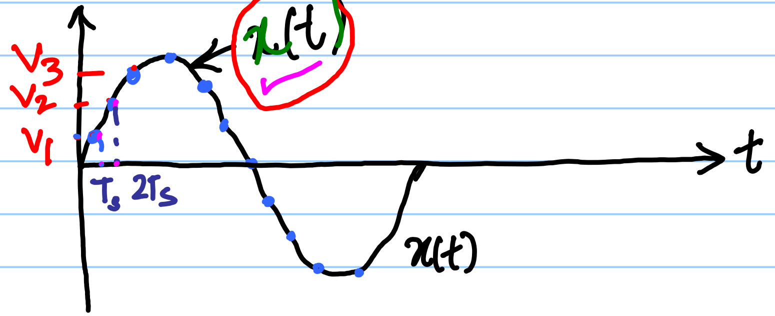
Note Title

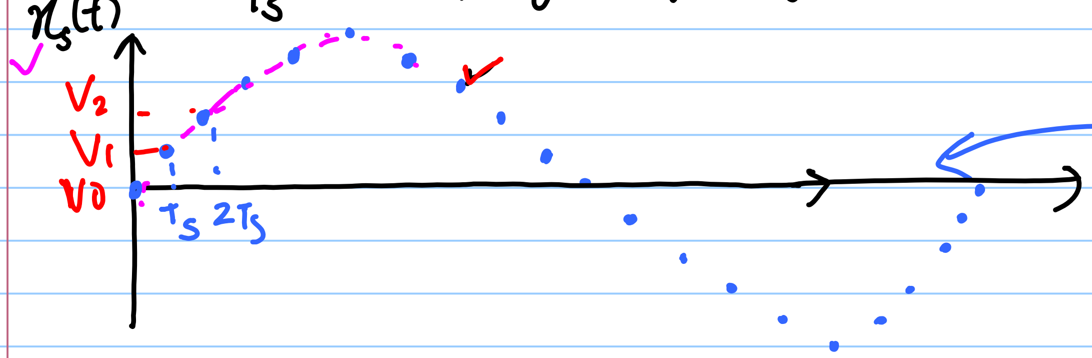
Lecture # 34

Analog Filters



t	Voltage
0	0
T_s ✓	$V_1 \rightarrow$ Bits
$2T_s$ ✓	$V_2 \rightarrow$ Bits
⋮	⋮

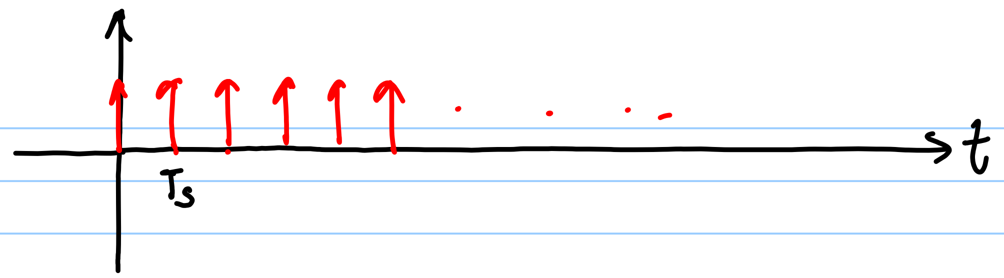
$f_s = \frac{1}{T_s}$ sampling frequency.



$$x_s(t) = x(nT_s)$$

$$= x(t) \times \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$ ✓

$$\sum_{k=-\infty}^{\infty} \delta(t-kT_s) = \sum a_n \cos(n\omega_s t) + b_n \sin(n\omega_s t)$$



$$\delta(t-kT_s) = \begin{cases} 1, & t = kT_s \\ 0, & \text{otherwise.} \end{cases}$$

$$\sum_{k=-\infty}^{\infty} \delta(t-kT_s) = \sum_{n=0}^{\infty} a_n \cos(n\omega_s t) + b_n \sin(n\omega_s t) \quad ; \; \omega_s = \frac{2\pi}{T_s}$$

$$= \frac{2}{T_s} \sum \cos(n\omega_s t)$$

$$= \frac{2}{T_s} \sum_{n=0}^{\infty} \frac{e^{jn\omega_s t} + e^{-jn\omega_s t}}{2}$$

$$= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{jn\omega_s t}$$

$$x_s(t) = x(t) \times \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

Fourier Series for periodic fn.

