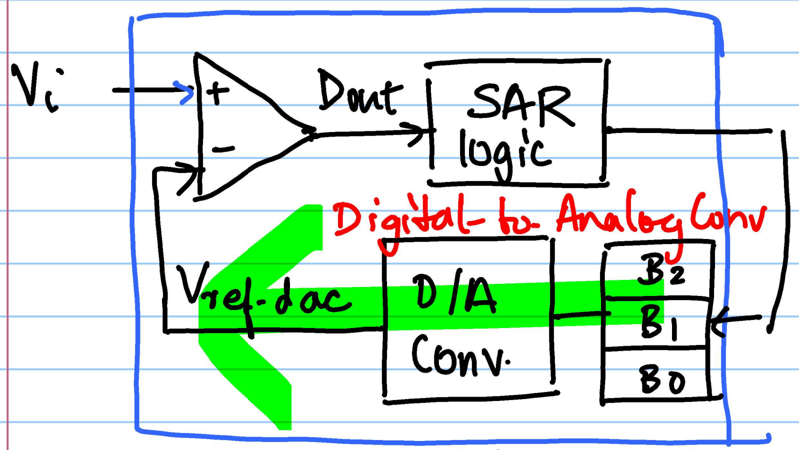
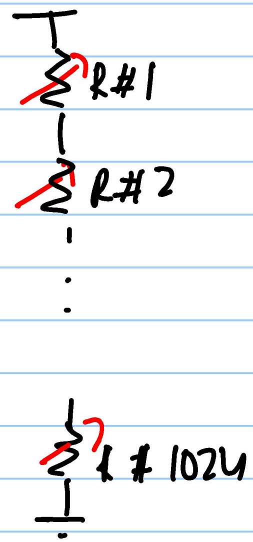
Lecture # 45



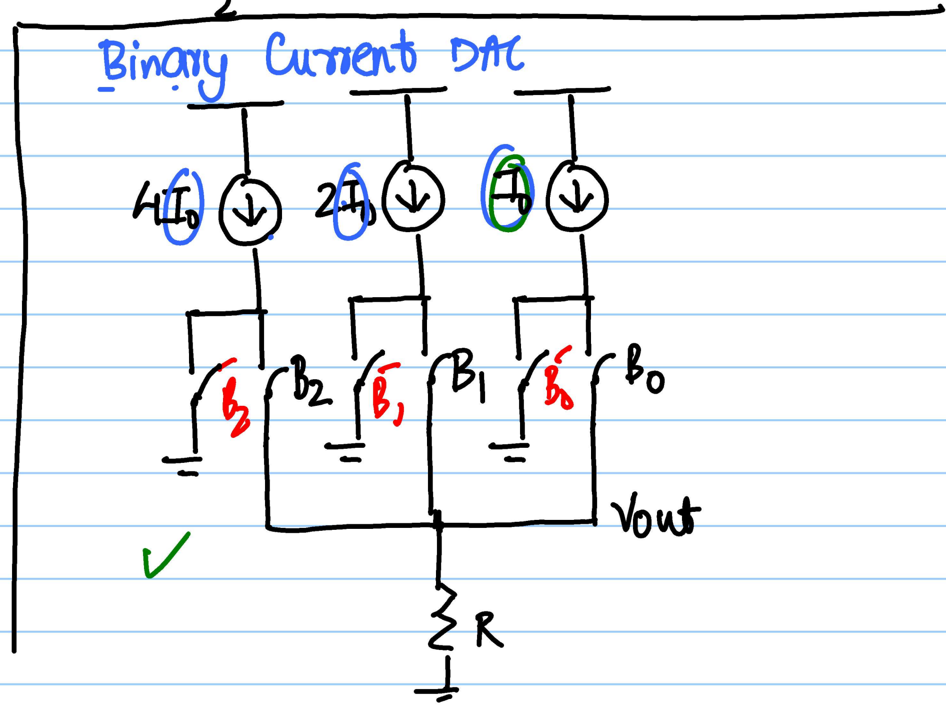
Clock Frequency
$(T = \frac{1}{f})$

- Change Vref-dac
- Give 'T' time for comparison

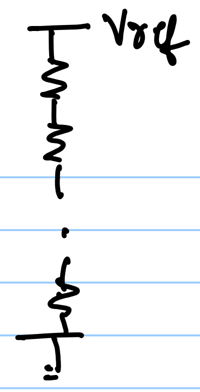
For N-bits ADC, resistor string requires 2^N resistors to generate $\frac{V_{ref}}{2^N} \times K$



