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Queue-Aware Optimal Resource Allocation for the LTE Downlink with Best M sub-band Feedback

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***Abstract*—We address the problem of optimal downlink resource allocation in an OFDMA system, in a scenario where very limited channel quality information (CQI) is available at the base-station. Our work is particularly applicable in the context of the LTE downlink, since the feedback mechanism we consider closely resembles one of the CQI reporting modes in LTE. Specifically, the users only report the indices of their best M sub-bands and an effective CQI corresponding to these best M bands. Our policy simultaneously performs optimal sub-band assignment and rate allocation, by taking into account channel quality as well as the queue backlogs of each user. The technical novelty of our work lies in exploiting a limit theorem on the best SNRs reported by the users, and combining it within a Lyapunov stability framework. We show that our policy is throughput maximizing among all policies which are constrained to the CQI mechanism considered. Numerical results indicate that in terms of throughput and average delay, our policy compares favorably to existing resource allocation policies such as proportional fair.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is employed in most of the emerging high data-rate wireless cellular standards such as Long Term Evolution (LTE) [1]. In this paper, we tackle the problem of optimal resource allocation on the downlink of an OFDMA system, in a scenario where very limited channel quality information (CQI) is available at the base-station (BS). Our work is particularly applicable in the context of an LTE downlink, since the feedback mechanism we consider closely resembles one of the CQI reporting modes in LTE.

In an OFDM system such as the LTE downlink, the available bandwidth (of say 20 MHz) is divided into several hundred sub-carriers (e.g., 512, 1024, or 2048). These sub-carriers need to be allocated to multiple user equipment (UEs). In practice, a *resource block* (RB) pair consisting of 12 contiguous sub-carriers and 14 OFDM symbols in time is the smallest resource allocation unit [2]. After accounting for unusable tones, this leaves us with about 50 to 100 RBs to allocate to the UEs.

In order to schedule the UEs opportunistically, the base-station, in principle, needs to obtain channel quality information from each UE, on each of the resource blocks. This is highly impractical, since it leads to an enormous amount of control overheads on the uplink. To overcome this, the UEs in an LTE system report CQI to the base-station in a very sparse manner.

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A. Related Work

Various reduced feedback mechanisms have been studied in the literature, in the context of resource allocation on the OFDM downlink. In [3], the CQI of each UE is fed back only for those sub-bands[1] whose quality is better than a certain threshold. The feedback overhead is further reduced in [4], where the UEs report one-bit per sub-band whenever the channel quality exceeds the threshold. In [5], an opportunistic feedback strategy is considered, wherein only the channel gains of a pre-specified number M of best sub-bands[2] are reported. A variation of this policy has been considered in [6], [7]. In [6], the UEs feedback the average gain of the best M sub-bands and the corresponding indices while in [7], each UE reports an Effective Exponential Signal-to-noise ratio Mapping (EESM) of the best M sub-bands and their respective indices. In effect, EESM translates the different SNRs on parallel channels into a single effective flat-fading SNR [8]. The throughput of adaptive modulation and coding based on EESM is analyzed in [9]. In [10], a reduced feedback scheme with different sub-band sizes for different UEs is studied. An opportunistic hybrid feedback scheme, where the number of sub-bands for feedback can be random, is studied in [11].

In this paper, we assume a CQI feedback mechanism similar to [6], [7], since it closely resembles one of the CQI reporting modes – namely, the UE-selected sub-band feedback mode – defined in the LTE standards [12]. Specifically, the UEs only report the indices of their best M sub-bands, where M is a small number (say 2 to 5), and the EESM corresponding to these best M bands.

Downlink resource allocation for OFDM systems has been studied from various perspectives in recent years. In [13], resource allocation in downlink OFDM is posed as a utility maximization problem, which includes proportionally fair resource allocation [14], [15] as a special case. The optimal power and sub-carrier allocation are then determined using convex duality techniques. While [13] assumed full CQI availability except for an estimation error term, [16] takes imperfect CQI into account by factoring for outages due to erroneous CQI at the

[1] A sub-band typically consists of one to three resource blocks.

[2] Throughout the paper, we refer to the sub-bands with the M highest SNRs as the 'best M sub-bands.'

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base-station. In [17], the authors consider opportunistic resource allocation in OFDM under various fairness constraints, and propose a Hungarian algorithm based solution. It is worth noting that [13], [16], [17] assume fully backlogged buffers (i.e., that the base-station always has data to send to the UEs), and do not consider any queuing dynamics.

There is a vast literature on optimal server allocation to constrained queuing systems with time-varying connectivities. Most of the literature in this area based on the landmark papers [18], [19] which introduced Lyapunov techniques for resource allocation. Subsequently, these Lyapunov methods, which explicitly take queue lengths into account for making resource allocation decisions, have been applied in various contexts including high-speed switches [20], satellites [21], wireless [22], and optical networks [23]. In addition to being inherently throughput maximizing, Lyapunov based resource allocation policies can also been used to ensure Quality of Service (QoS) metrics such as delay guarantees [24], [25] and fairness [26]. In [27], greedy algorithms with low delay are proposed for an OFDM-based cellular downlink.

In the above Lyapunov based resource allocation policies, the resource allocation decision is based on the UE's channel quality as well as queue backlogs, and these are typically assumed to be available perfectly and instantaneously at the base station. In contrast, [28] proposes a throughput optimal resource allocation algorithm under *delayed* channel information; their policy utilizes the conditional expectation of the channel quality, given the delayed measurements. In [29], a cross layer resource allocation policy which maximizes the throughput under delayed CQI and takes into account the channel *outage* event is proposed. There has also been recent work on low-complexity dynamic resource allocation for OFDM [30], [31] to ensure low delay, but these papers do not consider sparse CQI feedback.

B. Our Contributions

In the present paper, we propose a queue-aware resource allocation policy for the OFDM downlink that is optimized for the specific form of the CQI available at the base-station. As described earlier, we assume that the UEs only report the indices of their best M sub-bands, where M is a small number, and the EESM corresponding to these best M sub-bands. We develop a sub-band assignment and rate allocation algorithm which is throughput maximizing under this CQI scenario, when the total number of sub-bands is large. In other words, our algorithm is guaranteed to keep the queuing system stable for all traffic rates that can be stabilized by any resource allocation policy which is constrained to this CQI scenario.

One of the technical contributions of the paper lies in obtaining an explicit characterization of the outage probability on each of the M reported sub-bands. In order to obtain the outage probability expression, we exploit a 'Gumbel' limit theorem on the joint distribution of the best M sub-bands, which subsequently leads to an explicit expression for the conditional density, given the EESM. It is worth commenting that the Gumbel weak limit is an attractor for the extremal values of a fairly large family of distributions [32], so that our work does not crucially depend on the assumption that the sub-band gains are i.i.d. Rayleigh distributed. Another distinguishing feature of our resource allocation policy is that it naturally decouples for each sub-band, and does not entail solving any computationally intensive matching problems [17], [30].

II. System Model

Consider a downlink system with one BS and K UEs. The BS maintains a separate queue corresponding to each UE. Time is slotted, and the queue corresponding to i^{th} UE receives exogenous arrivals according to a random process. We denote the amount of data that enters queue i during time slot t by $A_i(t)$, and the queue length corresponding to the i^{th} UE during slot t by $Q_i(t)$. We assume that the arrival process $A_i(t)$ is i.i.d. from slot to slot, with mean λ_i and a finite second moment.

