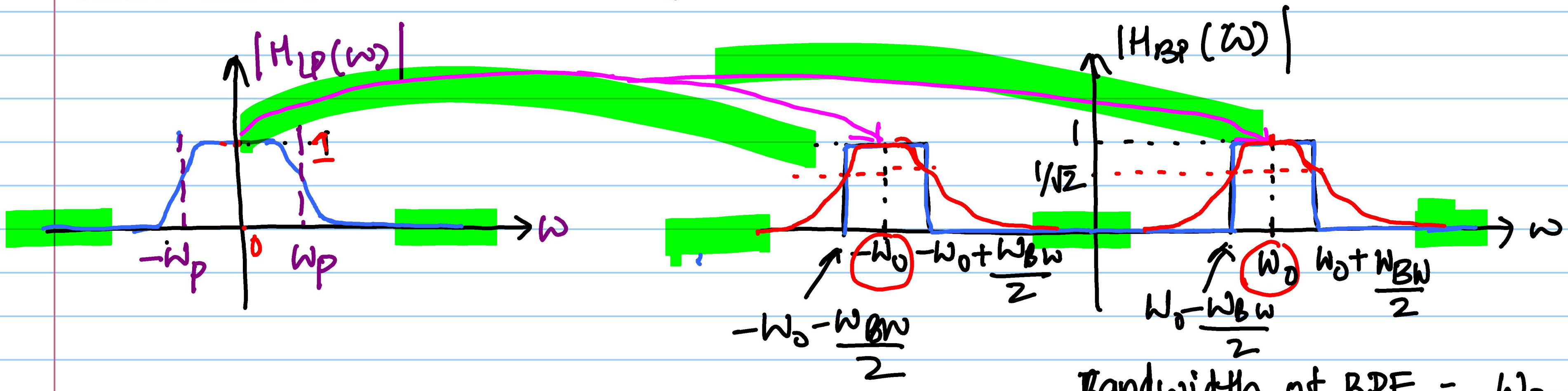
Lecture # 36

Bandpass Filter (BPF)



Bandwidth of BPF = ω_B

$H_{LP}(s)$ $\quad\quad$ $H_{BP}(s)$

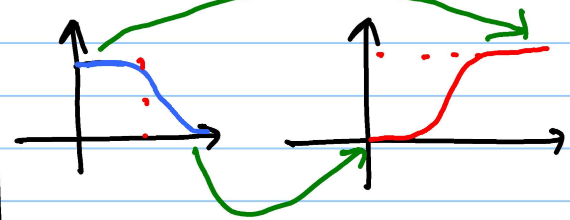
$H_{LP}(j\omega)$ $\quad\quad$ $H_{BP}(j\omega)$

$$s \longrightarrow Q\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)$$

LP $\quad\quad$ BP

$$Q\left(\frac{s^2 + \omega_0^2}{s \cdot \omega}\right)$$

$H_{LP}(s) \xrightarrow{s \to \frac{1}{s}} H_{HP}(s)$



$s = 0$
$s = \pm\infty$

$s = \pm j\omega_0$
$s = \pm\infty, 0$

$$H_{LP}(s) = \frac{1}{1+s/\omega_p} \longrightarrow H_{BP}(s) = \frac{1}{1+\frac{1}{\omega_p} Q\left(\frac{s^2+\omega_0^2}{s\omega_0}\right)}$$

$$H_{LP}\Big(\underbrace{Q\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)}_{=0}\Big) = H_{BP}(s)$$

$$= \frac{s\,\omega_0\,\omega_p/Q}{s^2+\omega_0^2 + \frac{s\,\omega_0\omega_p}{Q}}$$