$$= x(t) \times \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{jn\omega_s t}$$

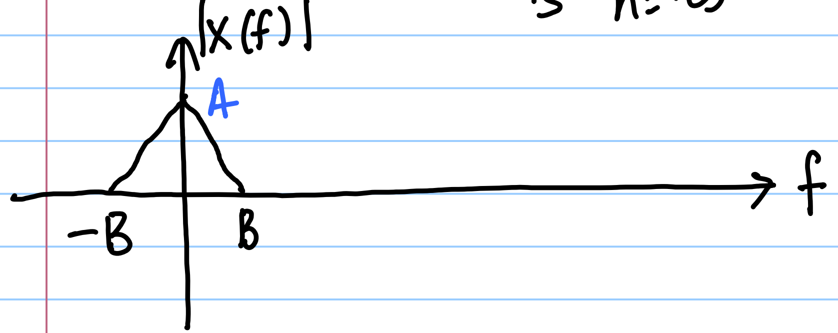
$$x_s(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} x(t) e^{jn\omega_s t}$$

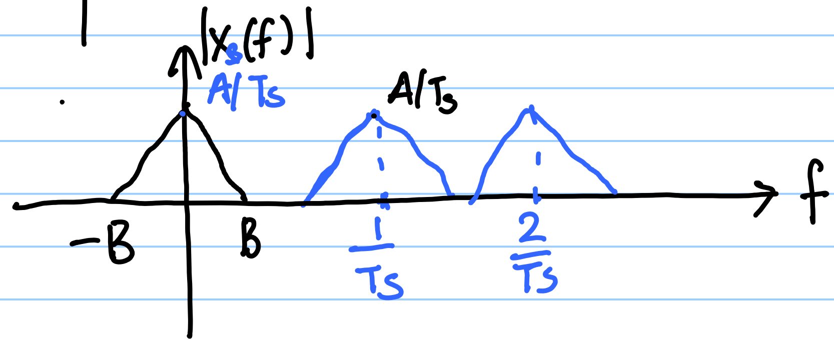
$$x(t) \xrightarrow{\mathcal{F}} X(j\omega)$$

$$x(t) \cdot e^{jn\omega_s t} \xrightarrow{\mathcal{F}} X(j(\omega - n\omega_s))$$

$$X_s(j\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(j(\omega - n\omega_s))$$

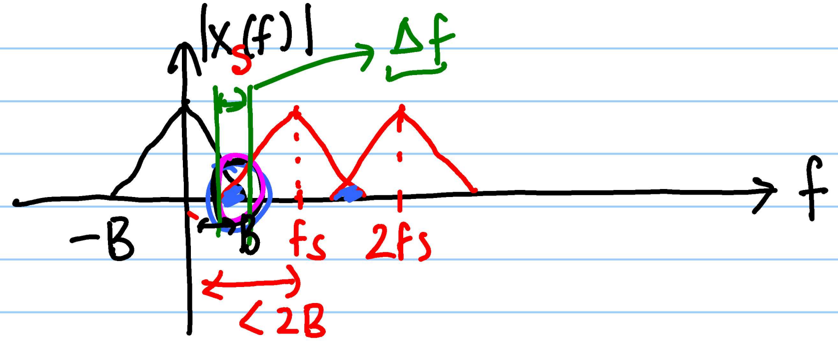
$$X_s(j2\pi f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(j2\pi\left(f - \frac{n}{T_s}\right)\right)$$