We assume that the channel between the BS and i^{th} UE is a frequency selective Rayleigh fading channel. We remark that this Rayleigh fading assumption is not crucial to our work, but it makes exposition easier. OFDM transmission with N_c sub-carriers is used. The SNR for the i^{th} UE on the j^{th} sub-carrier follows an exponential distribution. The average SNR for the i^{th} UE is denoted as $\gamma_{ave,i}$.

We assume that the downlink channel gains of the UEs are not known to the BS unless the UEs feedback their CQI to the BS. This corresponds to a scenario where the uplink and the downlink channels are not reciprocal, or a scenario where the UEs are not transmitting any data on the uplink, so that reciprocity (even if present) cannot be exploited. In order to reduce feedback overhead, we assume that the sub-carriers are grouped into N sub-bands in such a way that the channel can be approximated as flat-fading in each sub-band. Further, we consider the 'best M' feedback mechanism similar to [7], where each UE reports (i) the EESM corresponding to its best $M(\ll N)$ sub-bands according to SNR, and (ii) the indices of those sub-bands.

Let $\gamma_i^j(t)$ be the SNR on the j^{th} sub-band for the i^{th} UE in slot t, and $\gamma_i^{(j)}(t)$ be the SNR of the j^{th} best sub-band of the i^{th} UE in slot t, i.e., $\gamma_i^{(1)}(t), \gamma_i^{(2)}(t) \dots \gamma_i^{(N)}(t)$ are the ordered sub-band SNRs for i^{th} UE in descending order. The EESM for the best M sub-bands corresponding to the i^{th} UE in slot t, denoted γ_i^{eff}, is defined as [7]

$$\gamma_i^{eff}(t) = -\eta \ln \left(\frac{1}{M} \sum_{j=1}^{M} e^{-\frac{\gamma_i^{(j)}(t)}{\eta}} \right), \tag{1}$$

where η is a parameter that depends on the modulation and coding scheme (MCS). Hence, the i^{th} UE reports the following two quantities to the BS during each slot.

(i) The EESM γ_i^{eff},
(ii) The index set $I_i = \{i_1, i_2, \dots i_M\}$,

where i_j is the index of the j^{th} best sub-band of the i^{th} UE.

Since we are considering a downlink problem, the BS is assumed to know the instantaneous queue lengths $Q_i(t)$ for all the UEs.

III. Problem Formulation

In this section, we develop a mathematical formulation of the optimal resource allocation problem. As mentioned earlier, the following information is assumed to be available with the BS during time-slot t (for simplified notation, we omit t):

(i) The EESMs $\underline{\gamma}^{eff} = [\gamma_1^{eff}, \gamma_2^{eff}, \ldots, \gamma_K^{eff}]$

(ii) The index sets $\underline{I} = [I_1, I_2, \ldots, I_K]$

(iii) The queue length vector $\underline{Q} = [Q_1, Q_2, \ldots, Q_K]$.

Given this information, our aim is to come up with a resource allocation policy which can maximize throughput while keeping all queues at the BS stable. In order to make this statement precise, we develop some terminology and notation.

A resource allocation policy performs the following two operations in each slot.

- Sub-band assignment: For each sub-band j that is reported by at least one UE, the policy determines a unique UE to assign the sub-band. (Recall that a sub-band can be allocated to at most one UE due to interference considerations, whereas a UE can be allocated multiple sub-bands).
- Rate allocation: Given that j^{th} sub-band is assigned to i^{th} UE, determine the rate $r_{i,j}$ at which data transmission will take place on j^{th} sub-band.

From now on, we use the notation $[i,j]$ for the i^{th} UE - j^{th} sub-band pair. In the interest of simplicity, we restrict our attention to policies which allocate equal power to all scheduled sub-bands, although our framework can be modified to include optimal power allocation for different sub-bands. To be precise, define $\hat{I} = \cup_{i=1}^{K} I_i$ as the set of all sub-bands reported by at least one UE, and let $M' = |\hat{I}|$ denote the total number of such sub-bands. Assume that the BS has a power budget of P for transmissions during each slot. Then, the base station allocates power P/M' to each sub-band. Let $C_{i,j}$ be the instantaneous capacity of $[i,j]$. Under the above assumptions, we have

$$C_{i,j} = \log_2\left(1 + \frac{P}{M'}\gamma_i^j\right). \tag{2}$$

For a reliable communication over a sub-band, the rate assigned to $[i,j]$, $r_{i,j}$, should not exceed $C_{i,j}$. Given $\underline{\gamma}^{eff}$ and the index sets $\underline{I}$, we say $[i,j]$ is in outage if the rate allocated to $[i,j]$ is greater than $C_{i,j}$. The outage probability for $[i,j]$ when the assigned rate is $r_{i,j}$ is defined as follows:

$$P_{i,j}(r_{i,j}) = \mathbb{P}\left\{C_{i,j} < r_{i,j} \middle| \gamma_i^{eff}, I_i\right\}. \tag{3}$$

We define a natural metric, namely *goodput*, as the average successfully transmitted rate over a sub-band [33]. The goodput for $[i,j]$ when the assigned rate is $r_{i,j}$ is defined as follows:

$$G_{i,j}(r_{i,j}) = r_{i,j}(1 - P_{i,j}(r_{i,j})). \tag{4}$$

Next, we briefly review the queuing dynamics and stability considerations of the queuing system at the BS.

A. Stability considerations

The queue evolution equation for the i^{th} UE can be written as

$$Q_i(t+1) = \max\{Q_i(t) - \mu_i(t), 0\} + A_i(t), \tag{5}$$

where $A_i(t)$ and $\mu_i(t)$ are arrival and service processes of the i^{th} UE queue. Here, $\mu_i(t)$ is the amount of data served from the i^{th} UE queue during slot t, and can be written as

$$\mu_i(t) = \sum_{j=1}^{N} a_{i,j} r_{i,j} H_{i,j}(t),$$

where $a_{i,j}$ denotes the fraction of time the j^{th} sub-band is allocated to the i^{th} UE during slot t, and $H_{i,j}(t)$ is an indicator random variable which takes a value 1 whenever the transmission through $[i,j]$ during slot t is successful, and 0 otherwise. Thus, $\mathbb{P}\{H_{i,j}(t) = 0\} = P_{i,j}^N(r_{i,j})$. Further, it is clear that

$$\sum_{i=1}^{K} a_{i,j} \leq 1. \tag{6}$$

We will show later (in Proposition 1) that our optimal policy allocates a sub-band to at most one UE during each time-slot.

In the spirit of [34], we say that the queuing system at the BS is *strongly stable* if for each UE i,

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} E[Q_i(t)] < \infty. \tag{7}$$

Denote by $\mathcal{P}$ the family of all resource allocation policies which allocate equal power to all scheduled sub-bands, and have access only to the parameters $\underline{\gamma}^{eff}, \underline{I}$, and $\underline{Q}$ in order to make the resource allocation decisions during each slot. Let Λ be the *stability region* of the network, which is defined as (the closure of) the set of all arrival rates $\underline{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_K)$ for which there exists some policy $\Pi \in \mathcal{P}$ under which the queuing system is strongly stable.

