Outage Probability of Multiple-Input Single-Output (MISO) Systems with Delayed Feedback†

Venkata Sreekanta Annapureddy¹, Devdutt V. Marathe², T. R. Ramya³ and Srikrishna Bhashyam³

¹ Coordinated Science Laboratory
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
1308 West Main St. Urbana, IL 61801
E-mail: vannapu2@uiuc.edu

²Department of Electrical Engineering
California Institute of Technology
Pasadena, CA 91125.
E-mail: devdutt.marathe@gmail.com

³Department of Electrical Engineering
Indian Institute of Technology Madras
Chennai 600036, India
Phone: 91-44-22574439, Fax: 91-44-22570120
E-mail: {ee04d016,skrishna}@ee.iitm.ac.in

†This work was done at the Dept. of Electrical Engg., Indian Institute of Technology Madras. Part of this work was presented at the 40th Asilomar Conference on Signals, Systems, and Computers held in Oct-Nov 2006 at Pacific Grove, CA.
Abstract

We investigate the effect of the feedback delay on the outage probability of multiple-input single-output (MISO) fading channels. Channel state information at the transmitter (CSIT) is a delayed version of the channel state information available at the receiver (CSIR). We consider two cases of CSIR: (a) perfect CSIR and (b) CSI estimated at the receiver using training symbols. With perfect CSIR, under a short-term power constraint, we determine: (a) the outage probability for beamforming with imperfect CSIT (BF-IC) analytically, and (b) the optimal spatial power allocation (OSPA) scheme that minimizes outage numerically. Results show that, for delayed CSIT, BF-IC is close to optimal for low SNR and uniform spatial power allocation (USPA) is close to optimal at high SNR. Similarly, under a long-term power constraint, we determine the outage probability for BF-IC with temporal power control and USPA with temporal power control numerically. Again, BF-IC is close to optimal for low SNR and uniform spatial power allocation (USPA) is close to optimal at high SNR. With imperfect CSIR, we obtain an upper bound on the outage probability with USPA and BF-IC. Results show that the loss in performance due to imperfection in CSIR is not significant, if the training power is chosen appropriately. Also, the asymptotic diversity order with the imperfect CSIR is the same as with perfect CSIR.

I. INTRODUCTION

Channel State Information is very crucial in determining the performance of any wireless system. The minimum outage probability of multiple-input single-output (MISO) channels with perfect channel state information at the receiver (CSIR) and no channel state information at the transmitter (CSIT) is derived in [1]. For reasonably low outage probabilities, uniform spatial power allocation (USPA) across the spatial dimension is the optimal strategy. In most practical systems, two way communication is common and some form of CSI can be made available at the transmitter. Outage probability of MISO systems with perfect CSIT and CSIR is derived in [2]. It is shown that feeding back the CSI provides significant gain in the performance, and that beamforming to the direction of the channel is optimal and provides a constant SNR gain over no CSIT under short-term power constraint (i.e., transmit power is constant over each transmission interval). In the case of long-term average power constraint, it is also possible to adapt the transmission power level based on channel feedback (i.e., temporal power control). Outage can be reduced significantly by saving power when the channel is strong and using the saved power when the channel is worse. The optimum power allocation strategy to minimize
the outage probability over fading channels and MISO fading channels is determined in [3] and [2] respectively.

In practice, the feedback channel resources are seldom perfect enough to provide instantaneous and noiseless feedback. There is vast literature on the performance of multiple antenna systems with imperfect feedback. Under the short term power constraint, and for two cases of imperfect feedback namely mean feedback and covariance feedback, spatial schemes that a) minimize the outage probability are studied in [4], [5], and b) maximize the mutual information are studied in [6]. In [7], BER performance of spatial schemes in the presence of delayed feedback has been studied. Under a long-term power constraint, minimum outage probability with temporal power control for quantized CSIT has been studied in [8]. In practice, it is also not feasible to have a perfect estimate of CSIR. Usually, channel state information at the receiver is estimated using training symbols, and the resources used during the training period have to be accounted for. Outage probability with preamble based CSIR and quantized CSIT has been studied in [8]. In [9], maximizing mutual information in the presence of channel estimation error and delayed feedback has been studied.

In this paper, we focus on the effect of the delay in feedback on the performance from the point of view of outage probability. Using the delayed feedback model in [10], we solve the problem of minimum outage transmission over MISO channels under both short-term and long-term power constraints. Under a short-term power constraint, beamforming is optimal if the transmitter has perfect CSI. We analyze the loss in performance of beamforming due to the delay in the feedback and derive an analytical expression for the outage probability of beamforming with imperfect CSIT (BF-IC). Results show that BF-IC, which allocates total power in the direction of CSIT, is better at low SNR while USPA [1], which allocates equal power in all the directions and does not require any feedback, is better at high SNR. However, none of the above two strategies is optimal. The minimum outage transmission strategy for a given delay, optimal spatial power allocation (OSPA) is determined. OSPA involves beamforming along the spatial modes and optimal power allocation across the spatial modes. Numerical results show that BF-IC is very close to OSPA for low SNR while USPA is close to OSPA for high SNR. Since OSPA does not provide significant gain at any SNR, compared to the best of BF-IC and USPA, the cross-over SNR at which USPA
becomes better than BF-IC is important and can be used to switch between BF-IC and USPA. We present the equation to determine this cross-over SNR and solve it numerically.

