Note Title

# Lecture # 40

$H(s) = \frac{1}{1+s/\omega_p}$ ; $H_2(s) = H_1(s)\,H_1(s)$

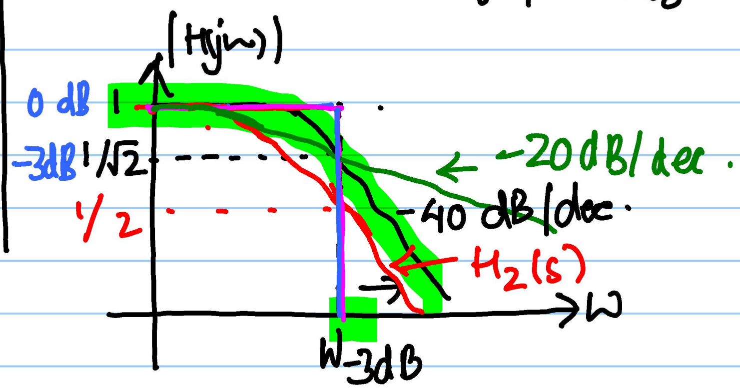
$= \frac{1}{(1+s/\omega_p)^2}$

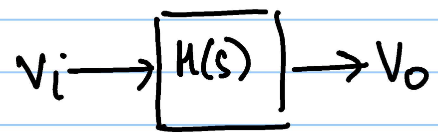
$= \frac{1}{1+\frac{2s}{\omega_p}+\frac{s^2}{\omega_p^2}}$

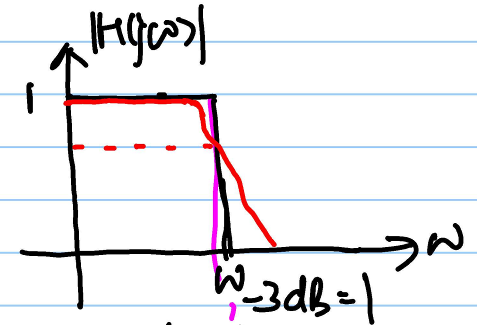
1. Passive filters
2. Active-RC filters
3. Gm-C filters

Ex. Sallen-Key filter
Rauch Filter

$H(s) = \frac{1}{1+\frac{s}{\omega_0 Q_p}+\frac{s^2}{\omega_0^2}}$



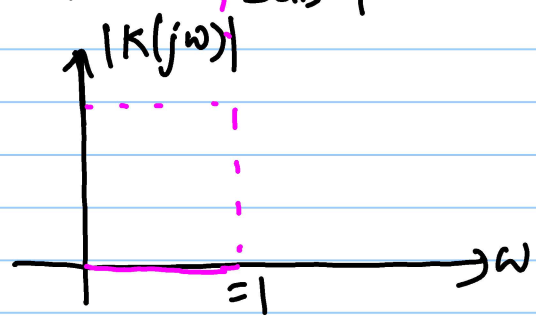




✓ $H(s) = \frac{1}{D(s)} = \frac{1}{1+K_0+K_1 s+K_2 s^2+\cdots+K_n s^n}$

$D(s) = 1+K(s)$

$|K(j\omega)| = |K_0+jK_1\omega-K_2\omega^2-jK_3\omega^3+K_4\omega^4+jK_5\omega^5|$



$$|H(j\omega)|^2 = \frac{1}{|D(j\omega)|^2}$$

$$= \left| \frac{1}{1 + k_0 + K_1 (j\omega) + K_2 (j\omega)^{2} + \cdots + K_n (j\omega)^n} \right|$$

$$= \left| \frac{1}{1 + j \sum_{p=0} \alpha_p \omega^{2p+1} (-1)^{p+1} + \sum_{q=0} \alpha_q \omega^{2q} (-1)^q} \right|$$