We find a resource allocation policy in $\mathcal{P}$ which is *throughput optimal*, in the same sense as in [34], i.e., it keeps the queuing system stable for *all* arrival rates in the interior of Λ.

IV. Throughput Optimal Resource Allocation Policy

During each time slot, the scheduler at the BS observes $\underline{\gamma}^{eff}, \underline{I}$, and $\underline{Q}$, and implements the following steps :

1) Determine $\hat{I} = \cup_{i=1}^{K} I_i$ and $M' = |\hat{I}|$.
2) for $j = 1$ to M' do
3) Determine $U_j = \{i | j \in I_i\}$.
4) Calculate an estimate of the outage probability $\hat{P}_{i,j}(r)$ as a function of r for each $i \in U_j$. (See equation (21) in Section V)
5) Calculate
$$r_{i,j}^* = \arg\max_r \{r(1 - \hat{P}_{i,j}(r))\} \ \forall\, i \in U_j.$$
6) Calculate
$$i(j) = \arg\max_{i \in U_j} \{Q_i(t) r_{i,j}^* (1 - \hat{P}_{i,j}(r_{i,j}^*))\}.$$
7) Assign j^{th} sub-band to $i(j)^{th}$ UE, and transmit at rate $r_{i(j),j}^*$.
8) end for

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A. Discussion

In the first step, the scheduler determines the set of all distinct sub-bands reported by the UEs. Then, for each such sub-band j, the scheduler determines the set U_j of all UEs who report that sub-band as being one of their best M sub-bands. In step 4, the outage probability on $[i, j]$ is computed, as explained in Section V. In step 5, the scheduler computes the rate that ensures the best goodput for each UE $i \in U_j$. Finally, in steps 6 and 7, the scheduler assigns j^{th} sub-band to the i^{th} UE that has the maximum queue-length goodput product.

Notice that the above algorithm assigns every reported sub-band to a unique UE. Also, no power is assigned to sub-bands that are not reported by any UE.

B. Lyapunov Analysis

In this section, we derive the optimal resource allocation policy as a Lyapunov drift minimizing policy, and prove that it is throughput optimal. We define the quadratic Lyapunov function

$$L(\underline{Q}(t)) = \sum_{i=1}^{K} (Q_i(t))^2,$$

and consider the *conditional Lyapunov drift*

$$\Delta(\underline{Q}(t)) = \mathbb{E}\left\{ L(\underline{Q}(t+1)) - L(\underline{Q}(t)) | \underline{Q}(t) \right\}.$$

We obtain the following inequality by squaring the both sides of (5).

$$\begin{aligned}(Q_i(t+1))^2 \leq & (Q_i(t))^2 + \left(\sum_{j=1}^{N} a_{i,j} r_{i,j} H_{i,j}(t) \right)^2 + (A_i(t))^2 \\ & - 2Q_i(t) \left(\sum_{j=1}^{N} a_{i,j} r_{i,j} H_{i,j}(t) - A_i(t) \right).\end{aligned}$$

Taking the sum over all the UEs and using the fact that the sum of squares of non-negative variables is less than or equal to the square of the sum, we get the following inequality.

$$\begin{aligned}L(\underline{Q}(t+1)) - L(\underline{Q}(t)) \leq & \sum_{i=1}^{K} (A_i(t))^2 \\ & + 2 \sum_{i=1}^{K} A_i(t) Q_i(t) \left(\sum_{j=1}^{N} \sum_{i=1}^{K} a_{i,j} r_{i,j} H_{i,j}(t) \right)^2 \\ & - 2 \sum_{i=1}^{K} Q_i(t) \sum_{j=1}^{N} a_{i,j} r_{i,j} H_{i,j}(t).\end{aligned} \tag{8}$$

Using (6), we get the following upper bound

$$\sum_{i=1}^{K} a_{i,j} r_{i,j} H_{i,j}(t) \leq \max_i \{C_{i,j}\} < \infty, \ \forall\, j.$$

Thus, taking conditional expectations and exploiting the independence of $A_i(t)$ and $Q_i(t)$, we get

$$\begin{aligned}\Delta(\underline{Q}(t)) \leq & B + 2 \sum_{i=1}^{K} Q_i(t) \lambda_i \\ & - 2 \sum_{i=1}^{K} \sum_{j=1}^{N} Q_i(t) a_{i,j} r_{i,j} (1 - P_{i,j}(r_{i,j})),\end{aligned} \tag{9}$$

where

$$B = \left(\sum_{j=1}^{N} \max_i \{C_{i,j}\} \right)^2 + \sum_{i=1}^{K} \mathbb{E}\left[A_i(t)^2 \right] < \infty.$$

We know from [34, Lemma 4.1] that the Lyapunov drift becoming negative for large queue backlogs is a sufficient condition for the strong stability of the queuing system. With this in mind, we seek the policy that maximizes the negative term on the right hand side of (9). We therefore formulate the optimal resource allocation problem as follows.

$$\max_{\{a_{i,j}\},\{r_{i,j}\}} \sum_{i=1}^{K} \sum_{j=1}^{N} Q_i(t) a_{i,j} G_{i,j}(r_{i,j}), \tag{10}$$

subject to

$$\begin{aligned}\sum_{i=1}^{K} a_{i,j} &\leq 1, \ \forall\, j, && (C1)\\ a_{i,j} &\geq 0, \ \forall\, i, j, && (C2)\\ r_{i,j} &\geq 0, \ \forall\, i, j. && (C3)\end{aligned}$$

We assume that it is possible to come up with modulation and coding schemes for any desired rate $r_{i,j}$. The solution is discussed next.

C. Minimizing the Lyapunov Drift

We now solve the optimization problem (10) and arrive at our resource allocation policy. First, note that for any sub-band allocation $\{a_{i,j}\}$, the objective function is maximized by choosing $r_{i,j} = r_{i,j}^*$, where $r_{i,j}^*$ maximizes the goodput $G_{i,j}(r_{i,j})$ of the i^{th} UE on the j^{th} sub-band. Such an $r_{i,j}^*$ can be shown to exist because: (1) $G_{i,j}(r) = r(1 - \hat{P}_{i,j}(r))$, (2) $(1 - \hat{P}_{i,j}(r))$ is monotonically decreasing in r, and (3) $G_{i,j}(r) \to 0$ as $r \to \infty$. This gives Step 5 of our policy in Section IV.

Now, the optimization problem in (10) reduces to the following linear program:

$$\max_{\{a_{i,j}\}} \sum_{i=1}^{K} \sum_{j=1}^{N} Q_i(t) a_{i,j} G_{i,j}(r_{i,j}^*), \tag{11}$$

subject to

$$\begin{aligned}\sum_{i=1}^{K} a_{i,j} &\leq 1, \ \forall\, j, && (C1)\\ a_{i,j} &\geq 0, \ \forall\, i, j. && (C2)\end{aligned}$$

Introducing non-negative Lagrange multipliers $\{\alpha_j\}$,$\{\beta_{i,j}\}$ for constraints $(C1)$-$(C2)$ respectively, the following condi-

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tions must be satisfied at the optimal solution (superscript $(\cdot)^*$ denotes optimal values).

$$Q_i(t)G_{i,j}(r^*_{i,j}) + \beta^*_{i,j} - \alpha^*_j = 0, \ \forall \ i, j. \tag{12}$$

$$\alpha^*_j \left(\sum_{i=1}^{K} a^*_{i,j} - 1 \right) = 0, \ \forall \ j. \tag{13}$$

$$\beta^*_{i,j} a^*_{i,j} = 0, \ \forall \ i, j. \tag{14}$$

Condition (12) is a gradient condition, and conditions (13) and (14) are complementary slackness conditions corresponding to the constraints $(C1)$ and $(C2)$, respectively [35].

