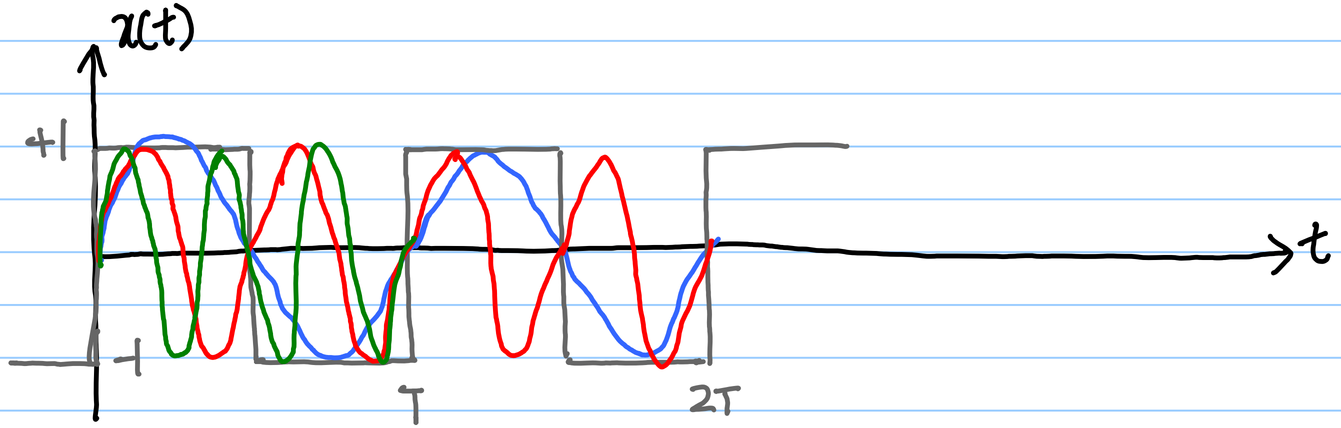
Note Title

Lecture # 33

Analog Filters



$x(t)$: square wave

$$x(t) = \sum a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \checkmark = \sum c_n \sin(n\omega_0 t + \phi_n)$$

where $\omega_0 = \frac{2\pi}{T}$

sinusoidal components at $n.\omega_0 = n.\frac{2\pi}{T}$: $n \geq 1$

$$n \cdot f = \frac{n}{T}$$

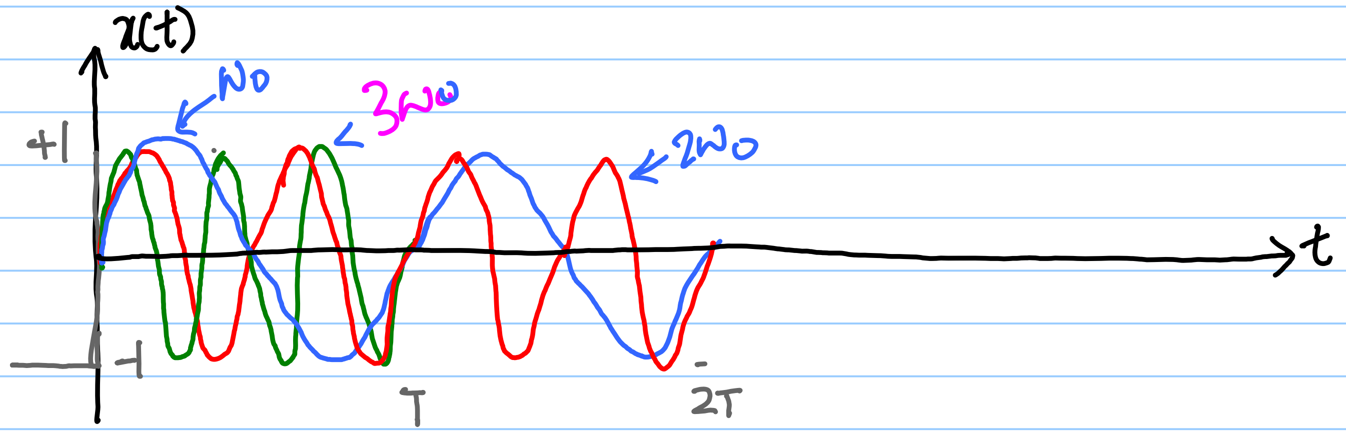
$x(t)$:

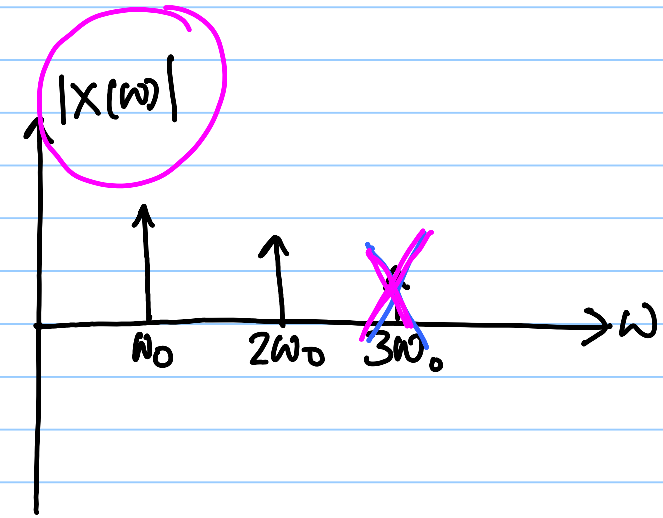
$$a_1 \sin(\omega_0 t) = a_1 \sin\left(2\pi \cdot \frac{t}{T}\right)$$

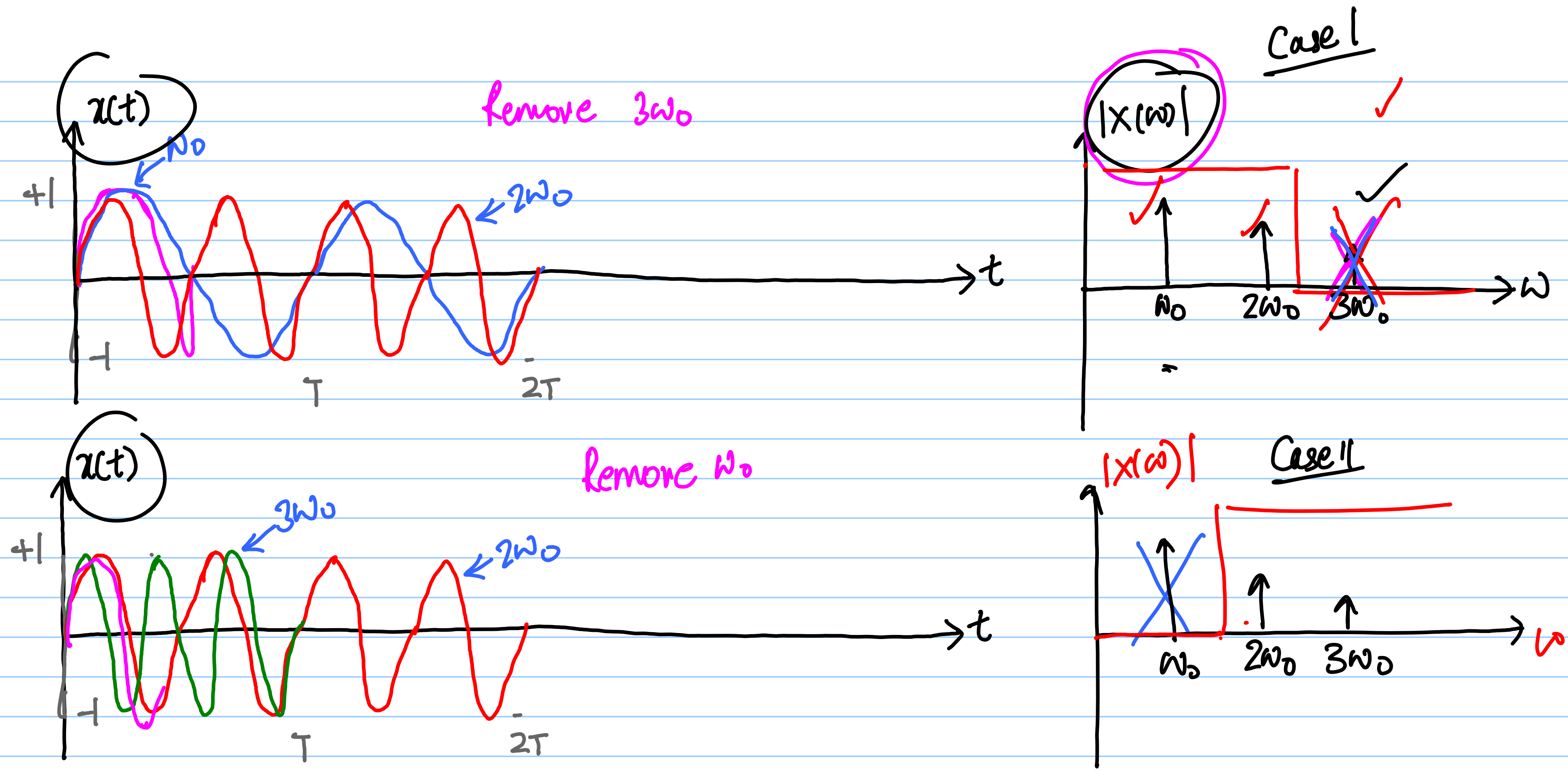
$$a_2 \sin(2\omega_0 t) = a_2 \sin\left(2\pi \frac{t}{T/2}\right)$$