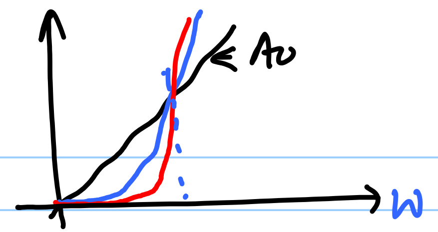
$$= \left( \frac{1}{(1 + \alpha_2 \omega^2 + \alpha_4 \omega^4 + \cdots)^2 + (\alpha_1 \omega + \alpha_3 \omega^3 + \cdots)^2} \right)^{1/2}$$

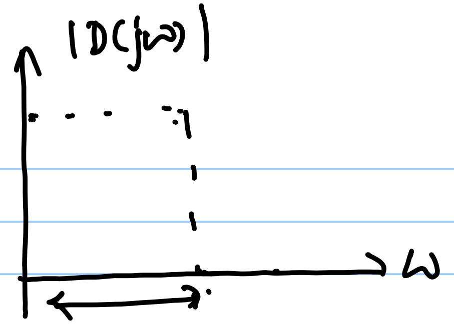
$$\omega^2 \; (\omega^2) \qquad \omega^2 (\alpha_1 + \alpha_3 \omega^2 + \cdots)^2$$

$$|D(s)|^2 = 1 + A(\omega^2)$$

$$= 1 + \underbrace{A_0 + A_1 \omega^2 + A_2 \omega^4 + A_3 \omega^6 + \cdots + A_n \omega^{2n}}$$

$$|H(j\omega)|^2 = \frac{1}{|D(j\omega)|^2}$$




$$|D(j\omega)| = 1.$$

$\omega_{-3dB} = 1$

✓, $A_0 = 0$, $A_1 = 0$, $A_2 = 0$, $A_{n-1} = 0$

$$|D(j\omega)|^2 = 1 + A_n (\omega^2)^n$$

$$|D(j\omega)|^2 = 1 + \omega^{2n}$$

$$\frac{d^q(|D(j\omega)|^2)}{d(\omega^2)^q} = 0$$

$q = 1, 2, 3, \ldots \ldots, n-1$

Maximally flat response within filter bandwidth.

"Butterworth filters"

$$|D(j\omega)|^2 = 1 + \omega^{2n}$$

$$|H(j\omega)|^2 = \frac{1}{1 + \omega^{2n}}$$

$$H(s) \underset{\omega = \frac{s}{j}}{\overset{s = j\omega}{\rightleftarrows}} H(j\omega)$$

if $H(s)$ has all L.H.P

$\Rightarrow$ $H(-s)$ has all R.H.P.

$$\underbrace{H(s)\,H(-s)} = \frac{1}{1 + \left(\frac{s}{j}\right)^{2n}} = \frac{1}{1 + \underbrace{(-1)^n s^{2n}}}$$

$$1 + (-1)^n s^{2n} = 0$$

$$s^{2n} = \frac{(-1)}{(-1)^n}$$

$\exp(x) = e^x$

$n$ = even

$$s^{2n} = -1 = \exp(j(2k\pi + \pi))$$

$$s = \exp\left(j\left(\frac{2k\pi}{2n} + \frac{\pi}{2n}\right)\right) \quad . \quad k = 0, 1, 2, \ldots, 2n-1$$

$n$ = odd

$$s^{2n} = 1 = \exp(j(2k\pi + 2\pi))$$

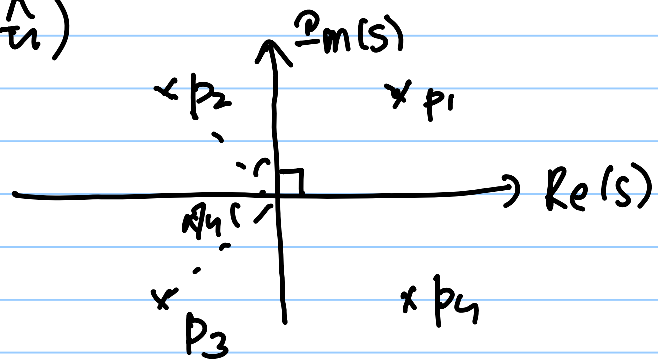
$$s = \exp\left(j\left(\frac{2k\pi}{2n} + \frac{2\pi}{2n}\right)\right) \quad k = 0, 1, 2, \ldots, 2n-1$$

H(s) is $n^{th}$ order fn.