Proposition 1. *The optimal sub-band allocation for problem (10) assigns a sub-band exclusively to a UE with the largest corresponding queue-length goodput product.*

Proof. It follows from (14) and (12) that if $a^*_{i,j} > 0$, then $\beta^*_{i,j} = 0$, and $Q_i(t)G_{i,j}(r^*_{i,j}) = \alpha^*_j$. On the other hand, if $a^*_{i,j} = 0$, then $\beta^*_{i,j} \geq 0$, i.e., $Q_i(t)G_{i,j}(r^*_{i,j}) \leq \alpha^*_j$. Therefore, to maximize the objective function, the j^{th} sub-band should be assigned to the UE with largest queue-length goodput product $Q_i(t)G_{i,j}(r^*_{i,j})$. If multiple UEs have the same queue-length goodput product for the same sub-band j, the sub-band can be shared in any arbitrary manner among these users without affecting optimality in terms of the objective function in (10). In such a scenario, without loss of generality, we will assume that one such UE is arbitrarily chosen and allocated sub-band j. From (13), $a^*_{i,j} = 1$ as long as the queue-length goodput product is positive for at least one UE. □

Proposition 1 shows that the optimal sub-band allocation assigns each reported sub-band j to the UE which has the maximum queue-length goodput product on the j^{th} sub-band. This is Step 6 of the policy in Section IV.

Since the proposed policy ensures the "most negative" Lyapunov drift among the class $\mathcal{P}$, it seems plausible that our policy should be able to stabilize the queuing system, whenever some policy in $\mathcal{P}$ can do so. The following theorem asserts that this is indeed true.

Theorem 1. *The resource allocation policy proposed in Section IV is asymptotically throughput optimal, i.e., given any arrival rate vector $\underline{\lambda}$ that is stabilizable by some policy $\Pi \in \mathcal{P}$, there exists an N such that, when the number of sub-bands is at least N, the proposed policy will stabilize the queues.*

The proof is relegated to Appendix A. Our policy is only asymptotically throughput optimal since in Step 4, the policy uses the limiting outage probability given by a limit theorem, instead of the actual outage probability which is difficult to compute.

V. Derivation of Outage Probability

In this section, we describe how the BS estimates the outage probability on $[i, j]$ in Step 4 of our algorithm, using only the parameters $\underline{\gamma}^{eff}$ and $\underline{I}$. We utilize a limit theorem on the order statistics of the SNRs to derive an expression for the conditional joint distribution of the SNRs on the best M sub-bands for each UE, given the EESM and the sub-band indices. For ease of exposition, we assume that the SNRs on the sub-bands of a given UE are i.i.d. exponentially distributed. This assumption will hold well in the case of Rayleigh fading in a rich multi-path environment, with number of paths comparable to the number of sub-bands. However, we remark that the limit theorem we are about to exploit holds for a fairly large class of distributions – namely, those which lie within the Gumbel domain of attraction [32]. Therefore, our policy remains asymptotically throughput optimal for this class of sub-band SNR distributions. The correlated sub-bands case is studied in the simulation results section (Section VI).

We first state a result which follows from [36, Theorem 15] regarding the order statistics of M extremal values, drawn from N i.i.d. exponential random variables .

Theorem 2. *Let $Z_1, Z_2, \ldots, Z_N$ be a sequence of i.i.d. unit exponential random variables, and $Z_{(1)}, Z_{(2)}, \ldots, Z_{(N)}$ be the corresponding order statistics in descending order. Then*

$$(e^{-\tilde{Z}_{(1)}}, e^{-\tilde{Z}_{(2)}}, \ldots, e^{-\tilde{Z}_{(M)}}) \xrightarrow{\mathfrak{D}} (Y_1, Y_2, \ldots Y_M),$$

as $N \to \infty$, where $\tilde{Z}_{(i)} = Z_{(i)} - \ln N$, $Y_i = \sum_{j=1}^{i} X_j$ and X_js are i.i.d. unit exponential random variables.

Proof. Note that $1 - e^{-Z_1}, 1 - e^{-Z_2}, \ldots, 1 - e^{-Z_N}$ is a sequence of i.i.d. standard uniform random variables. Now directly applying the [36, Theorem 15], the result follows. □

Lemma 1. *Let $\underline{Y}^{(n)} = (Y_1, Y_2, \ldots, Y_n)$ with the entries $Y_i = \sum_{j=1}^{i} X_j, \forall \ i = 1, \ldots, n$, where X_js are i.i.d. unit exponential random variables. The joint pdf of $\underline{Y}^{(n)}$ is given by*

$$f_{\underline{Y}^{(n)}}(y_1, y_2, \ldots, y_n) = e^{-y_n}, 0 \leq y_1 \leq y_2 \leq \ldots \leq y_n. \tag{15}$$

Proof. See Appendix B. □

Consider the i^{th} UE. Let $S_i^j = e^{-(\gamma_i^{i_j} - \ln N)}$ for $j = 1, \ldots, M$. Assuming that the number of sub-bands N is large, we first apply the limit theorem in Theorem 2 for the vector $\underline{S}_i^{(M)} = (S_i^1, S_i^2, \ldots, S_i^M)$. Then, using Lemma 1, we obtain the joint pdf of $\underline{S}_i^{(M)}$ as[3]

$$f_{\underline{S}_i^{(M)}}(s_1, s_2, \ldots, s_M) = e^{-s_M}, \quad 0 \leq s_1 \leq s_2 \leq \ldots \leq s_M.$$

Numerical results indicate that this approximation is good even for moderate values of N. Next, define $S_i^{eff} = e^{-(\gamma_i^{eff} - \ln N)}$. Choosing the parameter η in (1) as unity for simplicity, we have

$$S_i^{eff} = \frac{1}{M} \sum_{j=1}^{M} S_i^j.$$

Note that S_i^{eff} is known to the BS. Next, conditioned on $S_i^{eff} = s$ and $I_i = I$, $\underline{S}_i^{(M)}$ takes values only on the hyperplane $\frac{1}{M} \sum_{j=1}^{M} S_i^j = s$. Hence, we ignore the M^{th} best SNR

[3] We suppress the conditioning on the indices I_i in order to avoid cumbersome expressions.