$$a_3 \sin(3\omega_0 t) = a_3 \sin\left(2\pi \frac{t}{T/3}\right)$$

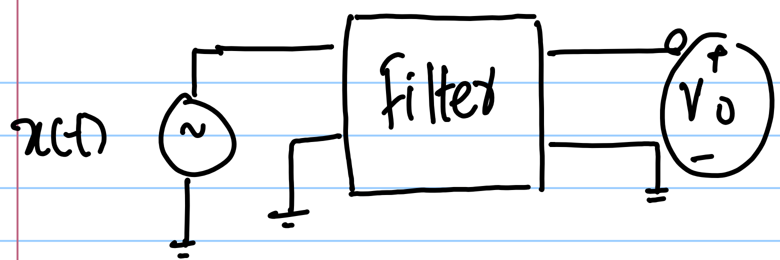
⋮







Removing a selected frequency component from a signal : filtering.



$x(t) = a_1 \sin(\omega_0 t) + a_2 \sin(2\omega_0 t) + a_3 \sin(3\omega_0 t)$

Case I $\quad V_o(t) \doteq a_1 \sin(\omega_0 t) + a_2 \sin(2\omega_0 t)$

Case II $\quad V_o(t) = \times + a_2 \sin(2\omega_0 t) + a_3 \sin(3\omega_0 t)$

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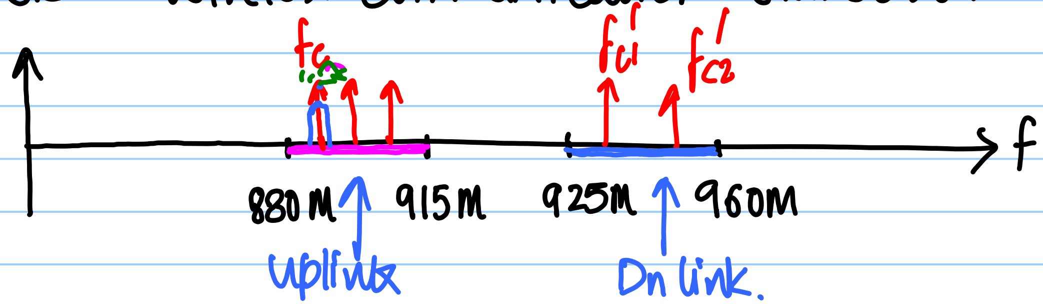
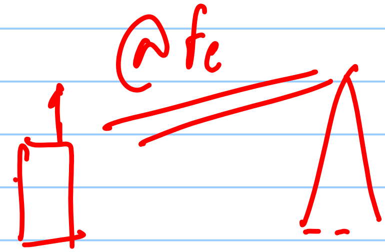
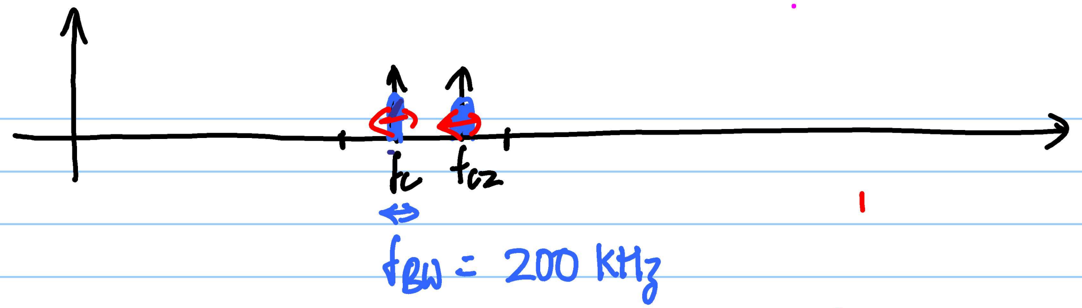
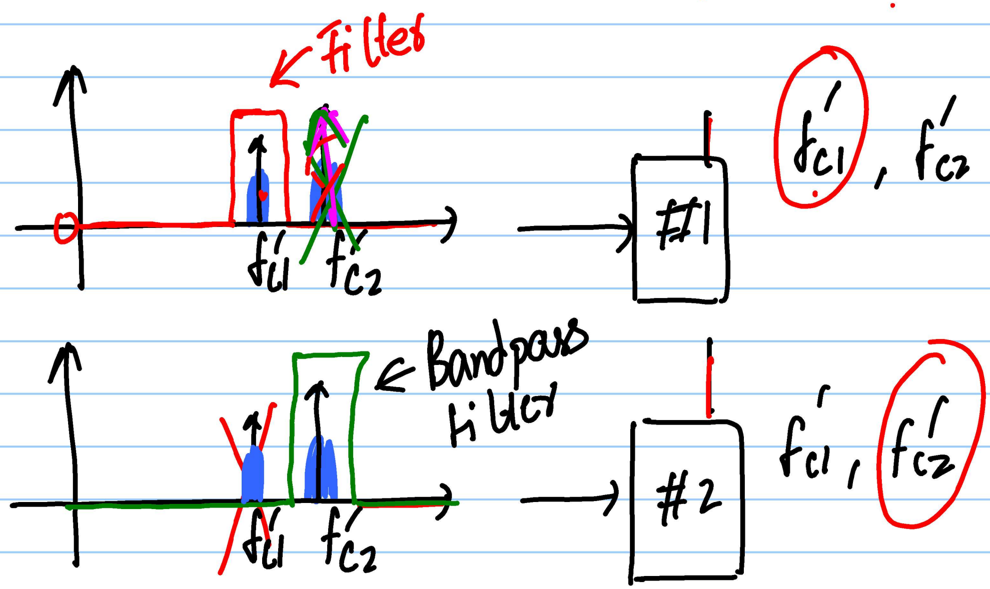
1. Communication

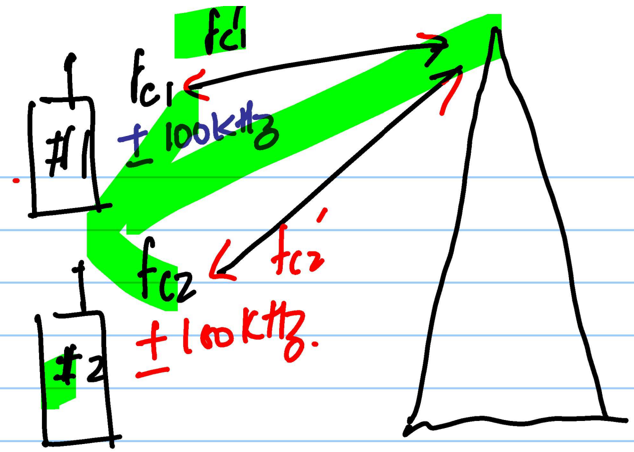
GSM wireless communication standard.