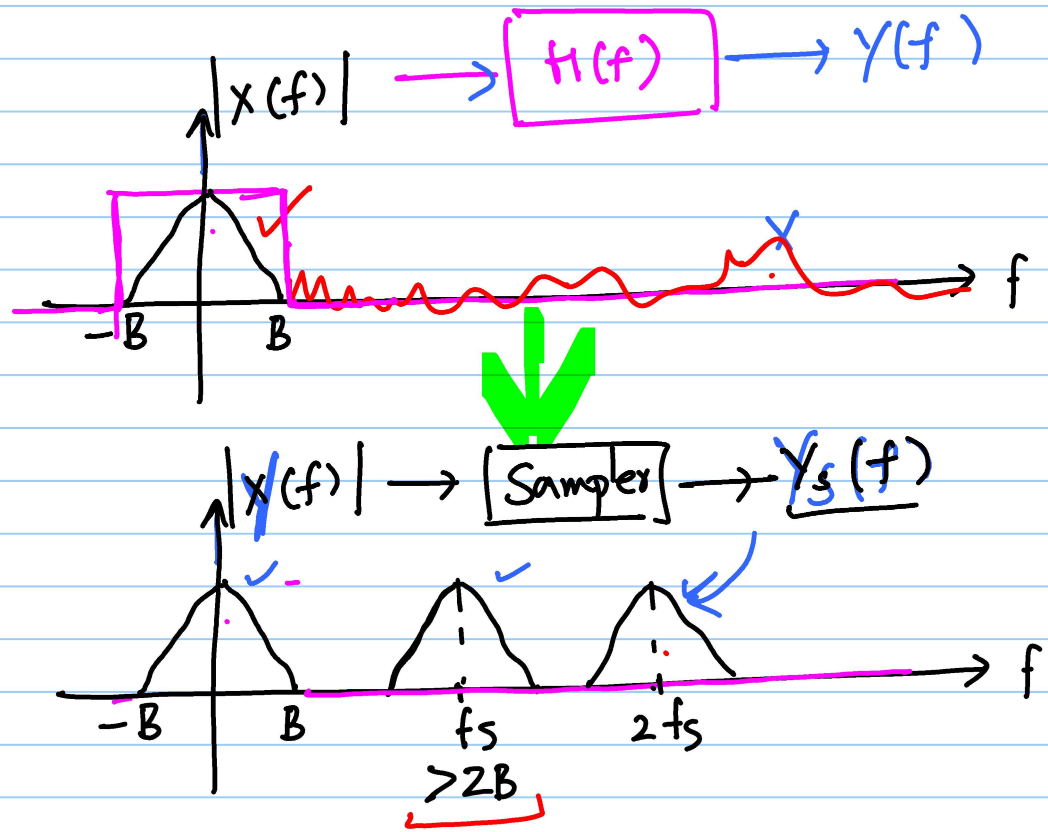




$$X_s(j2\pi f) = \frac{1}{T_s}\Big[\; X(f) + X\left(f-\frac{1}{T_s}\right) + X\left(f+\frac{1}{T_s}\right) + X\left(f-\frac{2}{T_s}\right) + X\left(f+\frac{2}{T_s}\right) + \cdots$$

