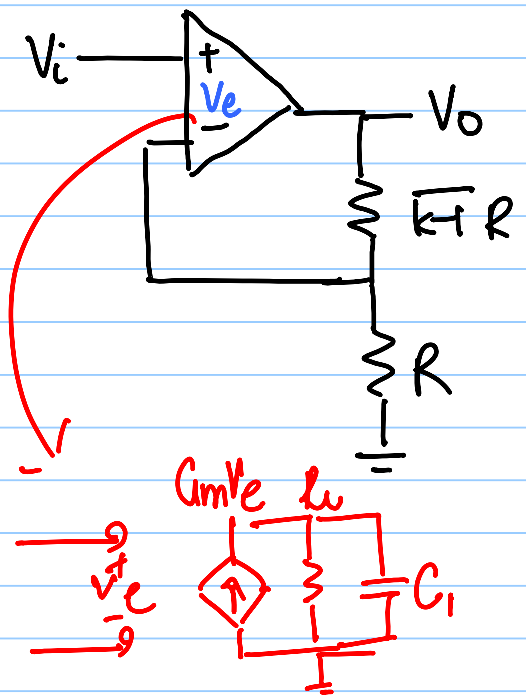
Lecture # 16



$$\frac{V_o}{V_e} = A(s) = \frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$ Gain of amp.

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{1+\frac{K}{A(s)}} = \frac{K}{1+\frac{K}{A_0}\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$

$$= \frac{K}{1+\frac{K}{A_0}} \times \frac{1}{1+\frac{s\,K/A_0}{(1+K/A_0)}\left(\frac{1}{p_1}+\frac{1}{p_2}\right) + \frac{s^2}{p_1 p_2}\,\frac{K/A_0}{1+K/A_0}}$$

$$\frac{V_o}{V_i} = \frac{A_{DC}}{1+\frac{s}{\omega_p Q_p}+\frac{s^2}{\omega_p^2}} = \frac{A_{DC}\cdot\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$A_{DC} = G_{m1} R_{o1} G_{m2} R_{o2}$

$$\omega_p = \omega_n = \sqrt{p_1 p_2 \left(1 + \frac{A_0}{K}\right)}$$

$$\frac{1}{\omega_p Q_p} = \left(\frac{1}{p_1} + \frac{1}{p_2}\right) \frac{1}{1 + A_0/K}$$

$$Q_p = \frac{1}{\sqrt{p_1 p_2 \left(1 + \frac{A_0}{K}\right)} \left(\frac{1}{p_1} + \frac{1}{p_2}\right) \frac{1}{1 + \frac{A_0}{K}}}$$

$$= \frac{1}{\left(\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}}\right)} \times \sqrt{1 + \frac{A_0}{K}}$$

$$\zeta = \frac{1}{2Q_p} = \frac{1}{2}\left(\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}}\right) \frac{1}{\sqrt{1 + \frac{A_0}{K}}}$$

ζ: damping coefficient.

$$H(s) = \frac{A_{DC}}{1 + \underbrace{\frac{s}{\omega_p Q_p}} + \frac{s^2}{\omega_p^2}}$$