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and calculate the joint CDF [4] of $(S_i^1, \ldots S_i^{M-1}, S_i^{eff})$ as follows.

$$\begin{aligned} & F_{S_i^1,\ldots S_i^{M-1},S_i^{eff}}(s_1,\ldots s_{M-1},s) & (16)\\ = \; & \mathbb{P}(S_i^1 \le s_1,\ldots,S_i^{M-1} \le s_{M-1}, S_i^{eff} \le s), & (17)\\ = \; & \mathbb{P}(S_i^1 \le s_1,\ldots,S_i^{M-1} \le s_{M-1}, S_i^{M} \le Ms - \sum_{j=1}^{M-1} s_j), \\ = \; & F_{\underline{S}_i^{(M)}}(s_1,\ldots s_{M-1}, Ms - \sum_{j=1}^{M-1} s_j). \end{aligned}$$

Now taking the partial derivatives with respect to $s_1, \ldots, s_{M-1}, s$, we get

$$\begin{aligned} & f_{S_i^1,\ldots S_i^{M-1},S_i^{eff}}(s_1,\ldots s_{M-1},s) & (18)\\ = \; & M f_{\underline{S}_i^{(M)}}(s_1,\ldots s_{M-1}, Ms - \sum_{j=1}^{M-1} s_j), & (19)\\ = \; & M e^{-(Ms - \sum_{j=1}^{M-1} s_j)}, & (20)\\ & \text{for} \;\; 0 \le s_1 \le s_2 \le \ldots \le Ms - \sum_{j=1}^{M-1} s_j, \end{aligned}$$

where the factor M in equation (19) is the Jacobian of the linear transformation relating the random vectors $\underline{S}_i^{(M)}$ and $[S_i^1, \ldots S_i^{M-1}, S_i^{eff}]$. Next, we calculate the conditional joint pdf of the best $M-1$ sub-bands given S_i^{eff}.

$$\begin{aligned} & f_{\underline{S}_i^{(M-1)}|S_i^{eff}=s}(s_1, s_2, \ldots, s_{M-1}) \\ & = \frac{f_{S_i^1,\ldots S_i^{M-1},S_i^{eff}}(s_1,\ldots s_{M-1},s)}{f_{S_i^{eff}}(s)}, \\ & = \frac{M e^{-(Ms - \sum_{j=1}^{M-1} s_j)}}{f_{S_i^{eff}}(s)}, 0 \le s_1 \le s_2 \le \ldots \le Ms - \sum_{j=1}^{M-1} s_j. \end{aligned}$$

Now, we can find the conditional marginal density $f_{S_i^j|S_i^{eff}=s, I_i=I}(s_j)$ for $j = 1, \ldots, M-1$ by integrating out the other variables in the above expression.

In order to compute the conditional pdf of S_i^M, $f_{S_i^M|S_i^{eff}=s}(s_M)$, we start with the joint CDF of $(S_i^{eff}, S_i^2, \ldots S_i^{M-1}, S_i^M)$ and repeat the above steps starting from equation (17).

The outage probability can thus be determined as

$$\begin{aligned} \hat{P}_{i,i_j}(r) &= \mathbb{P}(C_{i,i_j} \le r | \gamma_i^{eff} = \gamma, I_i = I), \\ &= \mathbb{P}(\log_2(1 + \frac{P}{M'}\gamma_i^{i_j}) \le r | \gamma_i^{eff} = \gamma, I_i = I), \\ &= \mathbb{P}(S_i^j \ge N e^{-\frac{2^r-1}{\frac{P}{M'}}} | S_i^{eff} = N e^{-\gamma}, I_i = I), \\ &= \int_y^N f_{S_i^j|S_i^{eff}=Ne^{-\gamma}, I_i=I}(s_j) ds_j. \end{aligned} \quad (21)$$

where $y = N e^{-\left(\frac{2^r-1}{\frac{P}{M'}}\right)}$. Closed form expressions for the outage probabilities (21) can be obtained through some tedious computations. We provide explicit expressions for the case $M = 3$ in Appendix C. The outage probabilities computed in (21) are used in Step 4 of the resource allocation algorithm.

[4] F(.) denotes Cumulative Distribution Function (CDF).

VI. Simulation Results

In this section, we present simulation results that demonstrate the throughput gains achieved by the proposed policy over other existing policies. We also demonstrate that the limiting approximation we use to obtain closed form outage probability expressions is a good approximation.

The proposed policy (labeled as "optimal" in the plots) is throughput optimal among all policies that use the limited channel feedback scheme described in Section II. Three important components of our policy are: (1) evaluation of the *conditional expected CQI* for each sub-band from the EESM, (2) evaluation of goodput while *accounting for outage probability*, and (3) optimal utilization of *queue length* information. To illustrate the importance of each component of our proposed policy, we compare the proposed policy with the following policies (each of the heuristic policies ignores *at least* one component of our proposed policy): (1) a throughput optimal policy with perfect CQI (labeled "Perfect CQI"), (2) a policy that uses queue length information but assumes that the reported EESM is the CQI for all the best M reported sub-bands (labeled "Heuristic 1"), (3) a policy that uses queue length information and evaluates the conditional expected CQI for the best M reported sub-bands *without* accounting for outage probability (labeled "Heuristic 2"), and (4) a proportionally fair rate allocation policy that uses the conditional expected CQI and goodput evaluation without using queue length information (labeled "PF").

A. Throughput optimality: Comparison of aggregate arrival rate

A single-cell OFDM downlink with $K = 100$ UEs is simulated. The number of subcarriers is 512 and there are 12 subcarriers in each sub-band. Two channel models are considered: (1) i.i.d. sub-bands, and (2) Correlated sub-bands resulting from a 6-path channel (i.e., $L = 6$) with an uniform power-delay profile where each path is Rayleigh fading. The arrival traffic for the i^{th} UE is assumed to be Poisson with with parameter λ_i. The channel feedback from each UE is assumed to be the best M sub-bands and EESM for these sub-bands.

Figures 1 and 2 show the average queue length (averaged across UEs and time slots) versus the aggregate arrival rate (i.e., sum of λ_i's) for the i.i.d. and correlated sub-band cases respectively. λ_i is chosen as $i\lambda$, i.e., each UE has a different arrival traffic rate, and λ is varied to change the arrival traffic load. Also, $M = 3$ and the number of sub-bands $N = 43$. It is clear that the proposed policy can support significantly higher arrival traffic for the same average queue length than the heuristic policies. The Perfect CQI policy is also shown to quantify the loss due to limited feedback. It is also clear that the proposed policy provides performance gains even in the correlated sub-band case. Similar results can been observed for $M = 4$ in Figures 3, 4 and 5.

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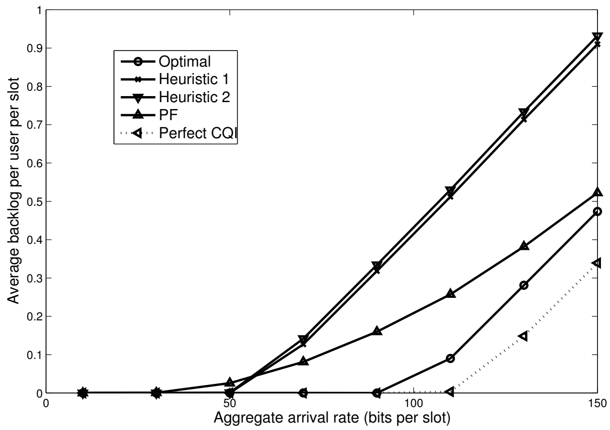

Fig. 1. i.i.d. sub-bands case: $M = 3, N = 43, L = 6, \eta = 1, \gamma_{ave,i} = 1 \, \forall \, i$.