Under a long-term power constraint, with perfect CSIT, the optimal beamforming (to the channel direction) and temporal power control strategy is obtained in [2]. However, with delayed CSIT, this policy is not optimal. We formulate the optimization problem to obtain the optimal spatio-temporal power control function to minimize the outage probability with delayed feedback. This is difficult to evaluate due to lack of a closed form expression for OSPA under a short term power constraint. Therefore, we evaluate the minimum outage probabilities for BF-IC and USPA with temporal power control by (a) first determining a condition that the power control function should satisfy using a result from the calculus of variations, and (b) solving for the power control function numerically. Again, BF-IC is better at low SNR while USPA is better at high SNR. The cross-over SNR when USPA becomes better than BF-IC can also be determined.

Finally, we also consider the effect of imperfect CSIR and extend the analysis with delayed feedback and perfect CSIR to the case of delayed feedback and imperfect CSIR. An upper bound on the outage probability with USPA and BF-IC with imperfect CSIR is obtained. The corresponding outage probabilities with perfect CSIR are the natural lower bounds. The loss in performance due to the error in estimation of CSIR is shown to be negligible if the training power is chosen optimally, and the asymptotic diversity gain remains the same even with imperfect CSIR compared to perfect CSIR.

The rest of the paper is organized as follows. In Section II, our system model is introduced. In Section III, under the short power constraint, outage probability with BF-IC and OSPA is determined and compared with USPA. In Section IV, under the long term power constraint, the optimal temporal power allocation schemes are determined for both USPA and BF-IC. In Section V, the analysis under short term power constraint is extended to imperfect CSIR. Finally, the results are presented in Section VI and summarized in Section VII.

II. SYSTEM MODEL

The MISO system with \(M\) transmit antennas and 1 receive antenna is, as usual, modeled as

\[
y = h^H x + z,
\]

(1)
where $\mathbf{h} \sim \mathcal{CN}(0, \mathbf{I})$ is a $M \times 1$ independent, identically distributed (i.i.d) and zero-mean circularly symmetric complex Gaussian channel vector, $\mathbf{x}$ is a $M \times 1$ channel input vector and $\mathbf{z}$ is zero-mean unit-variance additive white Gaussian noise (AWGN). We use a block fading model, where the channel coefficients are assumed to be fixed within a given duration, known as coherence interval. We assume high correlation between successive time durations. Using the Gaussian channel vector model, the delay in the feedback is captured by the correlation coefficient $\rho$ between CSIT and CSIR. We can relate the old channel and the actual channel as follows [10]:

$$
\mathbf{h} = \rho \mathbf{h}_{old} + \sqrt{1 - \rho^2} \mathbf{w},
$$

(2)

where $\mathbf{h}_{old}$ is the delayed CSIT, $\rho$ is a correlation coefficient, and $\mathbf{w} \sim \mathcal{CN}(0, \mathbf{I})$ is independent of $\mathbf{h}_{old}$. The gap between no CSIT ($\rho = 0$) and perfect CSIT ($\rho = 1$) is bridged using $\rho$. Lower the delay in the feedback, higher the value of $\rho$.

III. SHORT-TERM POWER CONSTRAINT

Assuming a short-term power constraint [3], such that the transmit power is not a function of time, the mutual information is given by

$$
I(\mathbf{x}; \mathbf{y}/\mathbf{h}, \mathbf{h}_{old}) = \log(1 + \mathbf{P}\mathbf{h}^H\mathbf{Q}\mathbf{h}),
$$

(3)

where $\mathbf{Q}$ is the input covariance matrix such that $Tr(\mathbf{Q}) = 1$ and $\mathbf{P}$ is the transmit power.

Consider the two extreme cases: zero feedback ($\rho = 0$) and instantaneous feedback ($\rho = 1$). For $\rho = 0$, where the transmitter does not have any knowledge of the channel state information, the diversity strategy with the power distributed equally among the $M$ orthogonal independent transmit directions, i.e., USPA is optimal [1], i.e., we have

$$
\mathbf{Q} = \frac{\mathbf{I}}{M},
$$

(4)

$$
P_{outUSPA}(M, R, P) = \Gamma_M \left( \frac{e^R - 1}{P/M} \right),
$$

(5)

where $P_{outUSPA}(M, R, P)$ is the outage probability (as defined in [1]) for a $M \times 1$ system using USPA corresponding to a transmit power constraint $P$ and rate $R$ (in nats/transmission), and
\( \Gamma_M(\cdot) \) is the incomplete Gamma function defined as
\[
\Gamma_M(x) = \frac{1}{(M-1)!} \int_0^x t^{M-1}e^{-t}dt.
\]

For \( \rho = 1 \), where the transmitter has perfect knowledge of the channel state information, beamforming is optimal [2], i.e., \( x = \frac{h}{\sqrt{h^Hh}}d \), where \( d \) is a scalar i.i.d. Gaussian input, and
\[
Q = \frac{hh^H}{h^Hh}.
\]
(6)

\[
P_{\text{out BF}}(M, R, P, \rho = 1) = \Gamma_M \left( \frac{e^R - 1}{P} \right).
\]
(7)

Outage performance for \( \rho = 1 \) is \( 10\log_{10}M \) dB better than the performance for \( \rho = 0 \). For \( 0 < \rho < 1 \), where we do not have perfect CSIT, we evaluate the outage performance of beamforming using the imperfect CSIT in Section III-A. We also determine the optimal spatial power allocation strategy that minimizes the outage probability in Section III-B and compare it with beamforming using the imperfect CSIT.