$n=2$ : $\quad s = \exp\left( j \left( \frac{\cancel{2}k\pi}{\cancel{4}\,2} + \frac{\pi}{4} \right) \right) \qquad k = 0, 1, 2, 3$

$$s = \underset{p_1}{e^{j(\pi/4)}}, \quad \underset{p_2}{e^{j(\pi/2 + \pi/4)}}, \quad \underset{p_3}{e^{j(\pi + \frac{\pi}{4})}}, \quad \underset{p_4}{e^{j(\frac{3\pi}{2} + \frac{\pi}{4})}}$$

L.H.P     L.H.P




$$e^{j\left(\frac{\pi}{2} + \frac{\pi}{4}\right)} = -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

$$e^{j(\pi + \pi/4)} = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

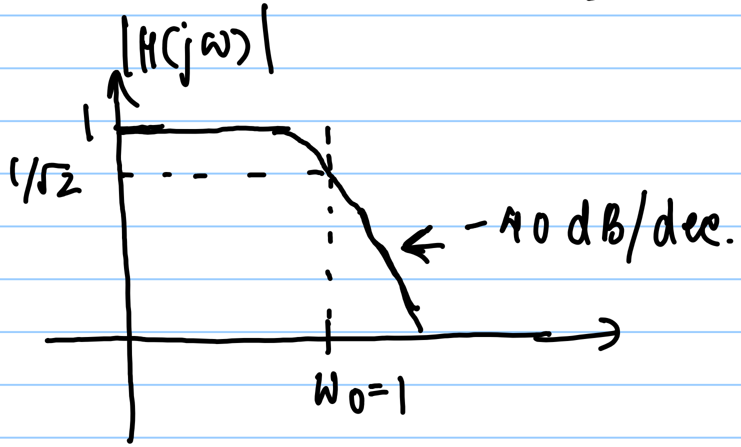
$$H(s) = \frac{1}{(s - p_2)\,(s - p_3)} = \frac{1}{\left(\left(s + \frac{1}{\sqrt{2}}\right) + \frac{j}{\sqrt{2}}\right)\left(\left(s + \frac{1}{\sqrt{2}}\right) - \frac{j}{\sqrt{2}}\right)}$$

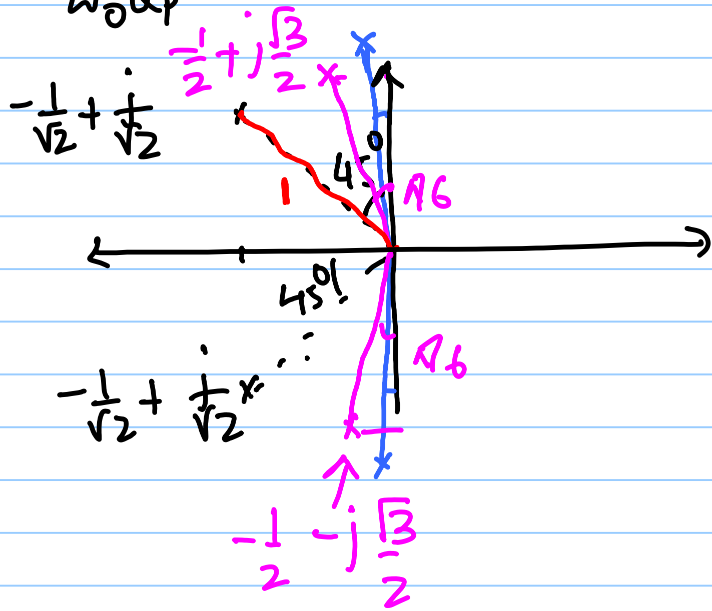
$$= \frac{1}{s^2 + \frac{1}{2} + \sqrt{2}s + \frac{1}{2}} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