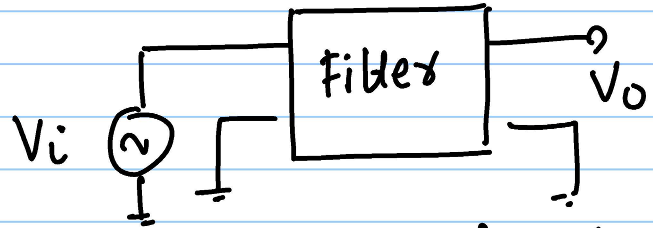




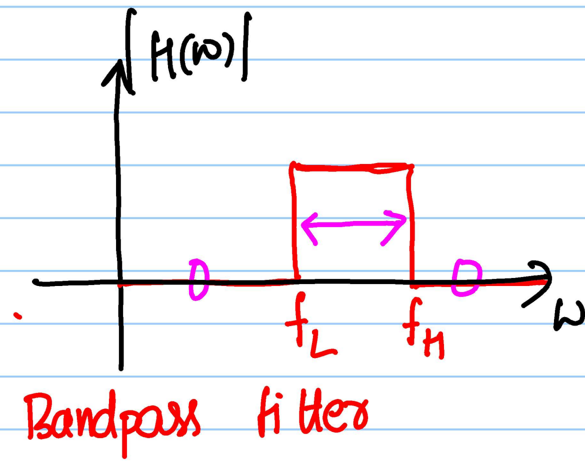

fc
fc2
fBW = 200 KHz


Filter
f'c1
f'c2
#1
f'c1, f'c2
Bandpass filter
f'c1
f'c2
#2
f'c1, f'c2


fc1
#1
fc1
± 100kHz
fc2
f'c2
#2
fc2
± 100kHz.


Vi
Filter
Vo

$$\frac{V_o}{V_i} = H(s)$$

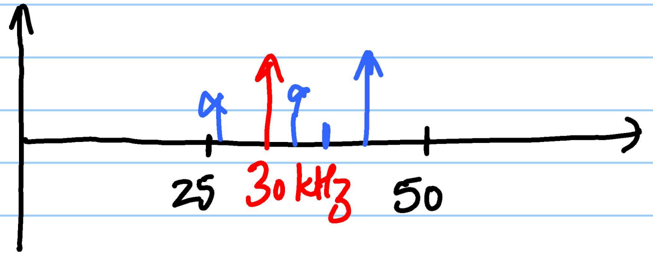

|H(ω)|
0
0
fL
fH
ω
Bandpass filter

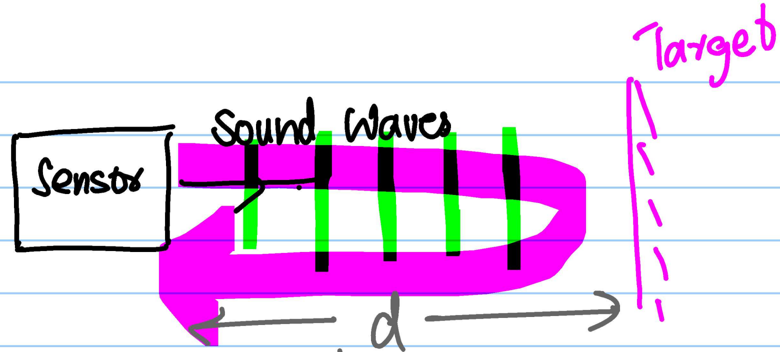
2. Sensors:

Ultra sonic sensors:

Electrical Signal ⇄ Sound wave

25 – 50 kHz



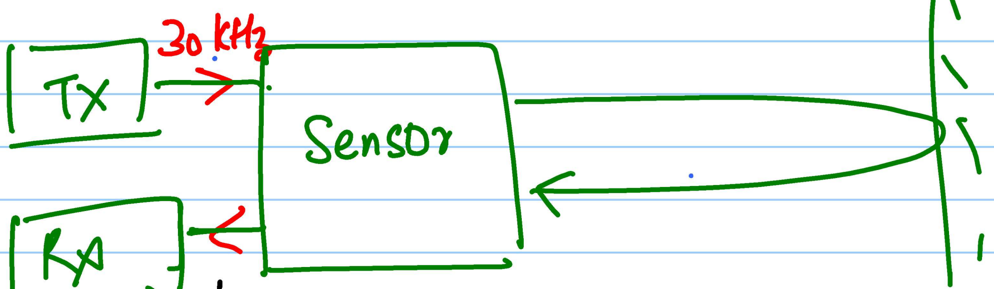


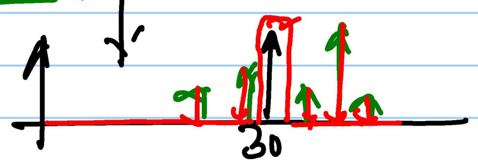
T_1 : soundwave leaves the sensor.