$$X\left(f-\frac{1}{T_s}\right)\Bigg|_{f=\frac{1}{T_s}} = X(0)$$

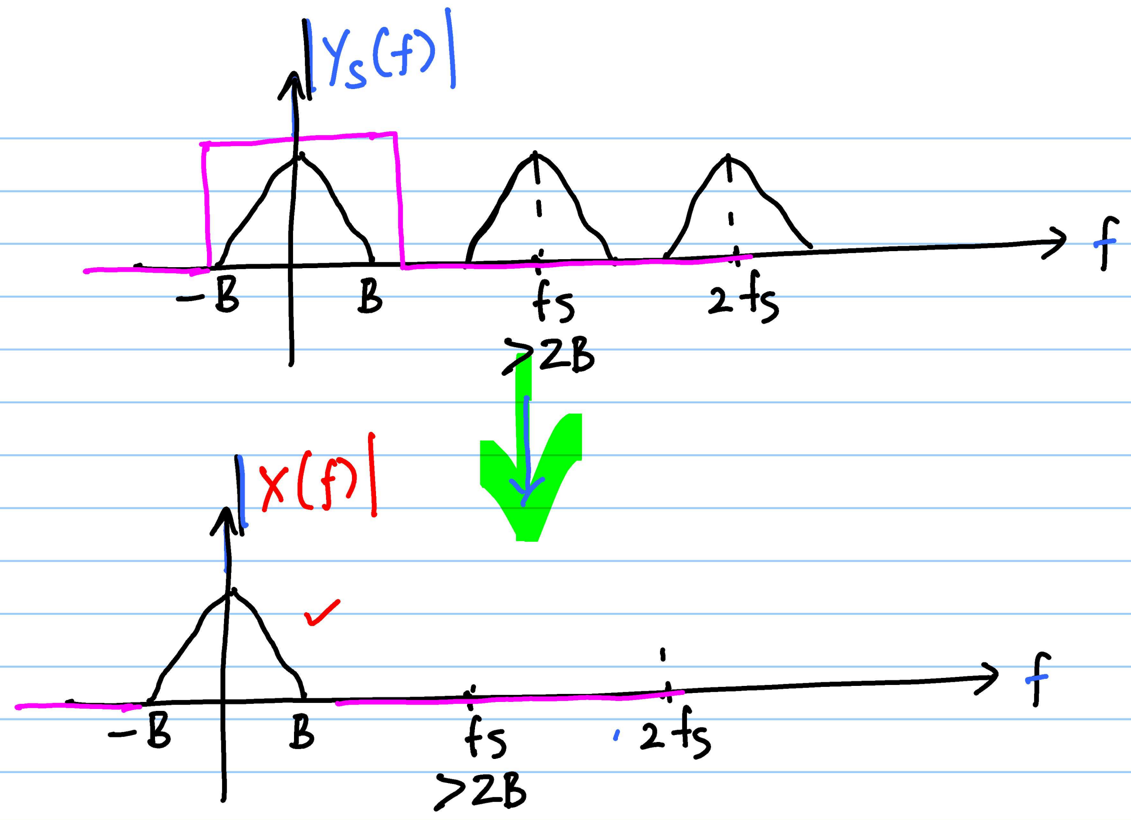




if $\frac{1}{T_s} = f_s < 2B$

$f_s > 2B$

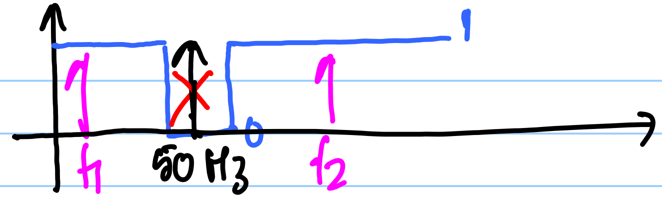
$y_s(t) \xrightarrow{\mathcal{F}} Y_s(f)$

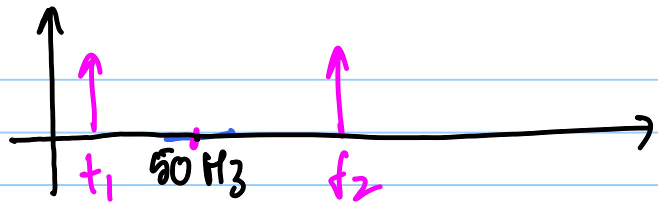


Analog-to-digital converter : low pass filter

$x(t) \longrightarrow$ $\boxed{\text{Low Pass Filter}}$ $\longrightarrow$ $\boxed{\text{Sampler}}$ $\longrightarrow$

$f_s > 2B$





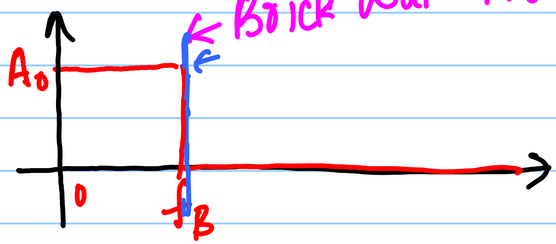
220V, 50 Hz $\longrightarrow$ $\boxed{\text{Electrical App.}}$ $\longrightarrow$ $V_o(t)$

50 Hz

Ideal filters:

Low Pass Filter (LPF)