Binary Current DAC

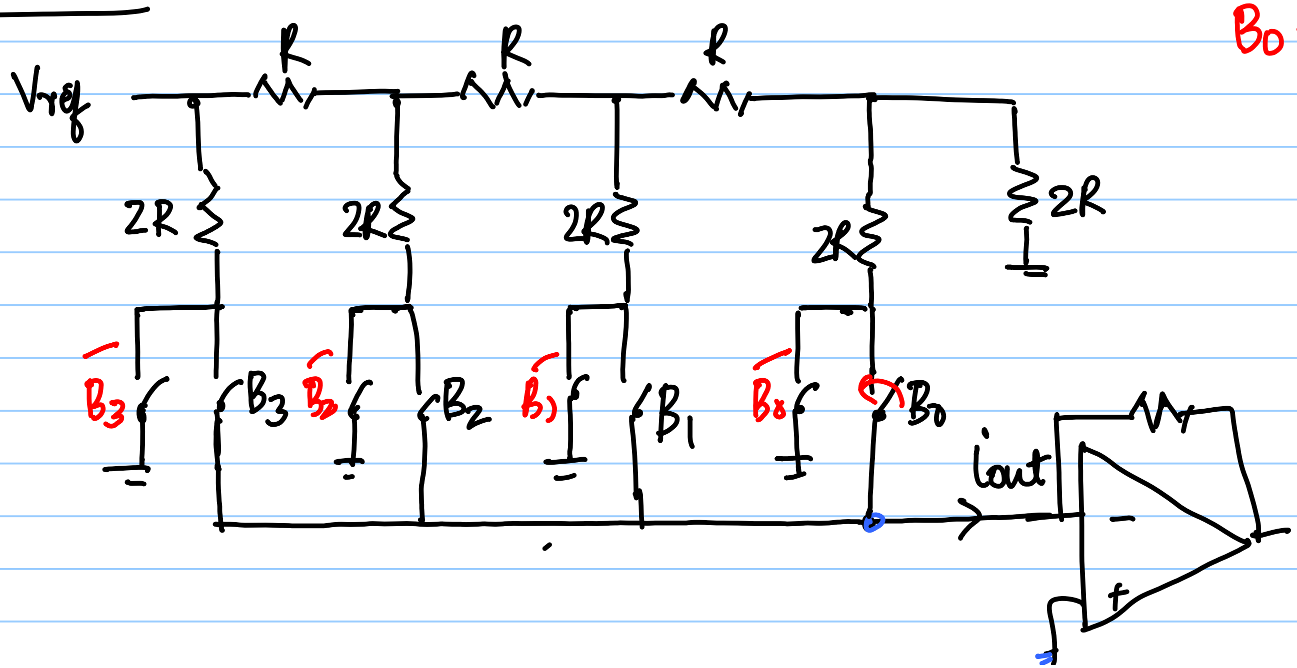


$$V_{out} = (4I_0 \cdot B_2 + 2I_0 \cdot B_1 + I_0 \cdot B_0)R$$
$$= I_0 R(2^2 \cdot B_2 + 2^1 B_1 + 2^0 B_0)$$
$$= \frac{V_{ref}}{8}(2^2 B_2 + 2^1 B_1 + 2^0 B_0)$$