# 2nd Order Butterworth filter.

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1} = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q_p} + 1}$$

$$\omega_0 = 1, \quad Q_p = \frac{1}{\sqrt{2}}$$



$n=3, \quad s = \exp\left( j \left( \frac{2k\pi}{2n} + \frac{2\pi}{2n} \right) \right)$

$= \exp\left( j \left( \frac{k\pi}{3} + \frac{\pi}{3} \right) \right) \qquad k = 0, 1, 2, 3, 4, 5$

$s = e^{j\pi/3}, \; e^{j(2\pi/3)}, \; e^{j\pi}, \; e^{j4\pi/3}, \; e^{j5\pi/3}, \; e^{j6\pi/3}$

R L L L R R.

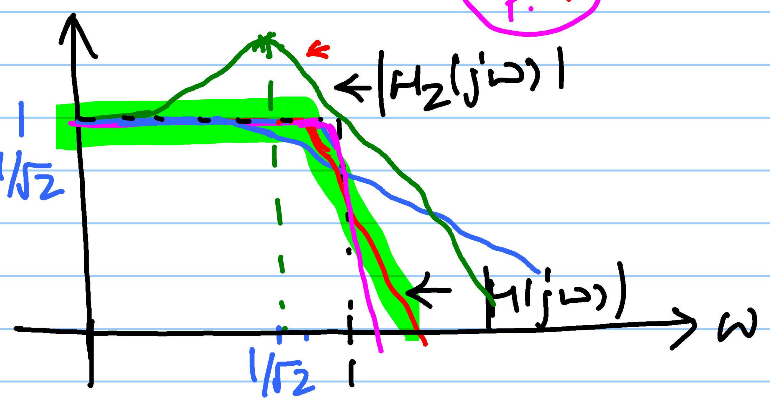
$e^{j\pi} = -1$

$e^{j2\pi/3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

$e^{j4\pi/3} = -\frac{1}{\sqrt{2}} - j\frac{\sqrt{3}}{2}$

$H(s) = \dfrac{1}{(s+1)\,(s^2+s+1)}$

$Q_p = 1$




$\boxed{H_1(s) = \dfrac{1}{s+1} \qquad H_2(s) = \dfrac{1}{s^2+s+1}}$

$$H_2(s) = \frac{1}{s^2+s+1}$$

$$|H_2(j\omega)|^2 = \frac{1}{(1-\omega^2)^2+\omega^2}$$

$$f(\omega) = (1-\omega^2)^2 + \omega^2$$

$$f'(\omega) = 2(1-\omega^2)(-2\omega) + 2\omega = 0$$

$$1 - 2(1-\omega^2) = 0$$

$$2\omega^2 = 1$$

$$\omega = \frac{1}{\sqrt{2}}$$

$$|H_2(j\omega)|\Big|_{\omega = 1/\sqrt{2}}$$

$$= \frac{1}{\left(1-\frac{1}{2}\right)^2 + \frac{1}{2}} = \frac{1}{\frac{3}{4}}$$

$$= \frac{4}{3}$$

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4th order filter.

$$s^{2n} = -1 = \exp\left(j\left(2k\pi + \pi\right)\right)$$

$s = \exp\left(j\left(\frac{\cancel{2}k\pi}{4\cancel{8}} + \frac{\phantom{x}}{8}\right.\right.$ $\qquad k = 0, 1, \ldots 7.$

R R L L L

$s : e^{j(\pi/8)},\ e^{j(\pi/4 + \pi/8)},\ e^{j(\frac{\pi}{2} + \pi/8)},\ e^{j(3\pi/4 + \pi/8)},\ e^{j(\pi + \pi/8)}$

$e^{j(\frac{5\pi}{4} + \pi/8)},\ e^{j(6\pi/4 + \pi/8)},\ e^{j(7\pi/4 + \pi/8)}$

L R R

$s = e^{j(\pi/2 + \pi/8)},\ e^{j(\frac{3\pi}{2} - \pi/8)},\ e^{j(\pi - \pi/8)},\ e^{j(\pi + \pi/8)}$