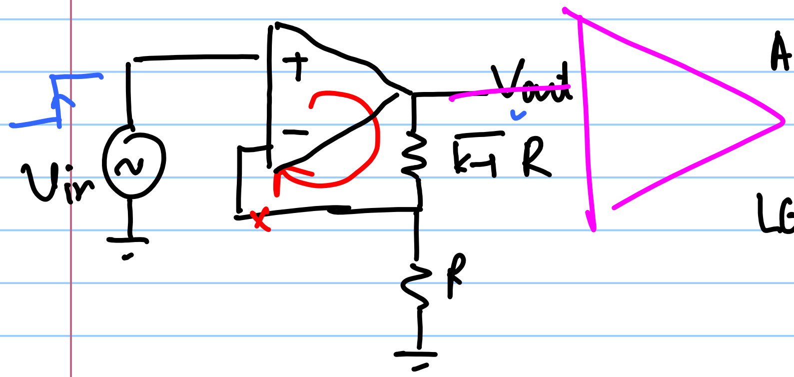
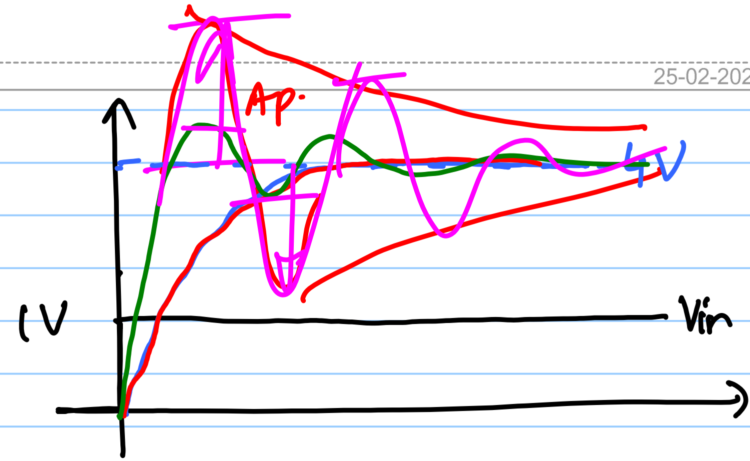
Lecture # 18



$$A(s) = \frac{A_0}{1+s/p_1}$$

$$LG(s) = \frac{A_0}{1+\frac{s}{p_1}} \times \frac{1}{K}$$



Second Order System

$$A(s) = \frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)}$$

$$LG = \frac{1}{K} \frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$

$$|LG| = \frac{A_0}{K} \frac{1}{\sqrt{1+\left(\frac{\omega_n}{p_1}\right)^2}\sqrt{1+\left(\frac{\omega_n}{p_2}\right)^2}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{\underbrace{s^2+2\zeta\omega_n s+\omega_n^2}}$$

$$\zeta \geq 1$$

$$\zeta = \frac{1}{2} \frac{1}{\sqrt{1+\frac{A_0}{K}}} \left(\sqrt{\frac{p_1}{p_2}}+\sqrt{\frac{p_2}{p_1}}\right)$$

$$\omega_u \approx \frac{A_0 p_1}{K}$$ ✓

Ex. $A_0 = 100$, $K = 4$

$$\zeta \approx \frac{1}{2} \frac{1}{\sqrt{\frac{A_0}{K}}} \left(\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}} \right)$$

$\zeta = 1$:

$$\zeta = \frac{1}{2} \sqrt{\frac{K}{A_0}} \left(\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}} \right)$$