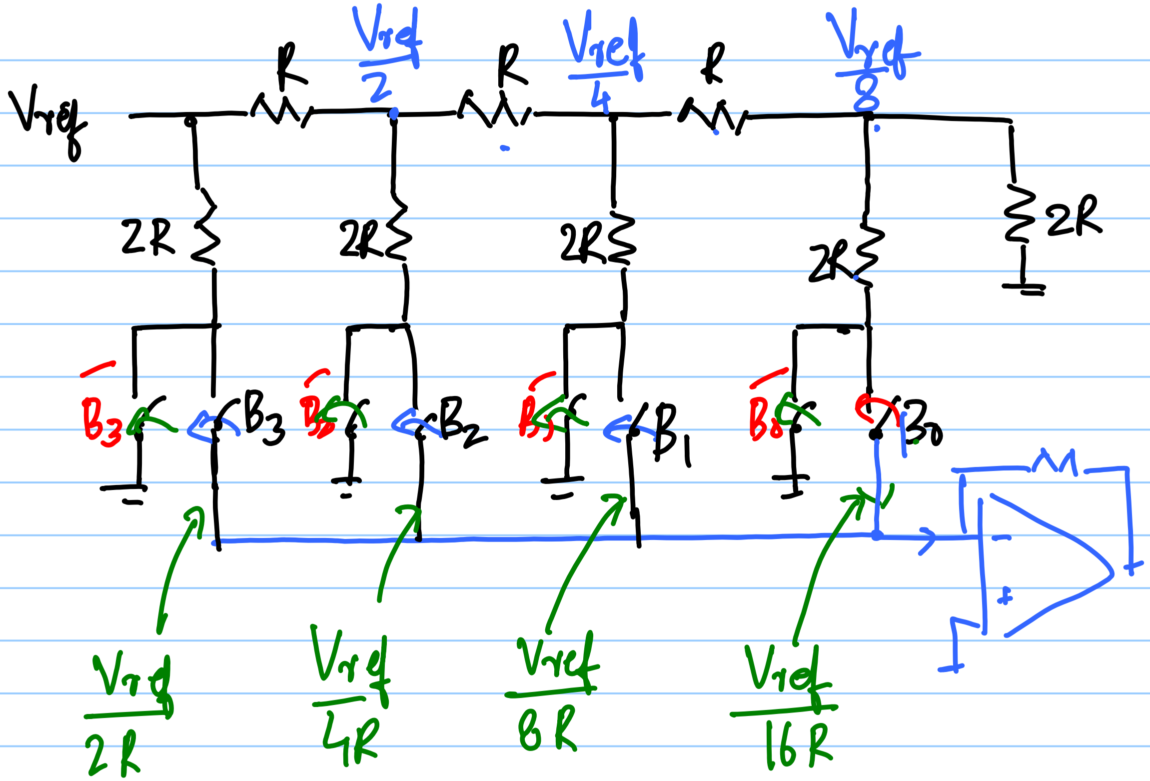


10-bit current DAC : 10 Current sources.