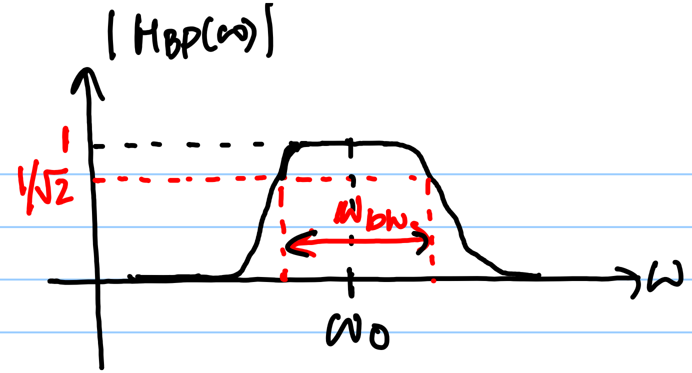
$$= \frac{s\,\omega_p/Q\omega_0}{\frac{s^2}{\omega_0^2} + \frac{s\,\omega_p}{Q\cdot\omega_0} + 1}$$

$$H_{BP}(s) = \frac{s\,\omega_p/Q\omega_0}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0}\frac{\omega_p}{Q} + 1} = \frac{s/\omega_0 Q'}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q'} + 1}$$ "2nd Order"

$$|H_{BP}(j\omega_0)| = 1$$

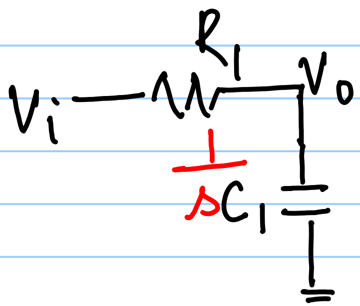
$$|H_{BP}(j\omega)|_{\omega=\omega_0+\omega_{BW}/2} = \frac{1}{\sqrt{2}}\,|H_{BP}(j\omega_0)| = 1$$

$$\omega_{BW} \approx \frac{\omega_0}{Q'} = \frac{\omega_0\,\omega_p}{Q}$$



$$\left(\frac{1}{\sqrt{2}}\right)^2 = \left|\frac{j\omega/\omega_0 Q'}{\left(1-\frac{\omega^2}{\omega_0^2}\right) + j\,\frac{\omega}{\omega_0 Q'}}\right|^2$$

$$\frac{\omega}{\omega_0} = \sqrt{1 + \frac{1}{2Q'^2} \pm \frac{1}{Q'}\sqrt{1+\frac{1}{4Q'^2}}}$$



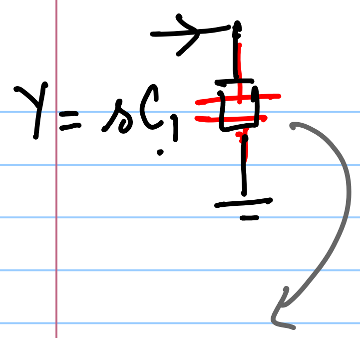
$$s \rightarrow Q\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)$$

$$Z = \frac{1}{sC_1} \longrightarrow \frac{1}{C_1}\,\frac{1}{Q\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)} = Z \quad ✓$$

$$sC_1 \longrightarrow s\,\frac{QC_1}{\omega_0} + \frac{QC_1\omega_0}{s}$$

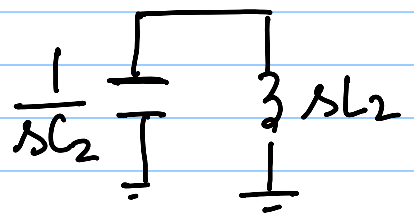
$$\frac{V_o}{V_i} = \frac{1}{1+sR_1C_1}$$