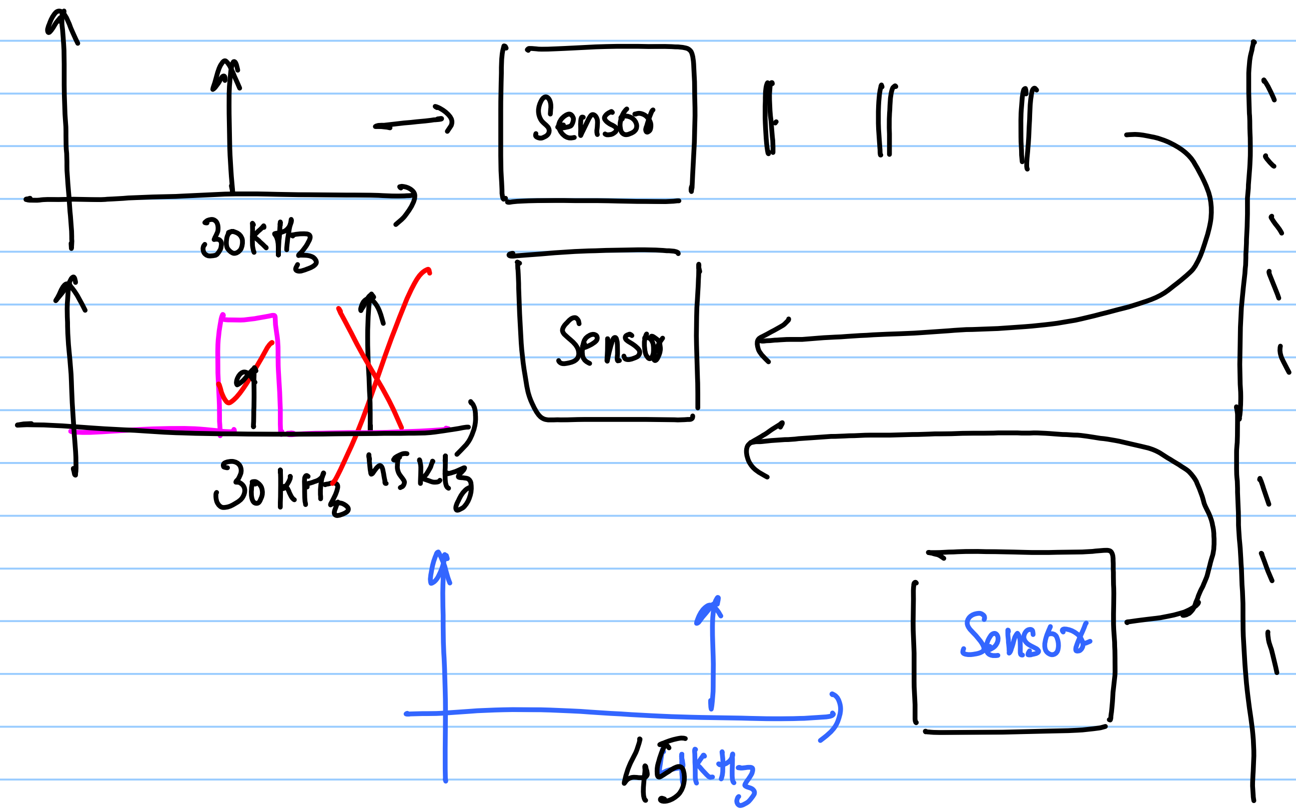
T_2 : sound wave comes back after reflection

$T_2 - T_1$ = Time of flight.