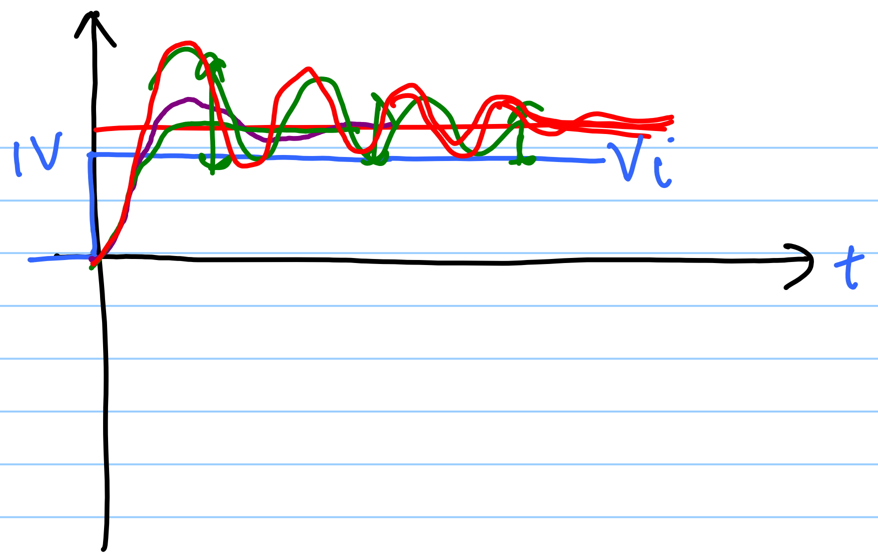
$$= \frac{A_{DC} \cdot \omega_p^2}{\omega_p^2 + \frac{s \cdot \omega_p}{Q_p} + s^2}$$

$$= \frac{A_{DC}\, \omega_p^2}{\omega_n^2 + 2\zeta\omega_n s + s^2} \checkmark$$

$$\frac{\omega_p}{Q_p} = 2\zeta\omega_n$$

$$\zeta = \frac{1}{2Q_p}$$

$$H(s) = \frac{A_{DC} \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{V_o}{V_i}$$



$$D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n$$

if $\zeta \geq 1$, real poles on L.H.P

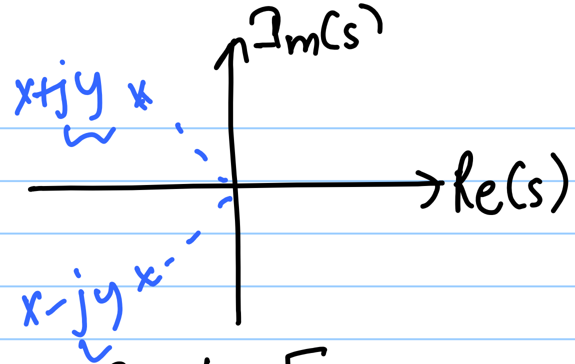
if $\zeta < 1$, $s = \left(-\zeta \pm j\sqrt{1-\zeta^2}\right)\omega_n$

$$H(s) = \frac{A_{DC}\omega_n^2}{(s+\omega_{p1})(s+\omega_{p2})}$$

$$V_o = \frac{A_{DC} \cdot \omega_n^2}{s \cdot (s+\omega_{p1})(s+\omega_{p2})}$$

$$v_o(t) = \alpha \cdot u(t) + \beta_1 e^{-\omega_{p1}t} + \gamma e^{-\omega_{p2}t}$$

$\zeta < 1$



$$v_0(t) = \qquad + \; e^{-\zeta\omega_n t} \times \left[\quad \sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right) + \quad \cos\left(\sqrt{1-\zeta^2}\,\omega_n t\right) \right]$$

$$\zeta = \frac{1}{2}\left(\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}}\right)\frac{1}{\sqrt{1+\frac{A_0}{K}}}$$

$$\zeta = 1 \Rightarrow 4\left(1+\frac{A_0}{K}\right) = \frac{p_1}{p_2} + \frac{p_2}{p_1} + 2$$