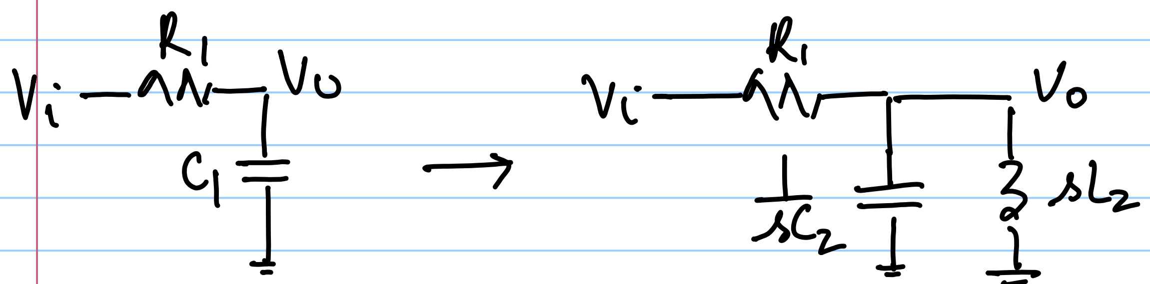
$$= \frac{1}{1+s/\omega_p} \longrightarrow \frac{s/\omega_0 Q'}{(s/\omega_0)^2 + \frac{s}{\omega_0 Q'} + 1} \quad ✓$$



$$Y = s \cdot \frac{QC_1}{\omega_0} + \frac{QC_1\omega_0}{s}$$

$$= s \cdot C_2 + \frac{1}{sL_2} \quad \text{where} \quad \underset{\sim}{C_2 = \frac{QC_1}{\omega_0}}, \quad \underline{L_2 = \frac{1}{QC_1\omega_0}}$$

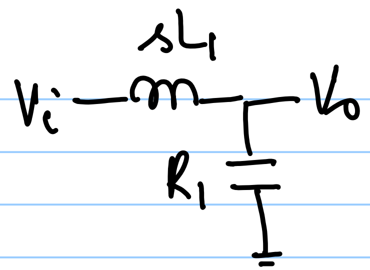




$$\frac{V_o}{V_i} = \frac{sL_2 / 1 + s^2 L_2 C_2}{R_1 + \frac{sL_2}{1 + s^2 L_2 C_2}}$$

$$= \frac{sL_2 / R_1}{s^2 L_2 C_2 + s\frac{L_2}{R_1} + 1}$$

$$= \frac{s / \omega_0 Q'}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q'} + 1}$$

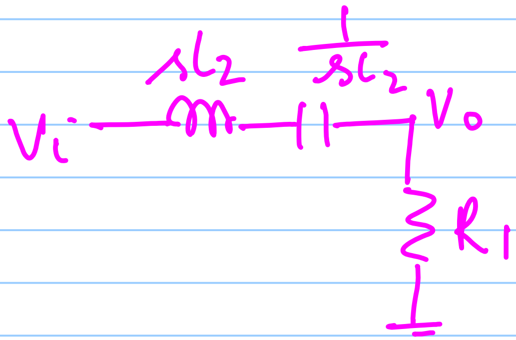


$$s \to Q\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)$$

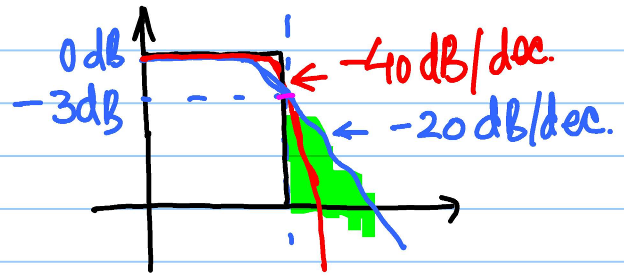
$$\frac{V_o}{V_i} = \frac{R_1}{R_1 + sL_1} = \frac{1}{1 + sL_1/R_1} = \frac{1}{1 + s/\omega_p}$$