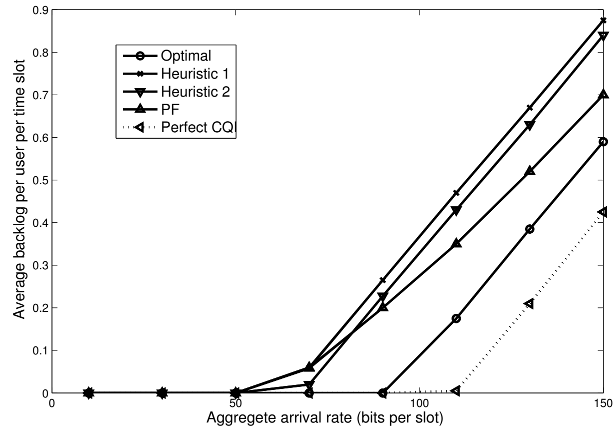

Fig. 3. i.i.d. sub-bands case: $M = 4, N = 43, L = 6, \eta = 1, \gamma_{ave,i} = 1 \, \forall \, i$.

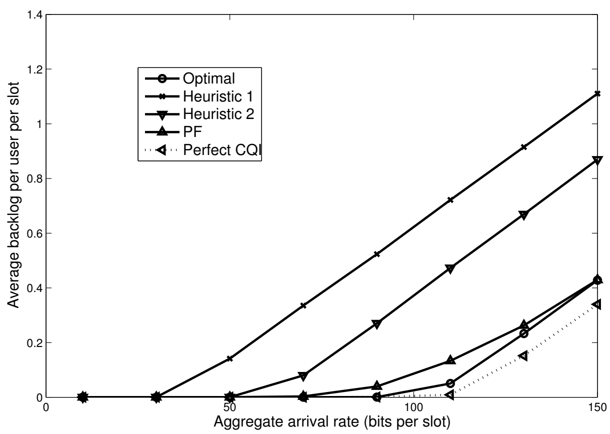

Fig. 2. Correlated sub-bands case: $M = 3$, $N = 43$, $L = 6$, $\eta = 1$, $\gamma_{ave,i} = 1 \, \forall \, i$.

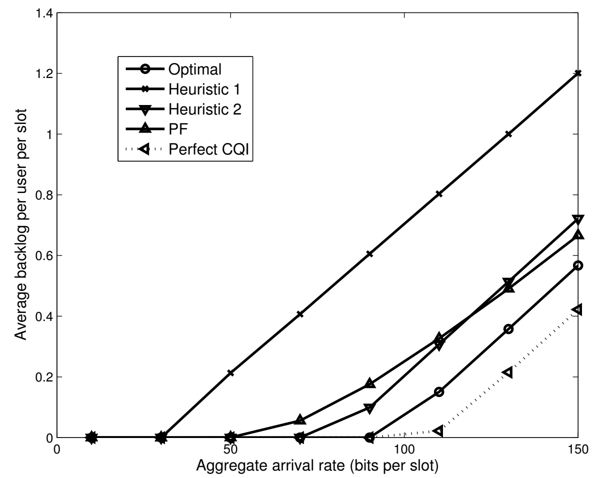

Fig. 4. Correlated sub-bands case: $M = 4$, $N = 43$, $L = 6$, $\eta = 1$, $\gamma_{ave,i} = 1 \, \forall \, i$.

B. Weak limit approximation: i.i.d. sub-bands case

The weak limit approximation improves as the number of sub-bands increases. Figure 6 shows the conditional CDF of the CQI of the best sub-band given a particular EESM for the best M sub-bands. Four cases of N (the total number of sub-bands) are shown. Note that the number of subcarriers is $12N$. It can be observed that the weak limit approximation is very good for $N = 22$ and $N = 43$.

C. Weak limit approximation: correlated sub-bands case

Figure 7 shows the conditional CDF of the CQI of the best sub-band given a particular EESM for the best M sub-bands. The sub-bands are correlated here since $L < N$. In this comparison, the number of paths L is approximately equal to half the number of sub-bands N. It can be observed that the correlated sub-band case is very similar to the i.i.d. sub-band case for this L/N ratio of approximately 0.5. Furthermore, as N increases the weak limit approximation is very good.

In Figure 8, L/N is approximately 0.25. It can be observed that the i.i.d. sub-bands case and correlated sub-bands case are not as close as in the case where L/N is approximately 0.5. However, it should be noted that even in this case the proposed throughput optimal policy provides performance gain over heuristic and PF policies (as seen in Figures 2, 4 and 5).

VII. Conclusions

We proposed a queue-aware policy for allocating sub-bands in the LTE downlink when each UE reports the best M sub-band indices and a single effective CQI for these bands. The throughput optimality of the proposed policy was shown using the Lyapunov stability framework. The policy assigns each sub-band to the UE with the best queue-length goodput product for that sub-band. The goodput was obtained by deriving analytical expressions for the conditional outage probability of each sub-band given the effective CQI. The conditional outage probability was derived by exploiting a limit theorem on the joint distribution of the SNR of the best sub-bands. The proposed policy supports significantly

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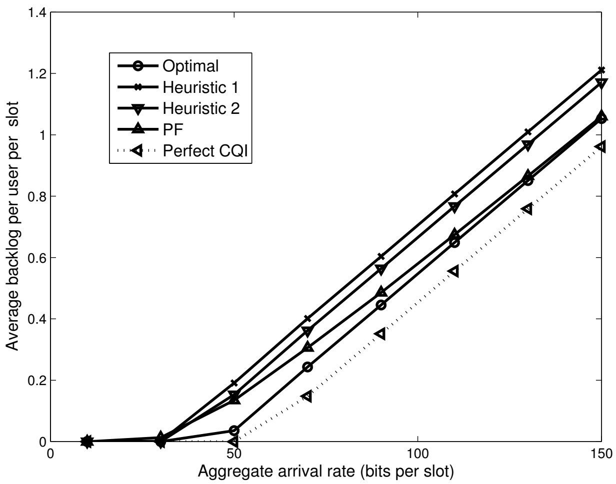

Fig. 5. Correlated sub-bands case: $M = 4$, $N = 22$, $L = 11$, $\eta = 1$, $\gamma_{ave,i} = 1 \; \forall \; i$.

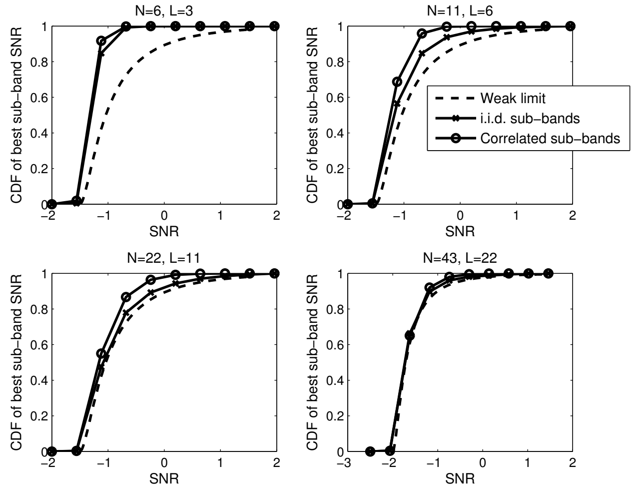

Fig. 7. Conditional CDF of best sub-band given $\gamma_{eff} = -1.5 + \log N$, $M = 3$, $\eta = 1$.