R2R DAC



$B_0 = 1$



$B_0 = B_1 = B_2 = B_3 = 1$

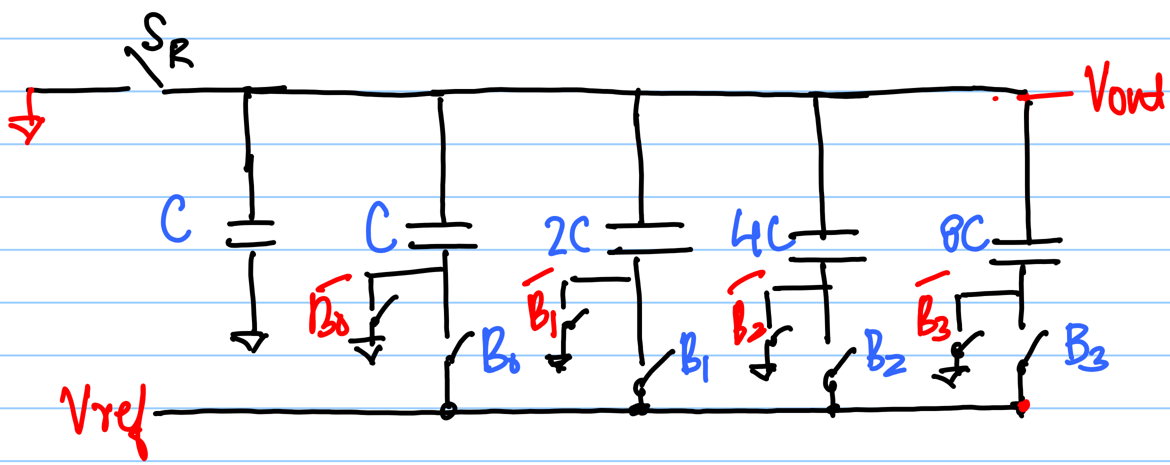
$B_0 = B_1 = B_2 = B_3 = 0$

$$i_{out} = B_3 \cdot \frac{V_{ref}}{2R} + B_2 \cdot \frac{V_{ref}}{4R} + B_1 \frac{V_{ref}}{8R} + B_0 \frac{V_{ref}}{16R}$$

$$= \frac{V_{ref}}{16R} \underbrace{\left[2^3 \cdot B_3 + 2^2 B_2 + 2^1 B_1 + 2^0 \cdot B_0 \right]}$$

$$0 \le i_{out} \le \frac{15}{16} \frac{V_{ref}}{R}$$

Capacitor DAC

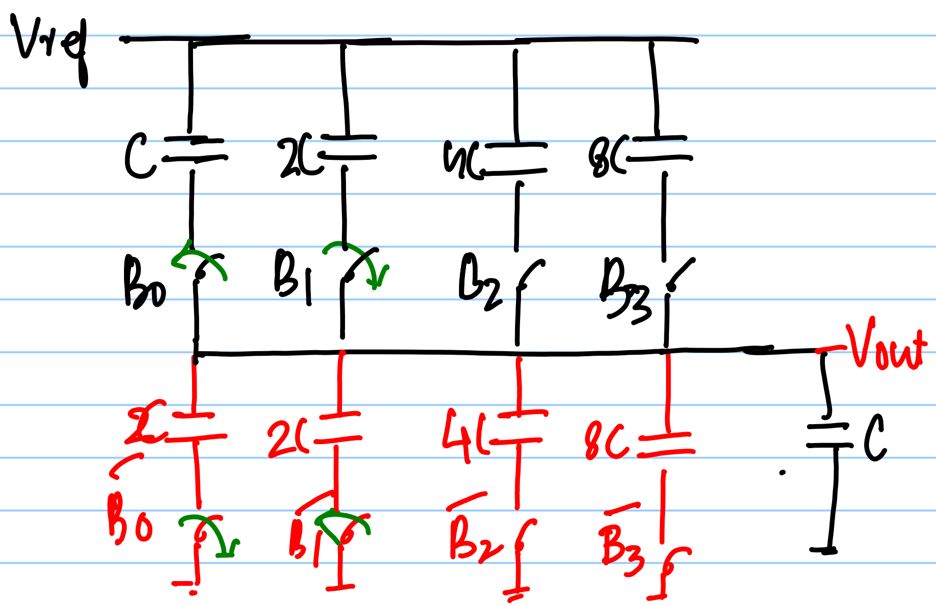


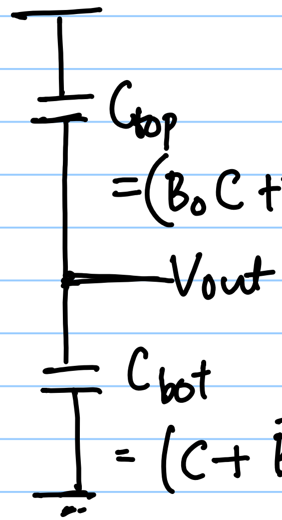
When S_R is high:

- Capacitor are reset.

