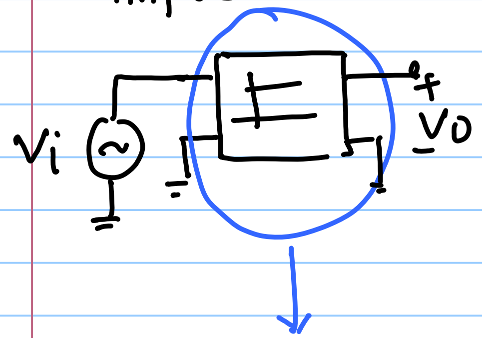
Note Title

Lecture # 35

Allpass filter.



$H(s) = \frac{V_o}{V_i} = \frac{1 + s/\omega_z}{1 + s/\omega_p} \rightarrow |H(j\omega)| = 1 \;\; \forall \omega$, if $\omega_z = \omega_p$ **Case1**.

$|H(j\omega)| = 1$

$H(s) = \frac{1 - s/\omega_z}{1 + s/\omega_p}$ **Case 2**

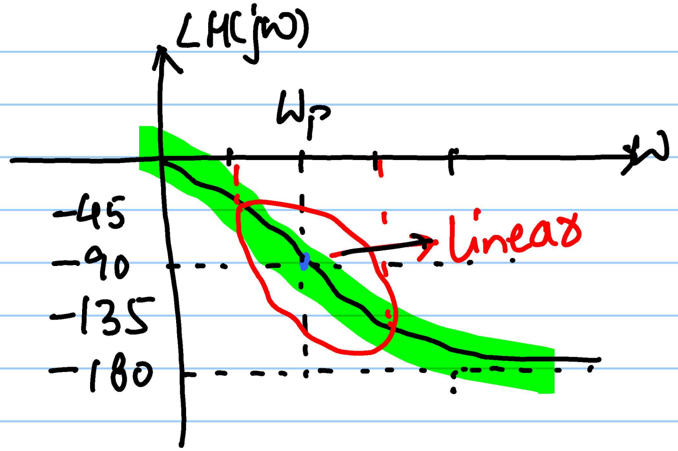
$|H(j\omega)|^2 = \frac{1 + (\omega/\omega_z)^2}{1 + (\omega/\omega_p)^2}$

$= 1$, if $\omega_z = \omega_p$

Case 1: zero is in L.H.P $\Rightarrow \measuredangle H(j\omega) = 0$

Case 2: zero is in R.H.P $\Rightarrow \measuredangle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega}{\omega_p}\right)$

$= -2\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$ for $\omega_z = \omega_p$