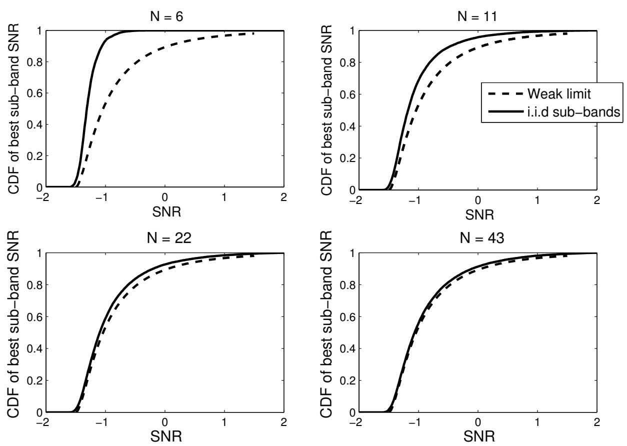

Fig. 6. Conditional CDF of best sub-band given $\gamma_{eff} = -1.5 + \log N$, $M = 3$, $\eta = 1$.

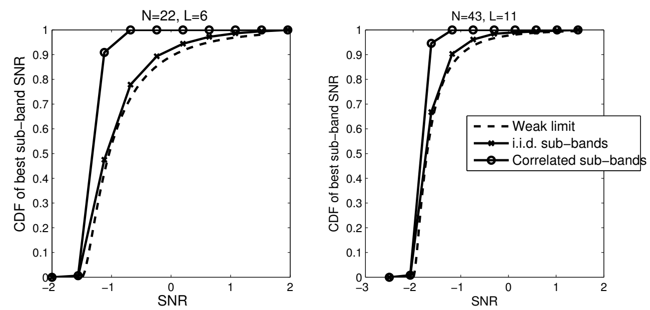

Fig. 8. Conditional CDF of best sub-band given $\gamma_{eff} = -1.5 + \log N$, $M = 3$, $\eta = 1$.

higher arrival traffic than existing policies like: (1) proportional fair allocation based on CQI that does not consider queue information, (2) queue-aware policies that use the effective CQI as the CQI of each sub-band, and (3) queue-aware policies that do not account for *outage* in the estimation of goodput.

APPENDIX A
PROOF OF THEOREM 1

Proof. If the arrival rate vector $\underline{\lambda}$ is stabilizable by some policy $\Pi \in \mathcal{P}$ then $\exists \underline{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_K)$ with $\epsilon_i > 0 \; \forall \; i$ such that

$$\lambda_i \leq \sum_{j=1}^{N} b_{i,j} r_{i,j} (1 - P_{i,j}(r_{i,j})) - \epsilon_i, \quad \forall \; i,$$

where $b_{i,j}$ is the fraction of j^{th} sub-band allocated to i^{th} UE and $r_{i,j}$ is the rate assigned to $[i, j]$ by policy Π. Next, we invoke Scheffé's Lemma [37], which asserts the uniform convergence of $|P_{i,j}(r) - \hat{P}_{i,j}(r)|$ to zero. Thus, for large N,

$$|P_{i,j}(r) - \hat{P}_{i,j}(r)| \leq \delta_{i,j}^N, \; \forall \; r, i, j,$$

where $\delta_{i,j}^N$ is a small positive number independent of r.. Hence, for large N,

$$1 - P_{i,j}(r) - \delta_{i,j}^N \leq 1 - \hat{P}_{i,j}(r) \leq 1 - P_{i,j}(r) + \delta_{i,j}^N \quad (22)$$

Since for every sub-band, our policy assigns $a_{i,j}^*$ and $r_{i,j}^*$ such that $\sum_{i=1}^{K} Q_i(t) a_{i,j}^* r_{i,j}^* (1 - \hat{P}_{i,j}(r_{i,j}^*))$ is maximized, the following inequality holds good $\forall \; j, \{b_{i,j}\}, \{r_{i,j}\}$.

$$\sum_{i=1}^{K} Q_i(t) b_{i,j} r_{i,j} (1 - \hat{P}_{i,j}(r_{i,j})) \leq \sum_{i=1}^{K} Q_i(t) a_{i,j}^* r_{i,j}^* (1 - \hat{P}_{i,j}(r_{i,j}^*)).$$

Using (22) we get,

$$\sum_{i=1}^{K} Q_i(t) b_{i,j} r_{i,j} (1 - P_{i,j}(r_{i,j}) - \delta_{i,j}^N) \leq \sum_{i=1}^{K} Q_i(t) a_{i,j}^* r_{i,j}^* (1 - P_{i,j}(r_{i,j}^*) + \delta_{i,j}^N).$$

Therefore, for our policy, the Lyapunov drift can be upper bounded as

$$\Delta(\underline{Q}(t)) \le B - \sum_{i=1}^{K} Q_i(t) \left(\epsilon_i - \sum_{j=1}^{N} (a^*_{i,j} r^*_{i,j} + b_{i,j} r_{i,j}) \delta^N_{i,j} \right).$$

Note that at most M of the $a^*_{i,j}$ and $b_{i,j}$ are non-zero for each user i which ensures that the summation is finite even if N is large. Thus, for any $\underline{\epsilon}$, there exists a large enough N for which

$$\epsilon_i - \sum_{j=1}^{N} (a^*_{i,j} r^*_{i,j} + b_{i,j} r_{i,j}) \delta^N_{i,j} > 0, \ \forall\ i,$$

which ensures that the Lyapunov drift becomes negative as queues grow. i.e., the proposed policy stabilizes all the arrival rates which can be stabilized by any other policy for large enough N. Hence it is asymptotically throughput optimal. □

APPENDIX B
PROOF OF LEMMA 1

Proof. Let $f_{X_i}(x_i)$ and $f_{Y_i}(y_i)$ denote the pdf of X_i and Y_i respectively. Thus,

$$f_{X_i}(x) = e^{-x}, x \ge 0, \forall i.$$

Consider $n = 2$.

$$\begin{aligned} f_{\underline{Y}^{(2)}}(y_1, y_2) &= f_{Y_1}(y_1) f_{Y_2|Y_1=y_1}(y_2), \quad 0 \le y_1 \le y_2, \\ &= f_{X_1}(y_1) f_{X_2}(y_2 - y_1), \\ &= e^{-y_2}, \qquad\qquad\qquad 0 \le y_1 \le y_2. \end{aligned}$$

Hence the lemma holds for $n = 2$. We use this as the basis for the following proof by induction. We assume that the (15) holds for given n. Then,

$$\begin{aligned} & f_{\underline{Y}^{(n+1)}}(y_1, y_2, \dots, y_{n+1}) \\ &= f_{\underline{Y}^{(n)}}(y_1, y_2, \dots, y_n) f_{Y_{n+1}|\underline{Y}^{(n)}=(y_1,y_2,\dots,y_n)}(y_{n+1}), \\ &= e^{-y_n} f_{X_{n+1}}(y_{n+1} - y_n), \\ &= e^{-y_{n+1}}, \qquad\qquad 0 \le y_1 \le y_2 \le \dots \le y_{n+1}. \end{aligned}$$

So by induction, the lemma holds for all $n \ge 2$. □

APPENDIX C
EXPRESSION FOR OUTAGE PROBABILITY FOR $M = 3$

The region for which the conditional joint pdf of S^1_i and S^2_i is non-zero is shown by the shaded area in Figure 9 We can find the marginal density as follows. Note that conditional pdfs are non-zero only for the specified region.