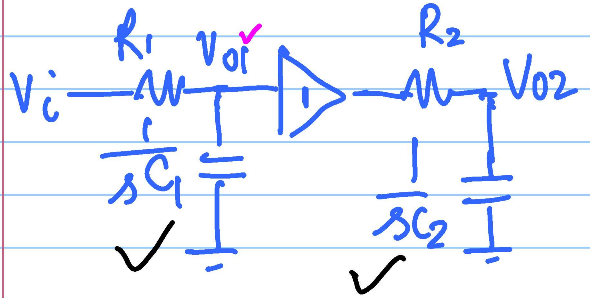
$$sL_1 \longrightarrow L_1\, Q\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right) = \frac{QL_1}{\omega_0} s + \frac{QL_1\omega_0}{s} = sL_2 + \frac{1}{sC_2}$$

$$\boxed{L_2 = \frac{QL_1}{\omega_0}, \quad C_2 = \frac{1}{QL_1\omega_0}}$$



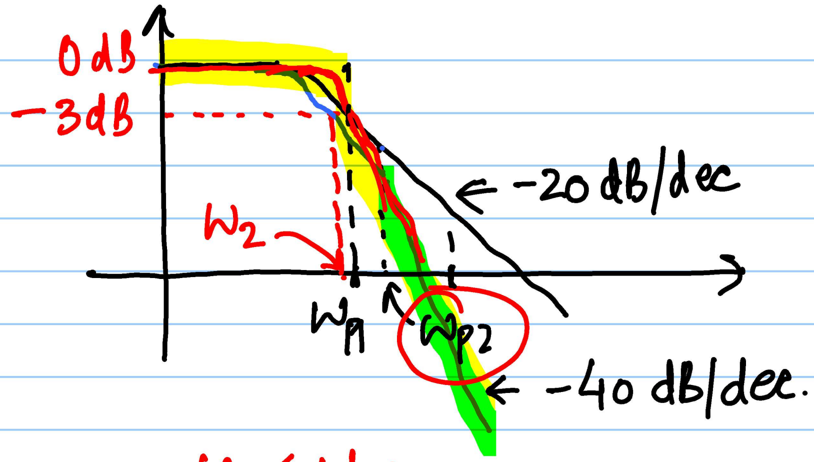
$$\omega_{BW} \approx \frac{\omega_0 \cdot \omega_p}{Q}$$



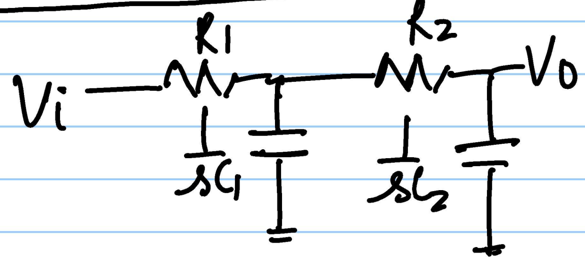


$$\frac{V_{02}}{V_i} = \frac{1}{1+sR_1C_1}\,\frac{1}{1+sR_2C_2}$$

$$= \frac{1}{1+s/\omega_{p1}}\,\frac{1}{1+s/\omega_{p2}}$$

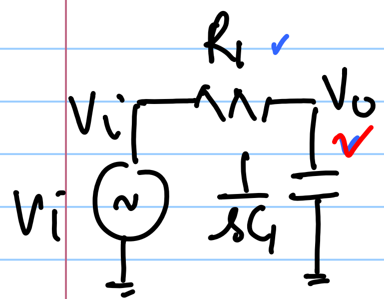


$\omega_2 < \omega_{p1}$



$$\frac{V_0}{V_i} = \frac{1}{1+s(C_2R_2+C_1R_1+C_2R_1)+s^2C_1C_2R_1R_2}$$

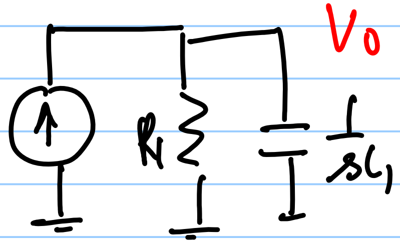
$$= \frac{1}{1+\frac{s}{\omega_p Q_p}+\frac{s^2}{\omega_p^2}}$$