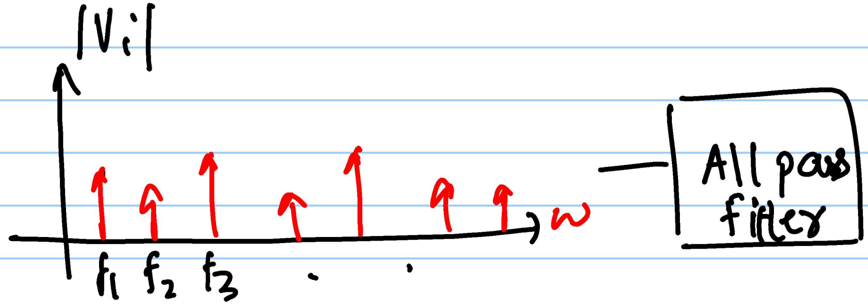


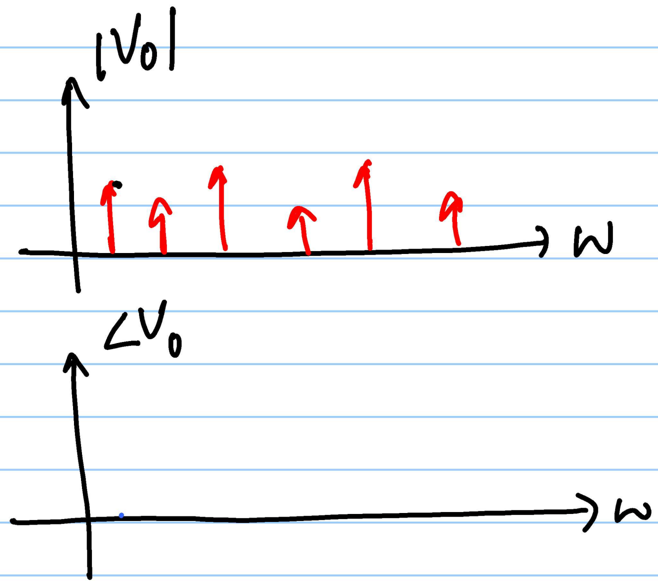
$V_i = a_1 \sin(2\pi f_1 t)$

$V_o = A \cdot a_1 \sin(2\pi f_1 t + \phi_1)$

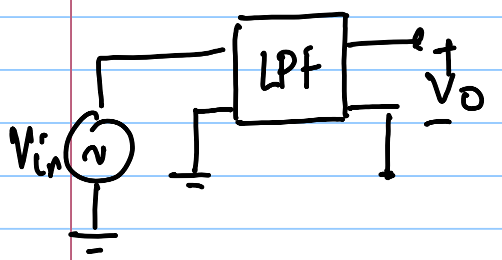
$V_o = V_i(t - \tau)$ ✓

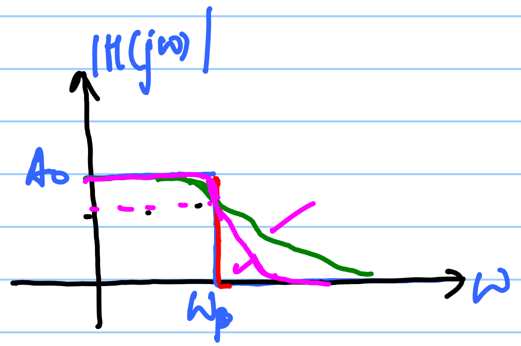
$\Delta\phi$ change.





Low Pass filter. (LPF)





$$\frac{V_o}{V_i} = H(s)$$

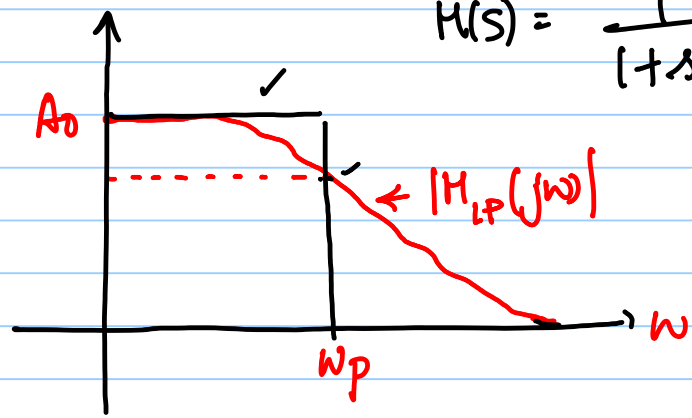
R, L, C : Passive filters.

R: impedance is constant

sL: increases w/ freq.

$\frac{1}{sC}$: decreases w/ freq.

$$H(s) = \frac{1}{1+s/\omega_p}$$



$$|H_{LP}(j\omega)|_{\omega=\omega_p} = \frac{1}{\sqrt{2}}|H_{LP}(j\omega)|_{\omega=0}$$

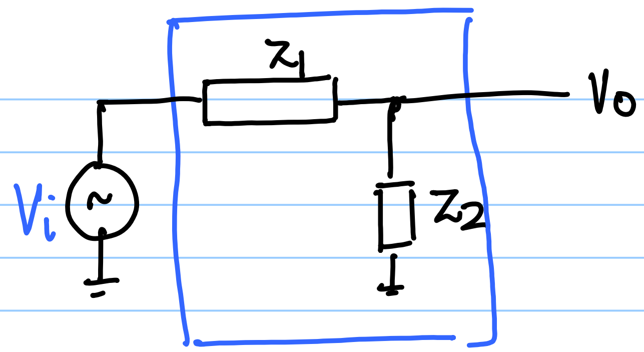
$$= \frac{A_0}{\sqrt{2}}$$

ω_p: -3dB bandwidth of the filter.

@ ω_p: $20\log_{10}\left(\frac{|H(j\omega)|_{\omega=\omega_p}}{|H(j\omega)|_{\omega=0}}\right)$

$= 20\log_{10}\left(\frac{1}{\sqrt{2}}\right)$

$= -3$ dB.