Brick wall filter



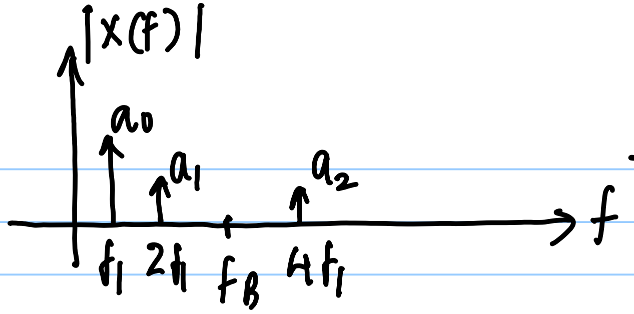
$X(f) \longrightarrow \boxed{\begin{matrix} LPF \\ (H(f)) \end{matrix}} \longrightarrow Y(f)$

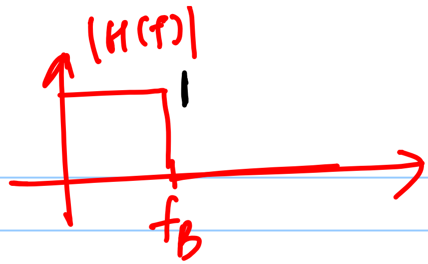
$$\frac{Y(s)}{X(s)} = H(s)$$

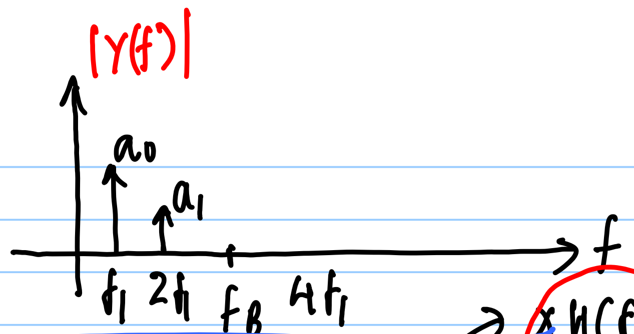
LPF with $|H(f)| = A_0 \; ; \; 0 \le f \le f_B$

$= 0 \; ; \; f > f_B$

✓ $\angle H(f) \Longrightarrow$ Group delay -







$$x(t) = a_0 \sin(2\pi f_1 t) + a_1 \sin(4\pi f_1 t)$$
$$+ a_2 \cancel{\sin(8\pi f_1 t)}$$

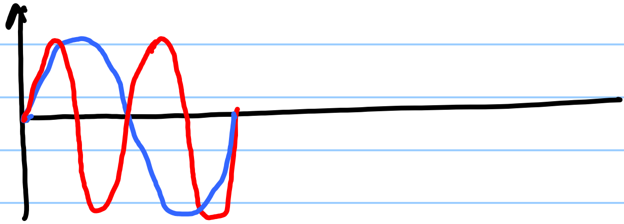
$$y(t) = a_0 \sin(2\pi f_1 t + \phi_0)$$
$$+ a_1 \sin(4\pi f_1 t + \phi_1)$$

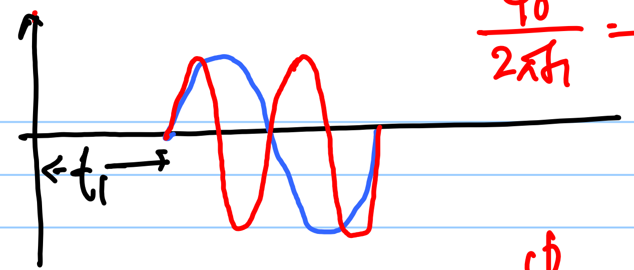
$\angle H(f_1)$

$\angle H(f_2)$

$$y(t) = a_0 \sin(2\pi f_1 t + \phi_0) + a_1 \sin(4\pi f_1 t + \phi_1)$$
$$= a_0 \sin\left(2\pi f_1 \left(t + \underbrace{\frac{\phi_0}{2\pi f_1}}\right)\right) + a_1 \sin\left(4\pi f_1 \left(t + \underbrace{\frac{\phi_1}{4\pi f_1}}\right)\right)$$
$$= x(t + t_1)$$