if $p_2 \gg p_1 \Rightarrow$

$$\cancel{\frac{p_1}{p_2}} + \frac{p_2}{p_1} + \cancel{2} = 4 \frac{A_0}{K}$$

$$\boxed{\frac{p_2}{p_1} \approx 4 \frac{A_0}{K}}$$

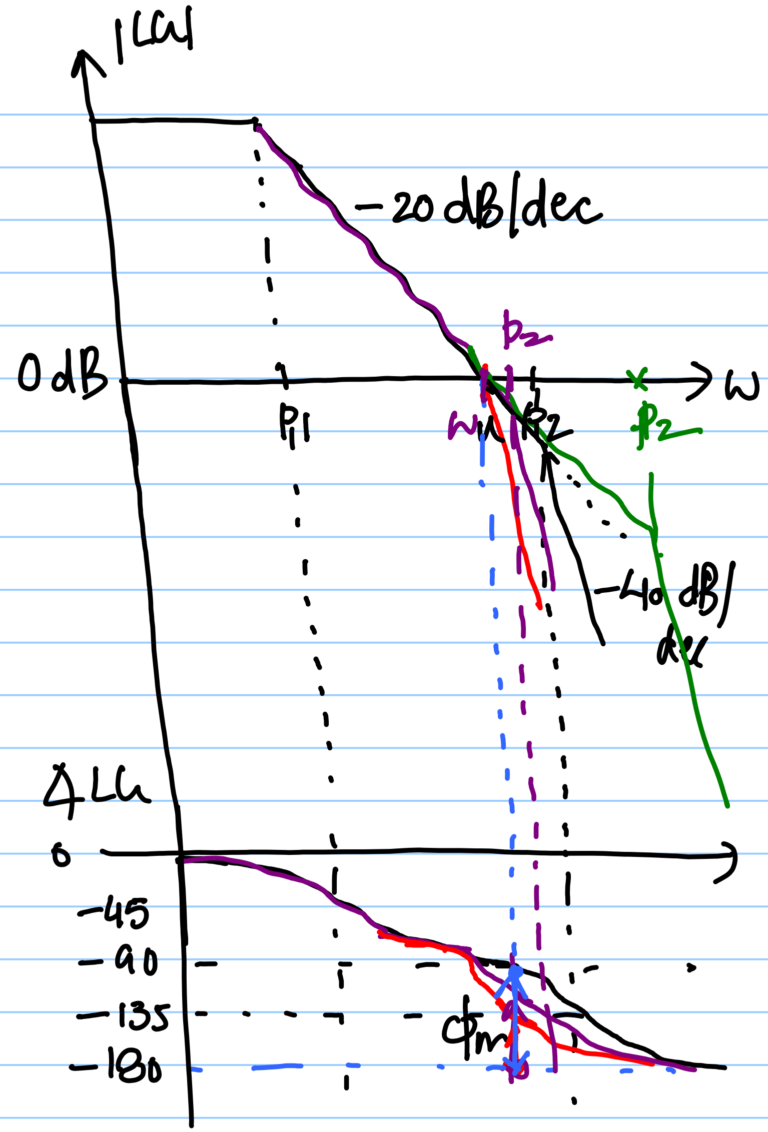
$$p_2 = 4 \underbrace{\frac{A_0 p_1}{K}}$$

$$= 4\omega_u =$$

$$\frac{p_1}{p_2} + \frac{p_2}{p_1} + 2 = 4\zeta^2 \frac{A_0}{K}$$

$$\frac{p_2}{p_1} = 4\zeta^2 \frac{A_0}{K}$$

✓ $p_2 = 4\zeta^2 \omega_u$



$$\Rightarrow \frac{\omega_u}{p_2} = \frac{1}{4\zeta^2}$$

$$\angle G = -\tan^{-1}\left(\frac{\omega}{p_1}\right) - \tan^{-1}\left(\frac{\omega}{p_2}\right)$$

$$\phi_m = \angle G(\omega_u) - (-180^\circ)$$

$$= 180^\circ - \underbrace{\tan^{-1}\left(\frac{\omega_u}{p_1}\right)}_{90} - \tan^{-1}\left(\frac{\omega_u}{p_2}\right)$$

$$= 90^\circ - \tan^{-1}\left(\frac{\omega_u}{p_2}\right)$$

$$\phi_m = 90^\circ - \tan^{-1}\left(\frac{1}{4\zeta^2}\right)$$

$$\phi_m \Big|_{\zeta=1} \doteq 76^\circ$$

$$p_2 = 2\omega_u \Rightarrow \zeta = \frac{1}{\sqrt{2}}$$

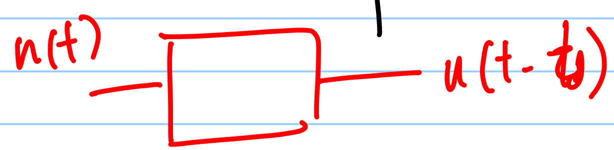
$$\phi_m = 90^\circ - \tan^{-1}\left(\frac{1}{2}\right) =$$