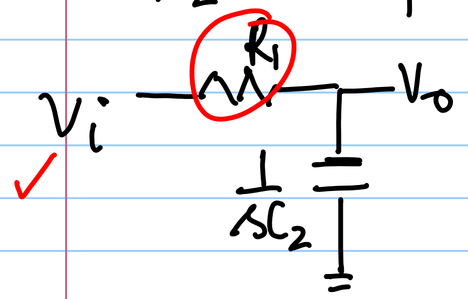


$$H(s) = \frac{V_o}{V_i} = \frac{Z_2}{Z_2 + Z_1} = \frac{1}{1 + \frac{Z_1}{Z_2}}$$

$$H(s) = \frac{1}{1 + s/\omega_p}$$

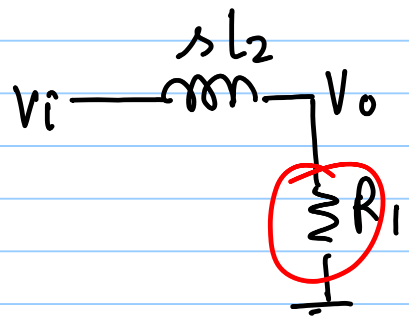
$$\frac{Z_1}{Z_2} = \frac{s}{\omega_p} = \frac{1}{\omega_p} \cdot s$$

Z_1	Z_2	Z_1/Z_2
R_1	R_2	R_1/R_2
R_1	$1/sC_2$	sC_2R_1 $= s/(1/R_1C_2)$
R_1	sL_2	R_1/sL_2
$1/sC_1$	R_2	$1/sC_1R_2$
$1/sC_1$	sL_2	$1/s^2L_2C_1$
sL_2	R_1	sL_2/R_1 $= s/(R_1/L_2)$
sL_2	$\frac{1}{sC_2}$	$s^2L_2C_2$



$$\frac{V_o}{V_i} = \frac{1}{1 + sR_1C_2} = \frac{1}{1 + s/(1/(R_1C_2))} = \frac{1}{1 + s/\omega_p}$$

$$\omega_p = \frac{1}{R_1C_2}$$



$$\frac{V_o}{V_i} = \frac{R_1}{R_1 + sL_2} = \frac{1}{1 + sL_2/R_1} = \frac{1}{1 + s/\omega_p}$$