When S_R is low

-





$C_{top} = (B_0 C + B_1 2C + B_2 4C + B_3 8C)$

$C_{bot} = (C + \bar{B}_0 C + \bar{B}_1 2C + \bar{B}_2 4C + \bar{B}_3 8C)$

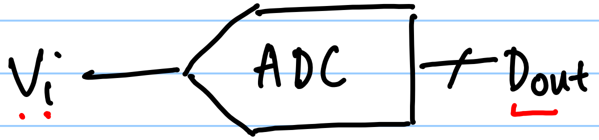
$$V_{out} = \frac{C_{top} V_{ref}}{C_{top} + C_{bot}} = \frac{(B_0 + 2B_1 + 4B_2 + 8B_3)\,C}{\underbrace{C + (1-\cancel{B_0})C + (1-\cancel{B_1})2C + (1-\cancel{B_2})8C + (1-\cancel{B_3})16C}_{C_{bot}} + (\cancel{B_0} + 2\cancel{B_1} + 4\cancel{B_2} + 8\cancel{B_3})\,C} \cdot V_{ref}$$

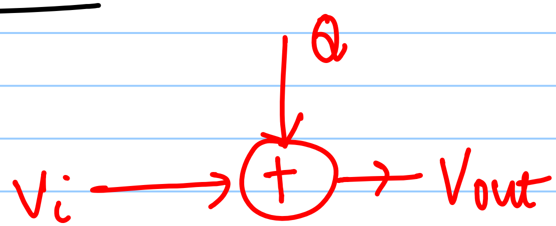
$$= \frac{C\,(8B_3 + 4B_2 + 2B_1 + B_0)}{16C} V_{ref}$$

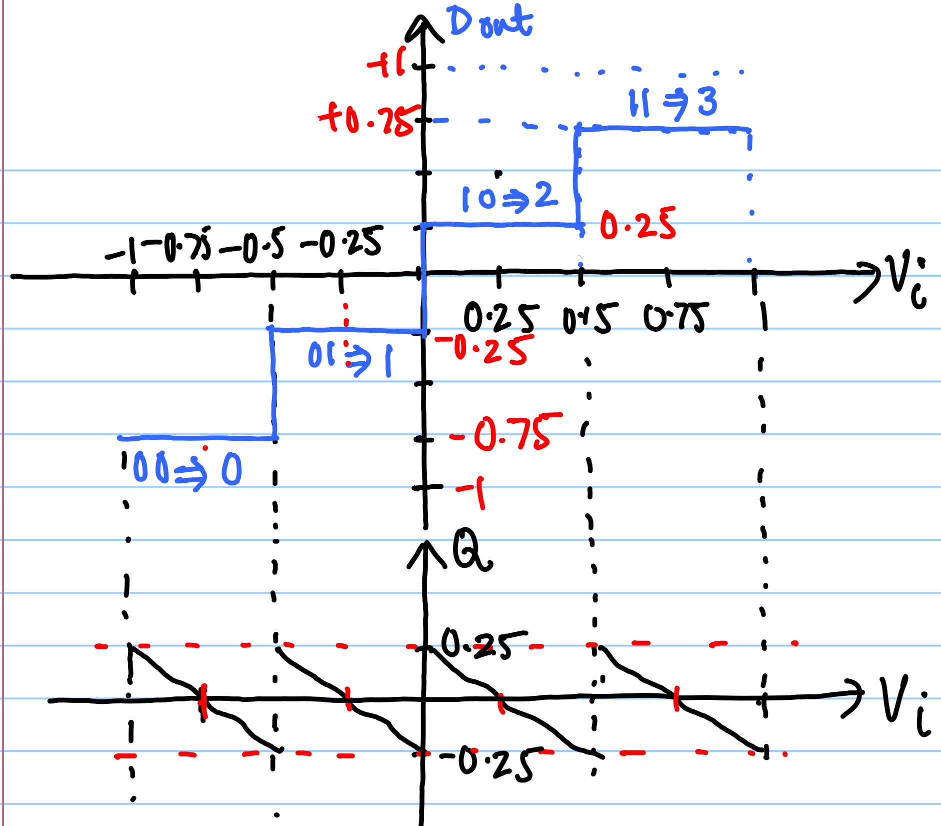
$$= \frac{V_{ref}}{16}\left(8B_3 + 4B_2 + 2B_1 + B_0\right)$$

Errors in A/D or D/A Converters.