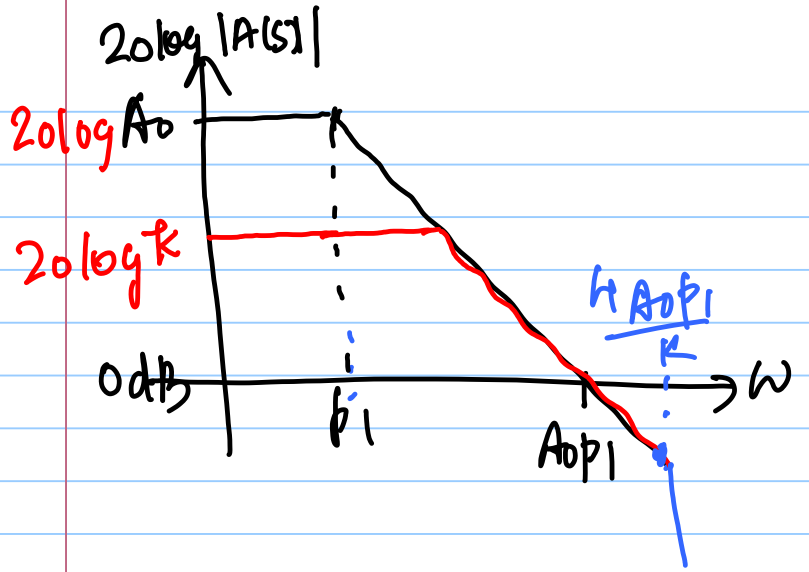
$$\frac{p_1}{p_2} + \frac{p_2}{p_1} = 4\left(1+\frac{A_0}{K}\right) - 2$$

$$\frac{p_1}{p_2} + \frac{p_2}{p_1} \approx 2 + \frac{4 \cdot A_0}{K}$$

if $p_1 \ll p_2$

$$\Rightarrow \frac{p_2}{p_1} \approx 4 \cdot \frac{A_0}{K}$$

$$p_2 \approx 4 \cdot \frac{A_0 p_1}{K}$$



$$A(s) = \frac{A_0}{1 + s/p_1}$$

$$A(S) = \frac{A_0^2}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)}$$

$$H(S) = \frac{K}{1 + \frac{A_0}{K}} \quad \frac{1}{1 + \frac{s}{p_1}}$$

$$H(S) = \frac{K}{1 + \frac{K}{A(s)}}$$

$$\wedge$$

$$1 = \frac{K}{\frac{s}{p_1 \cdot \frac{A_0}{K}}}$$

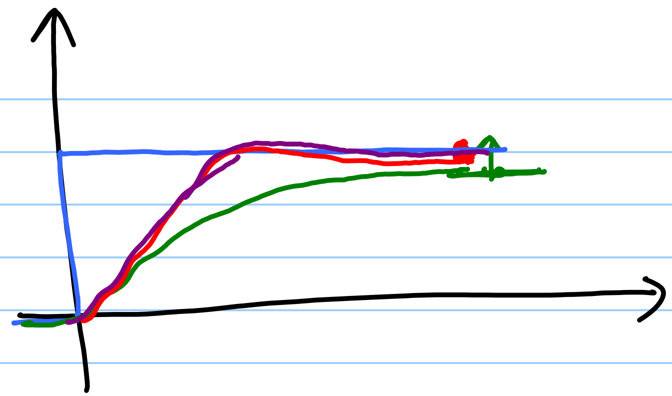
$$\omega_u = A_0 p_1$$

$$H(S) = \frac{K}{1 + \frac{K(1 + s/p_1)}{A_0}} = \frac{K}{\left(1 + \frac{K}{A_0}\right)\left(1 + \frac{\frac{K}{A_0} \cdot \frac{s}{p_1}}{1 + \frac{K}{A_0}}\right)}$$

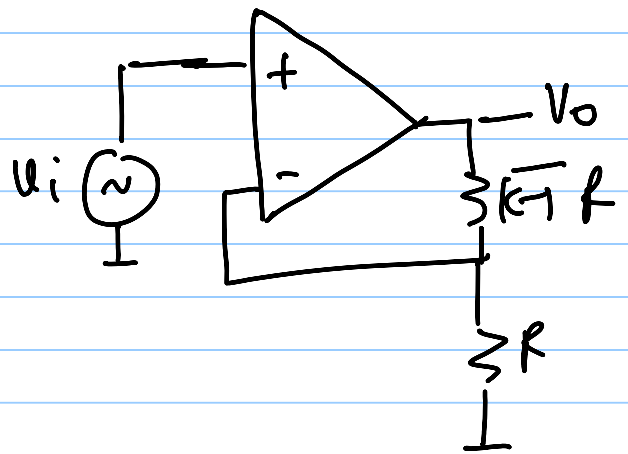
$$|H(s)| = \underbrace{\frac{K}{1 + K/A_0}} \quad \frac{1}{1 + \frac{s}{p_1} \times \underbrace{\frac{1}{\left(1 + \frac{A_0}{K}\right)}}}$$