$p_2 = \omega_u$: $\phi_m = 45^\circ$

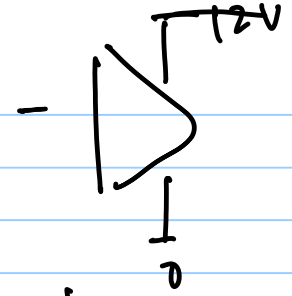
$$G_1(s) = \frac{A_0}{\left(1+\frac{s}{p_1}\right)K}$$

$$G_2(s) = \frac{A_0}{K\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$

$p_2 = 40\omega_u$, $\omega_u = \frac{A_0 p_1}{K}$



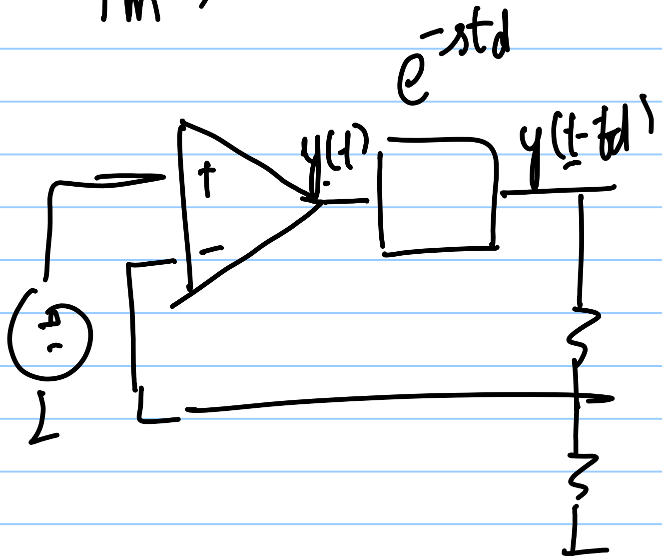
$$LG = \frac{A_0 \left(1+\frac{s}{p_2}\right)}{k\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)} \underbrace{e^{-st_d}}$$