1. Quantization Error.







$$|Q| \leq \frac{LSB}{2} = \frac{(1-(-1))/4}{2}$$

$$= 0.25$$

$$V_{out} = D_{out} \cdot \frac{V_{ref}}{2^2} - V_{dc}.$$

$$= \frac{V_{ref}}{4}\left(2 \cdot B_1 + 2^0 \cdot B_0\right) - 0.75$$

$V_i \longrightarrow D_{out}$
(V_{out})

$V_{out} = V_i + Q \Rightarrow$ Quantization Error.

$Q = V_{out} - V_i$

$-1 \leq V_i \leq -0.5$

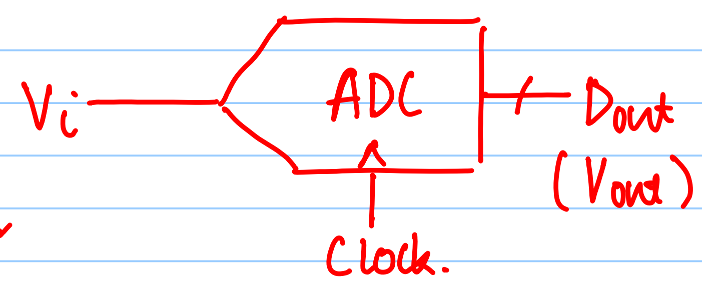
$-1 \leq V_{out} - Q \leq -0.5$

$-0.25 \leq -Q \leq 0.25$

$-0.25 \leq Q \leq \pm 0.25$

Δ = LSB size.

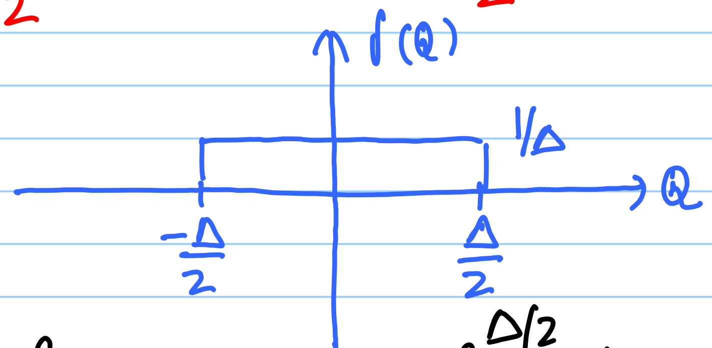
Quantization error, $|Q| \le \left(\frac{V_{ref,h} - V_{ref,l}}{2^N}\right)\frac{1}{2}$



Assumption: (q_0, q_1 . . q_p.)