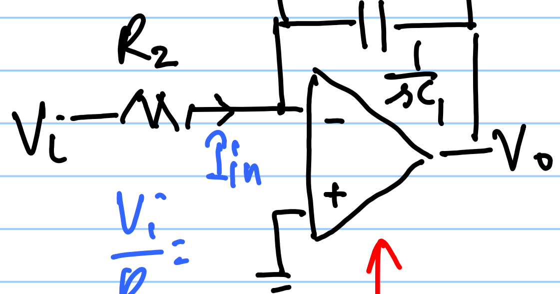


$$\frac{V_o}{V_i} = \frac{A_0}{1+sR_1C_1}$$

$$\omega_p = \frac{1}{R_1C_1}$$

$$I_{in} = \frac{V_i}{R_1}$$





$$\frac{V_i}{R_2} =$$

"Active filter"

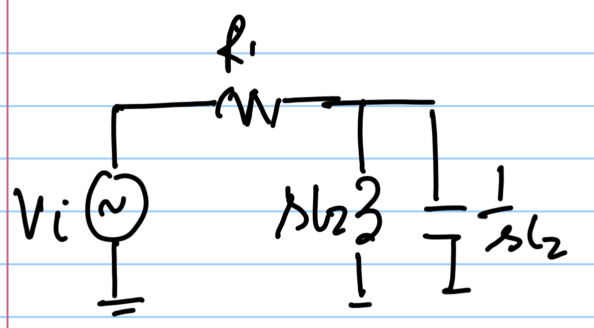
$$V_o = i_{in} \frac{R_1}{1+sC_1R_1}$$

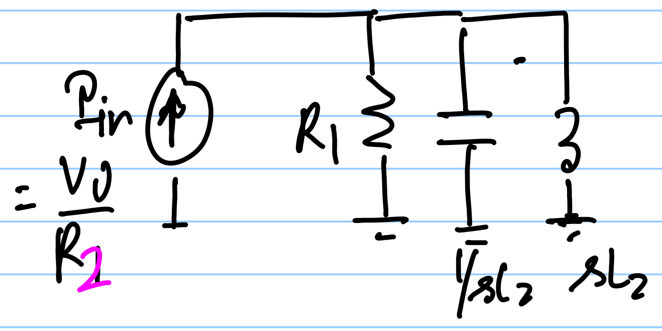
$$V_o = \frac{V_i}{R_2} \frac{R_1}{1+sC_1R_1}$$

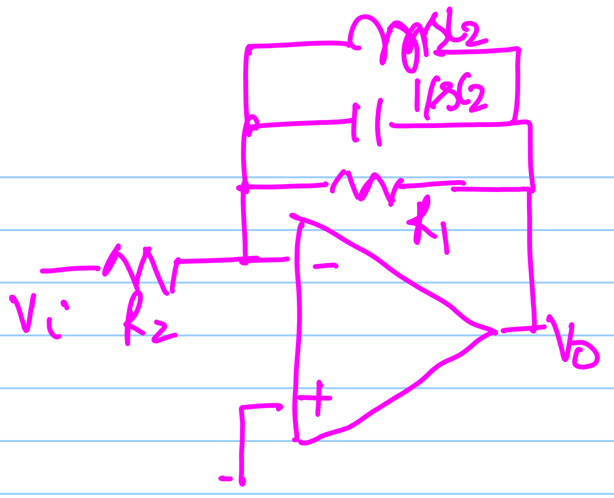
$$= V_i \frac{R_1/R_2}{1+sC_1R_1} \quad A_0$$

$$\frac{V_o}{V_i} = -\frac{R_2/R_1}{1+sC_1R_1}$$

$$= \frac{-A_0}{1+sC_1R_1}$$







$$\frac{V_o}{V_i} = -\frac{R_1}{R_2} \quad \frac{s/\omega_0 Q}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}$$