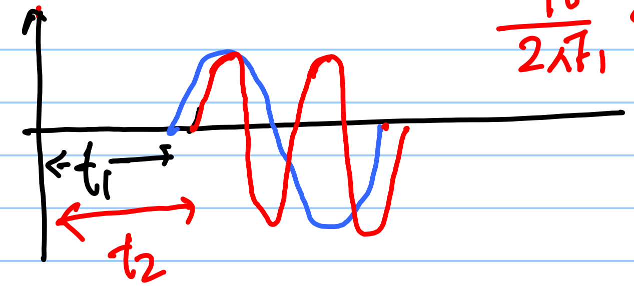
$$\frac{\phi_0}{2\pi f_1} = \frac{\phi_1}{4\pi f_1}$$





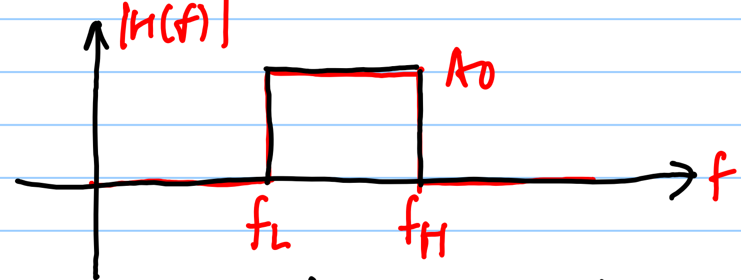
$\frac{\phi_0}{2\pi f_1} = \frac{\phi_1}{4\pi f_1}$ ✓

$\frac{\measuredangle H(f)}{2\pi f} = \frac{\measuredangle H(\omega)}{\omega} = \text{constant.}$



$\frac{\phi_0}{2\pi f_1} < \frac{\phi_1}{2\pi f_1}$ ✓

Band Pass filter (BPF)

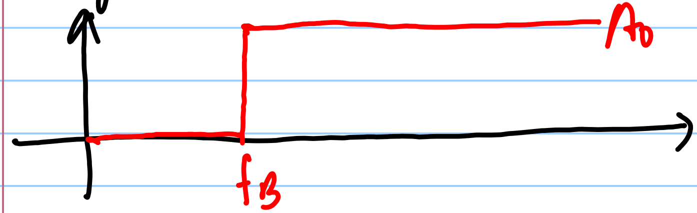


$|H(f)| = A_0, \quad f_L \leq f \leq f_H$

$= 0, \quad \text{otherwise.}$

Bandwidth $= f_H - f_L$

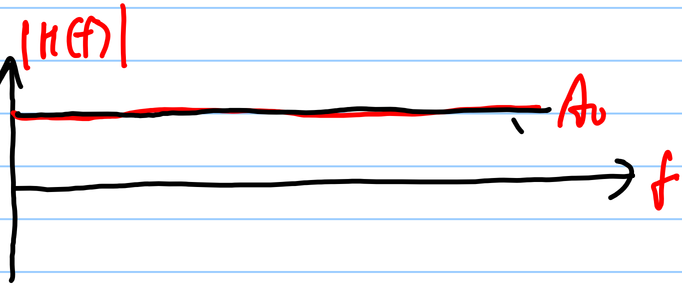
High Pass filter (HPF)



$|H(f)| = A_0, \quad f \geq f_B$

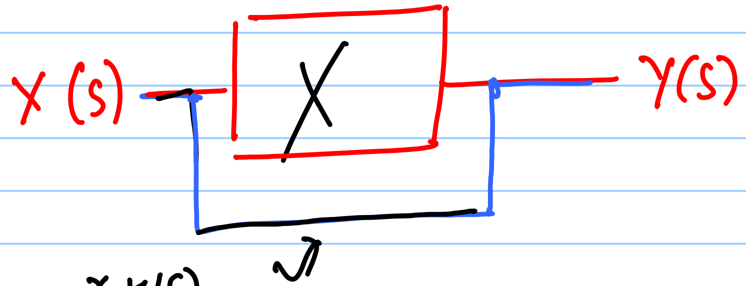
$= 0, \quad \text{otherwise}$

Band Reject Filter.



Allpass Filter

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{1+s/\omega_{p1}} \quad , \omega_p \text{ is large}$$



$$|H(s)| = 1 \quad ; \quad \omega \leq \omega_p$$

$$\measuredangle H(f) = -\tan^{-1}\left(\frac{\omega}{\omega_{p1}}\right)$$