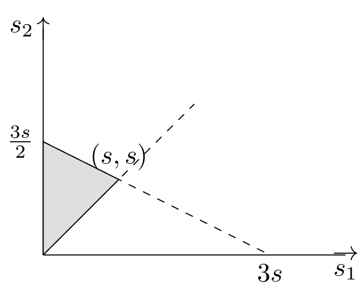

Fig. 9. Region for which the conditional joint pdf of S^1_i and S^2_i given $S^{eff}_i = s$ is non-zero

(i) Pdf of S^{eff}_i.
- For $0 \le s \le \infty$,

$$\begin{aligned} f_{S^{eff}_i}(s) &= \int_0^s \int_{s_1}^{\frac{3s-s_1}{2}} 3e^{-(3s-s_1-s_2)} ds_2 ds_1, \\ &= \frac{9}{2} e^{-s} + \frac{3}{2} e^{-3s} - 6e^{-\frac{3s}{2}}. \end{aligned}$$

(ii) Best sub-band SNR.
- For $0 \le s_1 \le s$,

$$\begin{aligned} f_{S^1_i|S^{eff}_i=s}(s_1) &= \int_{s_1}^{\frac{3s-s_1}{2}} \frac{3e^{-(3s-s_1-s_2)}}{f_{S^{eff}_i}(s)} ds_2, \\ &= \frac{6e^{\left(-\frac{3s}{2}+\frac{s_1}{2}\right)} - 6e^{2s_1}}{3 - 12e^{\frac{3s}{2}} + 9e^{2s}}. \end{aligned}$$

(iii) Second best sub-band SNR.
- For $0 \le s_2 \le s$,

$$\begin{aligned} f_{S^2_i|S^{eff}_i=s}(s_2) &= \int_0^{s_2} \frac{3e^{-(3s-s_1-s_2)}}{f_{S^{eff}_i}(s)} ds_1, \\ &= \frac{6e^{s_2}(-1 + e^{s_2})}{3 - 12e^{\frac{3s}{2}} + 9e^{2s}}. \end{aligned}$$

- For $s \le s_2 \le \frac{3s}{2}$,

$$\begin{aligned} f_{S^2_i|S^{eff}_i=s}(s_2) &= \int_0^{3s-2s_2} \frac{3e^{-(3s-s_1-s_2)}}{f_{S^{eff}_i}(s)} ds_1, \\ &= \frac{6e^{3s}(e^{-s_2} - e^{-3s+s_2})}{3 - 12e^{\frac{3s}{2}} + 9e^{2s}}. \end{aligned}$$

(iv) Third best sub-band SNR.

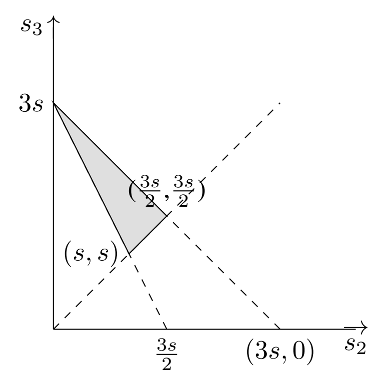

Fig. 10. Region for which the conditional joint pdf of S^2_i and S^3_i given $S^{eff}_i = s$ is non-zero

In-order to find the expression for $f_{S^3_i|S^{eff}_i=s}(s_3)$, we follow a similar procedure but by ignoring the best sub-band SNR. The joint pdf of $(S^{eff}_i, S^2_i, \dots, S^3_i)$ is given by

$$\begin{aligned} & f_{S^{eff}_i, S^2_i, \dots, S^3_i}(s, s_1, \dots, s_M) \\ = \;& M f_{\underline{S}^{(3)}_i}(3s - s_2 - s_3, s_2, s_3), \\ = \;& 3e^{-s_3}, \quad 0 \le 3s - s_2 - s_3 \le s_2 \le s_3. \end{aligned}$$

The conditional joint pdf can is given by

$$f_{S_i^2,S_i^3|S_i^{eff}=s,I_i=I}(s_2,s_3) = \frac{3e^{-s_3}}{f_{S_i^{eff}}(s)}, \quad (23)$$
$$\text{for} \quad 0 \le 3s - s_2 - s_3 \le s_2 \le s_3.$$

The region specified in (24) is shown in Figure 10. The conditional marginal pdf is obtained as follows.

- For $s \le s_3 \le \frac{3s}{2}$,

$$f_{S_i^3|S_i^{eff}=s}(s_3) = \int_{\frac{3s-s_3}{2}}^{s_3} \frac{3e^{-s_3}}{f_{S_i^{eff}}(s)} ds_2, = \frac{9e^{-s_3}(s_3-s)}{3e^{-3s} - 12e^{-\frac{3s}{2}} + 9e^{-s}}.$$

- For $\frac{3s}{2} \le s_3 \le 3s$,

$$f_{S_i^3|S_i^{eff}=s}(s_3) = \int_{\frac{3s-s_3}{2}}^{3s-s_3} \frac{3e^{-s_3}}{f_{S_i^{eff}}(s)} ds_2, = \frac{3e^{-s_3}(3s-s_3)}{3e^{-3s} - 12e^{-\frac{3s}{2}} + 9e^{-s}}.$$

The outage probabilities $\hat{P}_{i,i_j}(r)$ for $j = 1, \ldots, 3$ can be obtained as follows. let $y = Ne^{-\left(\frac{2^r-1}{\frac{P}{M'}}\right)}$.

(i) Best sub-band.

- For $0 \le y \le s$,

$$\hat{P}_{i,i_1}(r) = \frac{3e^{2s} + e^{2y} - 4e^{\left(\frac{3s}{2}+\frac{y}{2}\right)}}{1 - 4e^{\frac{3s}{2}} + 3e^{2s}}.$$

(ii) Second best sub-band.

- For $0 \le y \le s$,

$$\hat{P}_{i,i_2}(r) = \frac{2e^s}{1 + 2e^{\frac{s}{2}} + 3e^s} + \frac{(e^s - e^y)(e^s + e^y - 2)}{1 - 4e^{\frac{3s}{2}} + 3e^{2s}}.$$

- For $s \le y \le \frac{3s}{2}$,

$$\hat{P}_{i,i_2}(r) = \frac{2e^{-y}(e^{\frac{3s}{2}} - e^y)^2}{1 - 4e^{\frac{3s}{2}} + 3e^{2s}}.$$

(iii) Third best sub-band.

- For $s \le y \le \frac{3s}{2}$,

$$\hat{P}_{i,i_3}(r) = \frac{3e^{-3s} - 12e^{-\frac{3s}{2}} - 9se^{-y} + 9ye^{-y} + 9e^{-y}}{3e^{-3s} - 12e^{-\frac{3s}{2}} + 9e^{-s}}.$$

- For $\frac{3s}{2} \le y \le 3s$,

$$\hat{P}_{i,i_3}(r) = \frac{3e^{-3s} - 3e^{-y} + 9se^{-y} - 3ye^{-y}}{3e^{-3s} - 12e^{-\frac{3s}{2}} + 9e^{-s}}.$$

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