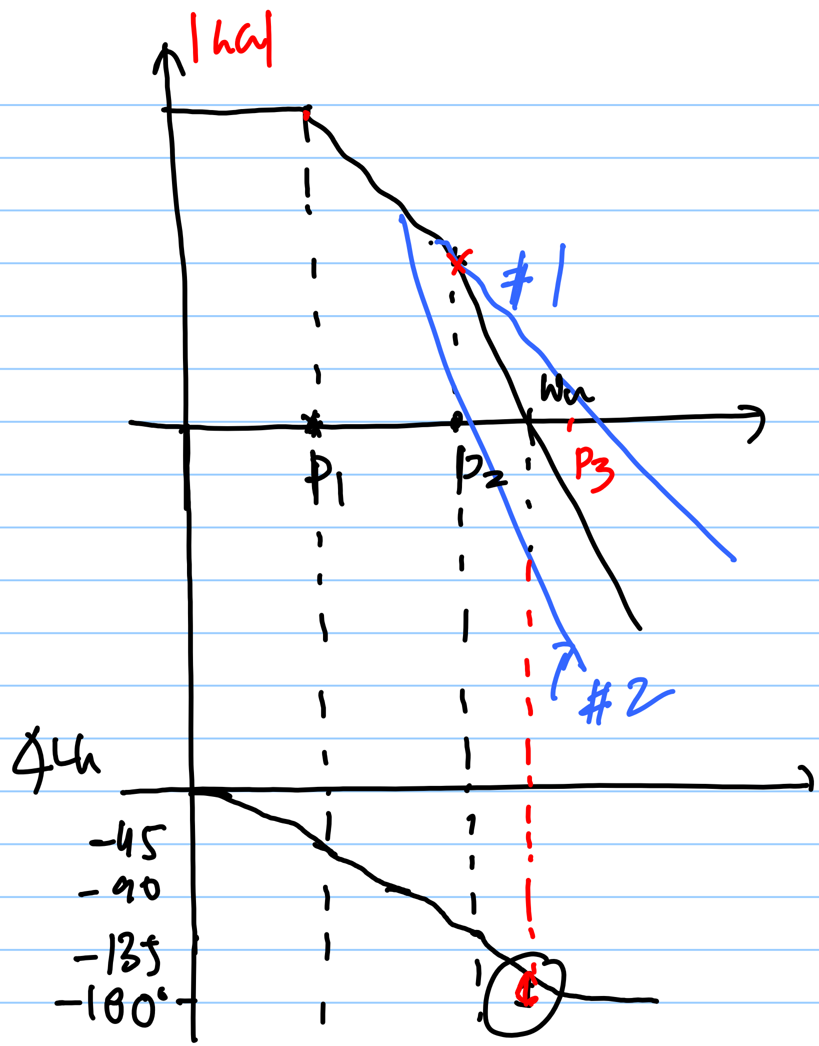


$$|LG| = 1 \Rightarrow \frac{A_0}{k\left(\frac{\omega}{p_1}\right)\left(\frac{\omega}{p_2}\right)} = 1$$

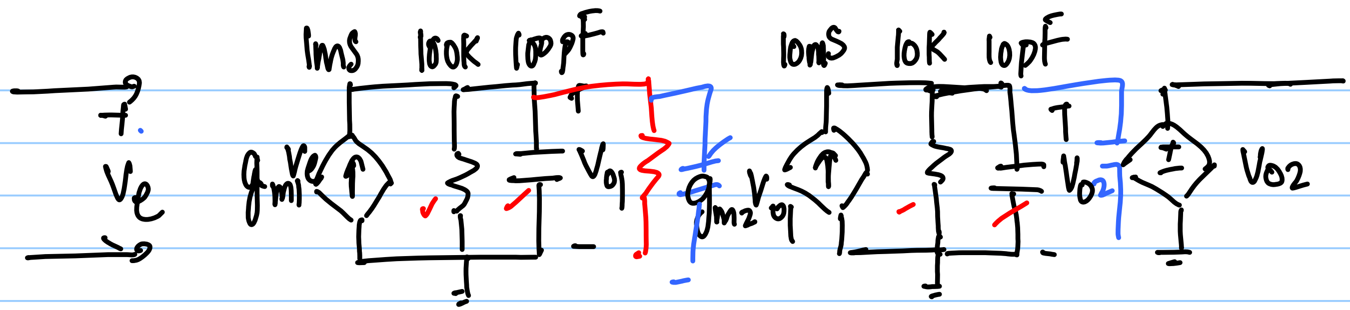
$$\omega = \sqrt{\frac{A_0}{k} p_1 \cdot p_2}$$

$$\phi_m > 0$$





Ex:



$$g_{m1}R_{o1} = 100$$

$$p_1 = \frac{1}{100\times10^3\times100\times10^{-12}} = 10^5$$

$$g_{m2}R_{o2} = 100$$

$$p_2 = \frac{1}{10\times10^3\times10\times10^{-12}} = 10^7 \text{ rad/s}$$

$$\omega_u = \frac{A_0 p_1}{K} = \frac{10^4\times10^5}{4} = 2.5\times10^7 \text{ rad/s}$$

$$\omega_u = \sqrt{\frac{A_0}{K}p_1 p_2} = \sqrt{\frac{10^4}{4}\times10^5\times10^7} = \frac{10^8}{2} = 5\times10^7$$

$$\phi_m = 180^\circ - \tan^{-1}\left(\frac{5\times10^7}{10^5}\right) - \tan^{-1}\left(\frac{5\times10^7}{10^7}\right) = 11.4^\circ$$