$$p_1' \approx \frac{A_0 p_1}{K}$$

$$H(s) = \frac{A_{DC}\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$



$$\lim_{t \to \infty} \epsilon = \lim_{t \to \infty, A_0 \to \infty} V_0(t) - \lim_{t \to 0} v_0(t)$$



$$A(s) = \frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)}$$

$$\frac{V_0}{V_i} = \frac{k}{1 + \frac{k}{A(s)}}$$

$$= \frac{k}{1 + \frac{k}{A_0}\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)}$$

$$D(s) = 1 + \frac{k}{A_0}\left(1 + s\left(\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}\right) + s^2\left(\frac{1}{p_1 p_2} + \frac{1}{p_2 p_3} + \frac{1}{p_1 p_3}\right) + \frac{s^3}{p_1 p_2 p_3}\right)$$

$$A(s) = \frac{A_0}{\left(1+\frac{s}{p_1}\right)^3} \quad \checkmark$$

$$D(s) = 1 + \frac{k}{A_0}\left(1+\frac{s}{p_1}\right)^3$$

$$\left(1+\frac{s}{p_1}\right) = \left[-1 \cdot \frac{A_0}{K}\right]^{1/3} = \left(\frac{A_0}{K}\right)^{1/3}\left[e^{j(2n\pi+\pi)}\right]^{1/3}$$

$$\frac{s}{p_1} = -1 + \left(\frac{A_0}{K}\right)^{1/3}\left[e^{j(\pi/3 + 2n\pi/3)}\right]$$

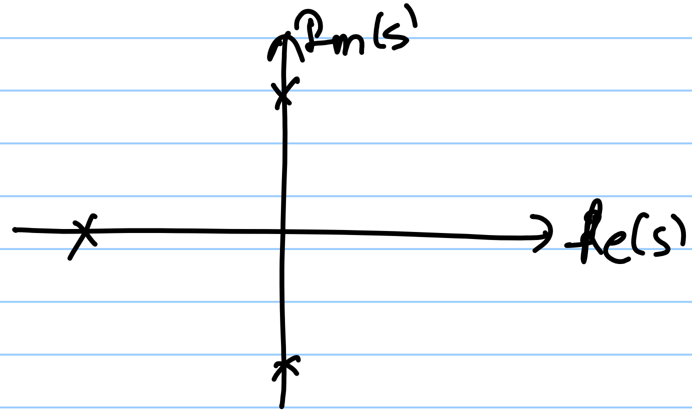
$$e^{j\pi/3} \qquad n=0 \qquad e^{j\pi/3} = \frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\frac{s}{p_1} = -1 + \left(\frac{A_0}{K}\right)^{1/3}$$

$$e^{j\pi} \qquad n=1 \qquad e^{-\pi/3} = \;\vdots$$

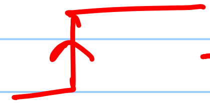
$$e^{-j\pi/3}$$

$$\frac{A_0}{K} = 8 \quad \Rightarrow \quad \frac{s}{p_1} = -1 + 2\left(e^{j\pi},\ \underline{\tfrac{1}{2} \pm j\tfrac{\sqrt{3}}{2}}\right)$$



$$H(s) = \underline{\frac{1}{s^2 + \omega_0^2}}$$

$s = j\omega_0$



$\rightarrow \quad \frac{1}{s}$