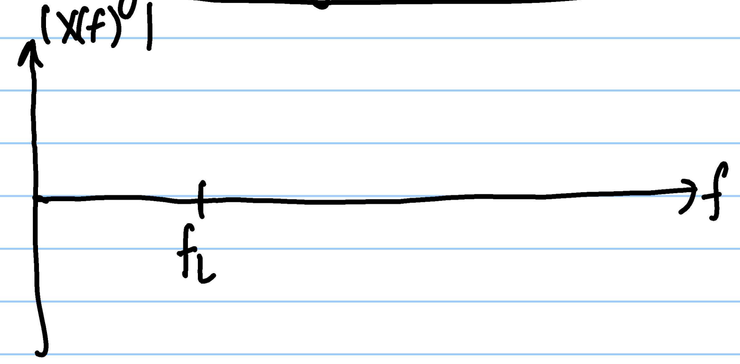
$d = speed \times (T_2 - T_1)$






Sensor
30kHz
Sensor
30 kHz
45kHz
Sensor
45kHz

3. **Analog-to-Digital Conversion.**



$x(t)$: High quality audio signal.

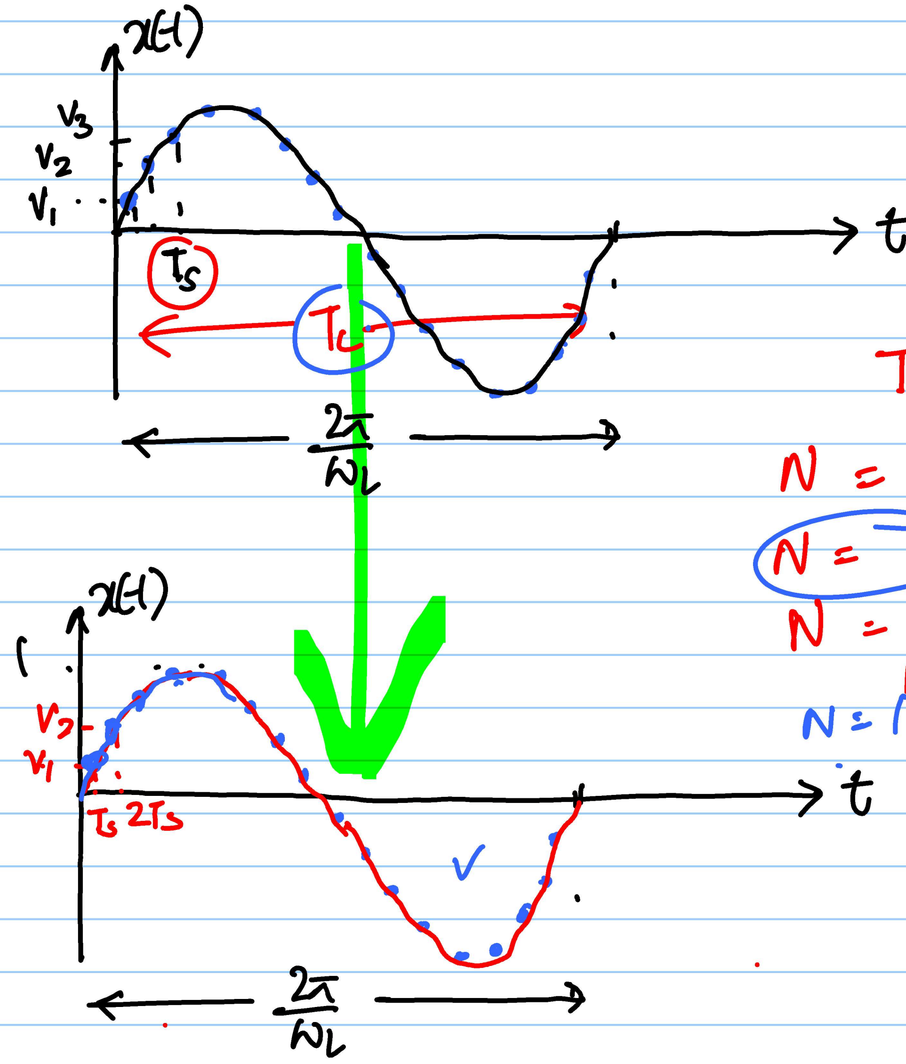
$|X(f)| \le f_L$

T_s : V_1 = 0.012569432 ✓

$2T_s$: V_2 :

$3T_s$: V_3 — 0.0125694 ✓

$4T_s$: — 0.0125 ✓



$T_L = NT_s$

$N = 1$

$N = 2$

$N = 16$

$N = 1024$

# of bits used to save voltage values → resolution of signal

$NT_s \overset{?}{\neq} T_L$: filters to reject unwanted signals.