- Uniform probability distribution function.
- $E[Q]=0$

$-\frac{\Delta}{2} \le Q[k] \le \frac{\Delta}{2}$



$D_{out}[n] = V_i[n] + Q[n]$,

$V_i(0) \longrightarrow D_{out}(0), Q(0)$ ✓

$V_i(1) \longrightarrow D_{out}(1), Q(1)$ ✓

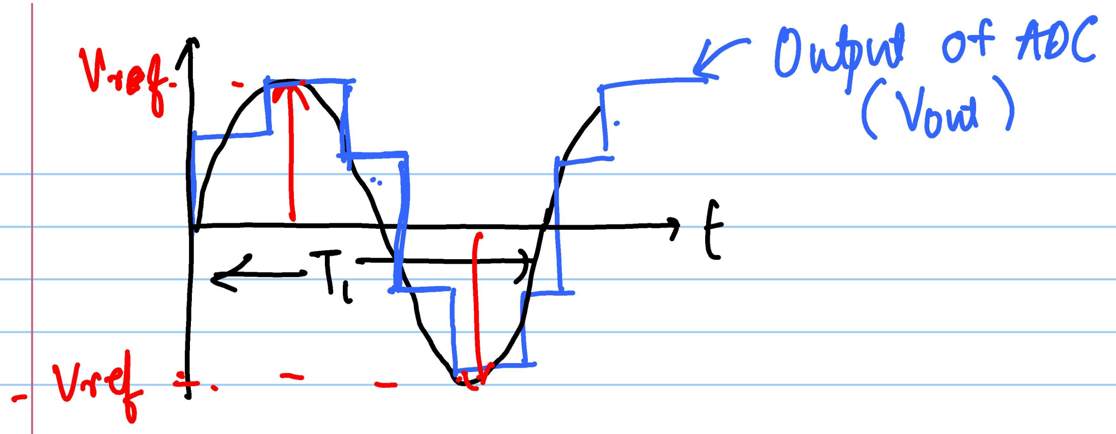
$\vdots$

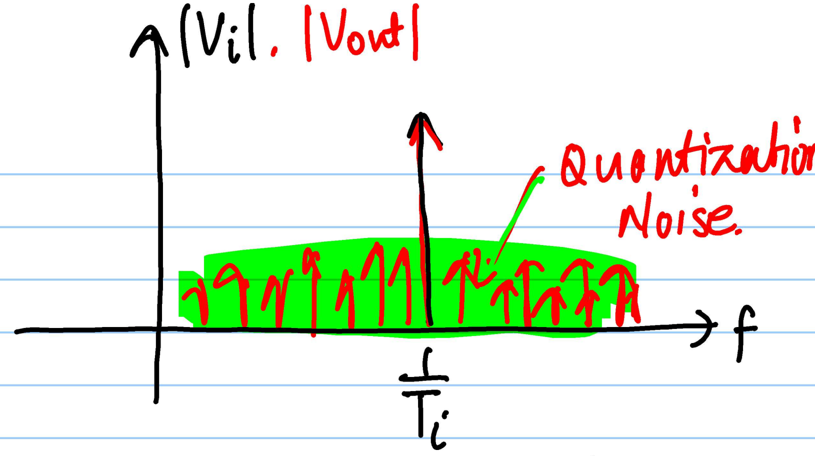
$V_i(p) \longrightarrow D_{out}(p), Q(p)$,

$$\int f(Q)\cdot dQ = \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta}\cdot dQ = 1$$

$$E[Q^2] = \int_{-\Delta/2}^{\Delta/2} Q^2 \cdot \frac{1}{\Delta} \cdot dQ = \frac{1}{\Delta} \left. \frac{Q^3}{3} \right|_{-\Delta/2}^{\Delta/2} = \frac{1}{3\cdot\Delta} \frac{2\Delta^3}{8} = \frac{\Delta^2}{12}$$

$E[Q^2] = \Delta^2/12$ ✓





Signal to Noise Ratio (SNR) $= 10\log_{10}\left(\frac{\text{Signal Power}}{\text{Quantization Noise Power}}\right)$

(Signal-to-Quantization Ratio)

$$E[Q^2] = \frac{\Delta^2}{12} = \frac{\left[\left(V_{ref} - (-V_{ref})\right)/2^N\right]^2}{12}$$

$$= \frac{1}{2^{2(N-1)}}\,\frac{1}{12} \times V_{ref}^2$$

$$\text{Signal power} = \frac{V_{ref}^2}{2}$$

$$\underset{[dB]}{SQNR} = 10\log_{10}\left(\frac{\cancel{V_{ref}^2}/2}{\cancel{V_{ref}^2}/12 \times 2^{2(N-1)}}\right)$$

$$= 10\log_{10}\left(\frac{\overset{3}{\cancel{12}} \times 2^{2N}}{\cancel{4} \times 2}\right)$$

$$= 10\log_{10}\left(2^{2N}\right) + 10\left[\log_{10}(3) - \log_{10}(2)\right]$$

$$= 2N \times 0.301 \times 10 + 10\left(0.477 - 0.301\right)$$

$$= 6.02N + 1.76$$

SNR [dB] = $6.02N + 1.76$