$$\omega_p = \frac{R_1}{L_2} = \frac{1}{C_2 R_1}$$

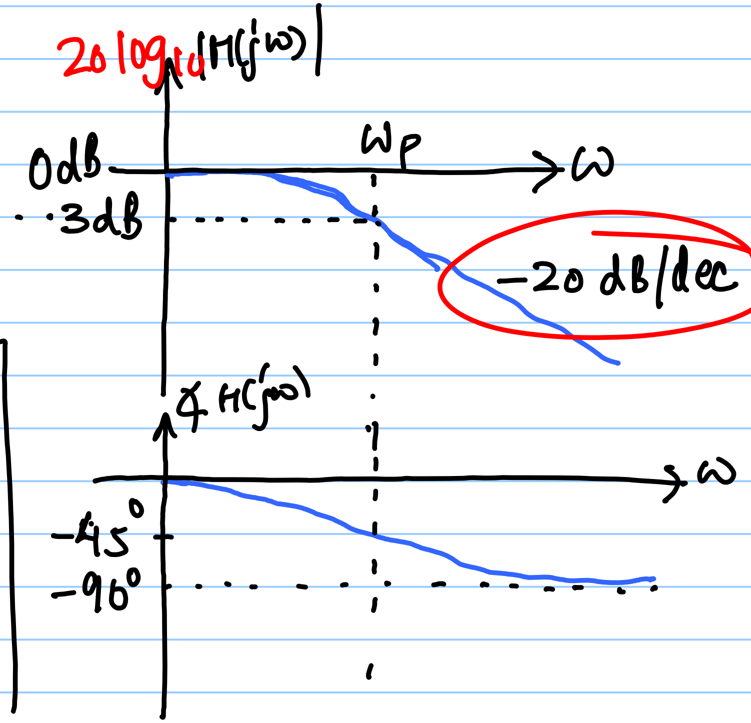
$$H(s) = \frac{1}{1 + s/\omega_p}$$

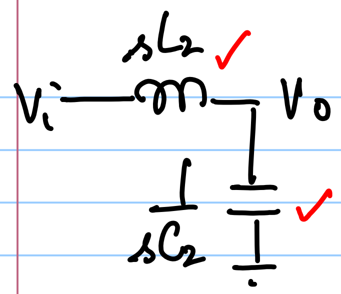
$$|H(0)| = 1 \rightarrow 0 \text{ dB}$$

$$|H(j\omega_p)| = \frac{1}{\sqrt{2}} \rightarrow -3\text{dB}$$

@ low freq or $s=0$, $\frac{Z_1}{Z_2} = 0$

@ high freq. $\frac{Z_1}{Z_2} \rightarrow \infty$

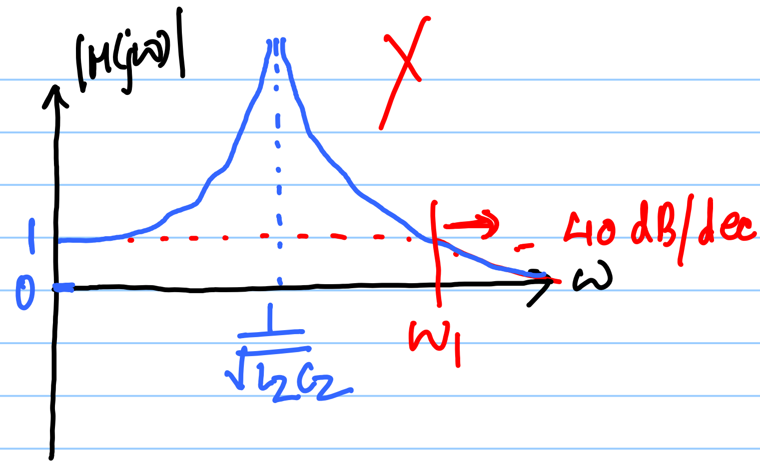




$$H(s) = \frac{V_o}{V_i} = \frac{1}{1+s^2 L_2 C_2}$$

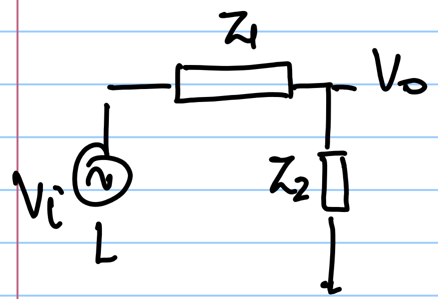
$$|H(0)| = 1$$

$$|H(\infty)| \to 0$$

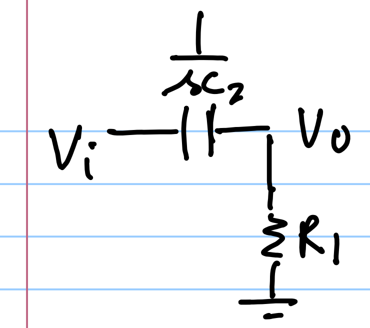


High Pass filter. (HPF)

$$H_{HP}(s) = \frac{V_o}{V_i} = \frac{s/\omega_p}{1+s/\omega_p} = \frac{s}{\omega_p+s} = \frac{1}{\frac{\omega_p}{s}+1} = \frac{1}{1+\frac{Z_1}{Z_2}}$$

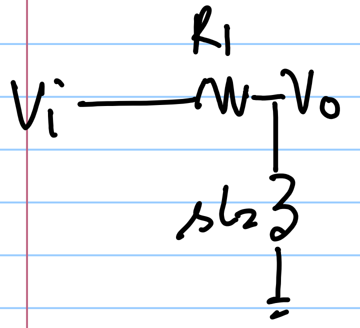


$$\frac{Z_1}{Z_2} = \frac{\omega_p}{s}$$



$$\frac{V_o}{V_i} = \frac{sC_2R_1}{1+sC_2R_1}$$

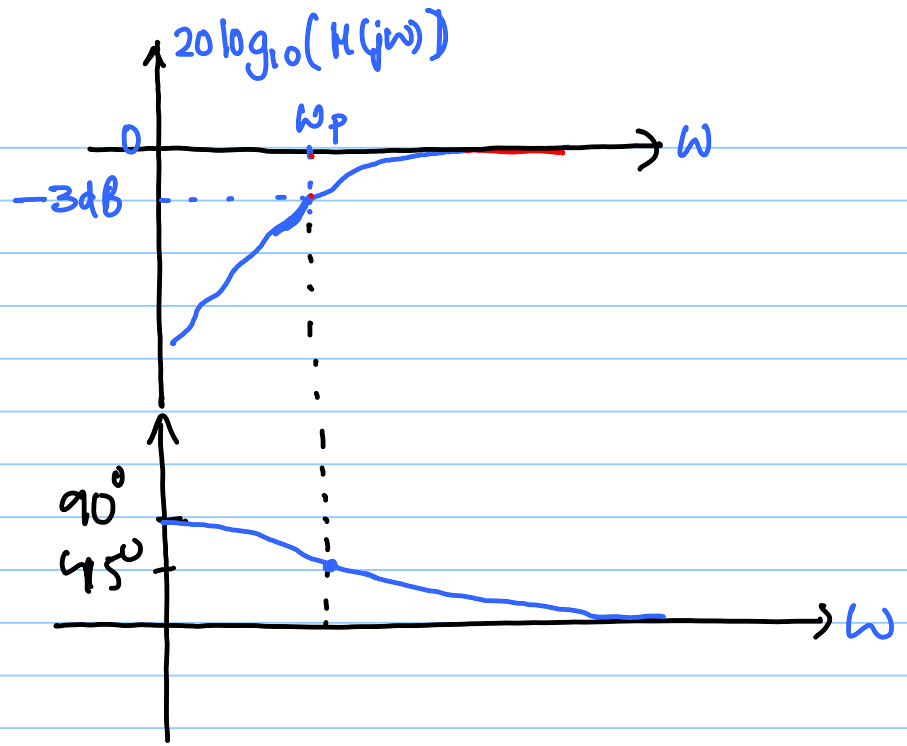
$$\omega_p = 1/R_1C_2$$



$$\frac{V_o}{V_i} = \frac{sL_2}{sL_2+R_1}$$

$$= \frac{sL_2/R_1}{1+sL_2/R_1}$$

$$\omega_p = \frac{R_1}{L_2}$$



$$|H(j\omega)|_{\omega=\omega_p} = \frac{1}{\sqrt{2}} \, |H(j\omega)|_{\omega\to\infty}$$

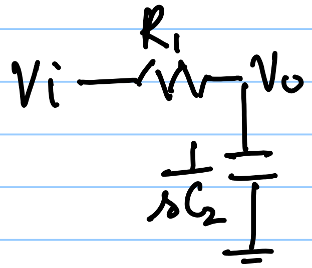
$$\left|\frac{j\omega/\omega_p}{1+j\omega/\omega_p}\right| = \frac{1}{\sqrt{2}} \times 1$$

@ $\omega = \omega_p$.

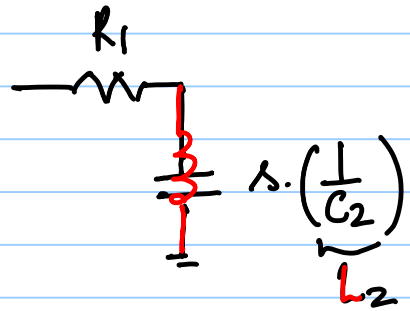
$$\left|\frac{j\cdot 1}{1+j\cdot 1}\right| = \frac{1}{\sqrt{2}}$$

$$\angle H(j\omega) = 90^\circ - \tan^{-1}\frac{\omega}{\omega_p}$$

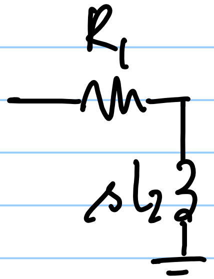
Order of the filter : # of poles in filter transfer function.



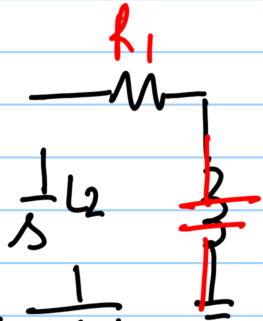
$$\xrightarrow[LP \to HP]{s \to \frac{1}{s}}$$



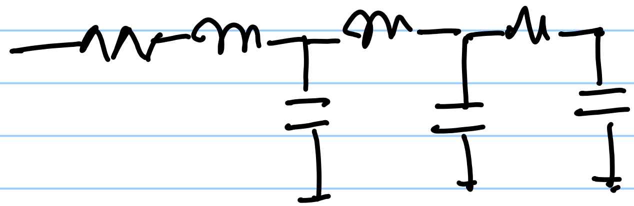
$$\frac{sL_2}{R_1 + sL_2} = \frac{sL_2/R_1}{1 + sL_2/R_1} = \frac{s/R_1C_2}{1 + s/R_1C_2}$$



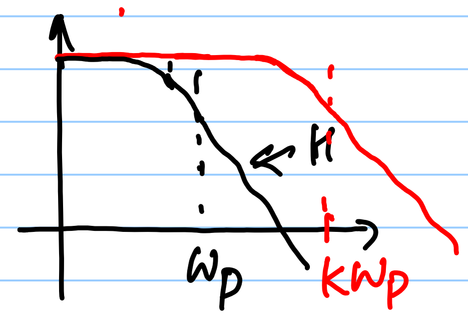
$$\xrightarrow[HP \to LP]{s \to \frac{1}{s}}$$



$$\vdots \ \frac{1}{sC_2} = \frac{1}{s(1/L_2)}$$



$$s \to \frac{1}{s}$$



$$H(s) = \frac{1}{1 + s/\omega_p} \xrightarrow{s \to s/K} H_n(s) = \frac{1}{1 + s/K\omega_p}$$

$$\left| H_n(k\omega_p) \right| = \left| \frac{1}{1 + j\frac{k\omega_p}{k\omega_p}} \right| = \left| \frac{1}{1+j} \right|$$