A. Beamforming using imperfect CSIT (BF-IC)

In this section, the loss in performance due to the presence of the delay in the feedback is analyzed and an expression for the outage probability (equation (14)) is derived. This is a simple extension of beamforming from perfect CSIT to the imperfect CSIT case, where beamforming is performed using the imperfect CSIT assuming that it is the actual channel. Therefore, we have \( x = \frac{h_{\text{old}}}{\sqrt{h_{\text{old}}^Hh_{\text{old}}}}d \), where \( d \) is a scalar i.i.d. Gaussian input, and
\[
Q = \frac{h_{\text{old}}h_{\text{old}}^H}{h_{\text{old}}^Hh_{\text{old}}},
\]
(8)

Substituting (8) in (3) and denoting the feedback SNR \( h_{\text{old}}^Hh_{\text{old}} \) by \( \gamma \), we get
\[
I(x; y/h, h_{\text{old}}) = \log \left( 1 + P \frac{h_{\text{old}}^Hh_{\text{old}}h_{\text{old}}^Hh}{\gamma} \right).
\]
(9)

Now, we derive the outage probability for the specific model described in equation (2). Note that \( \gamma \) is Gamma distributed with the pdf given by
\[
f_\Gamma(\gamma) = \frac{\gamma^{M-1}e^{-\gamma}}{(M-1)!},
\]
(10)
The expression for the mutual information for a given $h_{old}$ can be simplified as follows.

$$h^H h_{old} h_{old}^H = \left| h^H h_{old} \right|^2 = \left| (\rho h_{old} + \sqrt{1 - \rho^2} w^H h_{old}) \right|^2$$

$$= \frac{(1 - \rho^2)}{2} \left| \frac{2\rho^2}{(1 - \rho^2)} + \sqrt{2} \frac{w^H h_{old}}{\sqrt{\gamma}} \right|^2. \quad (11)$$

Hence, the mutual information given $h_{old}$ can be simplified as

$$I(x; y/h_{old}) = \log \left( 1 + P \frac{(1 - \rho^2)}{2} A \right), \quad (12)$$

where $A = \sqrt{\delta + \sqrt{2} \frac{w^H h_{old}}{\sqrt{\gamma}}^2}$. Note that $\frac{w^H h_{old}}{\sqrt{\gamma}}$ given $h_{old}$ is a zero mean complex Gaussian random variable with variance $||h_{old}||^2 = 1$. Thus, $A$ given $\gamma$ is a non-central chi-square ($\chi^2$) random variable with two degrees of freedom and parameter $\delta$. Observe that the distribution of mutual information given $h_{old}$ depends on only $\gamma = ||h_{old}||^2$. Therefore we have the following expression for the outage probability for a given $\gamma$.

$$\Pr(\text{outage} / \gamma) = \Pr \left( \log \left( 1 + P \frac{(1 - \rho^2)}{2} A \right) < R \right)$$

$$= F_{(\text{nc-}\chi^2, 2, \delta)}(2\beta), \quad (13)$$

where $\beta = \frac{e^{R-1} - 1}{P}$, and $F_{(\text{nc-}\chi^2, 2, \delta)}(\cdot)$ is the CDF of a non-central chi-square random variable with two degrees of freedom and parameter $\delta$. The overall probability of outage can be simplified as

$$P_{out}^{\text{BF-IC}}(M, R, P, \rho) = \int_0^\infty f_\Gamma(\gamma) \Pr(\text{outage} / \gamma) d\gamma \quad (14)$$

$$= \frac{1}{(1 + \mu)^M} \sum_{i=0}^{M-1} \binom{M-1}{i} \mu^i \Gamma(i+1) \left( \frac{e^{R-1}}{P} \right).$$

The derivation of equation (14) is shown in the appendix. Note that $(1 + \mu)^M = \sum_{i=0}^{M-1} \binom{M-1}{i} \mu^i$. Therefore, the result (14) can be interpreted as the weighted average of $\Gamma_K \left( \frac{e^{R-1}}{P} \right)$, which is the outage probability of a $K \times 1$ MISO system with perfect CSIT, where $K$ varies from 1 to $M$. Therefore, at high SNR, we expect the outage probability with BF-IC to be dominated by the first term ($K = 1$), which decays as $\frac{1}{\text{SNR}}$.

The asymptotic diversity gain at infinite SNR, defined as

$$d = \lim_{\text{SNR} \to \infty} \frac{\log P_{out}}{\log \text{SNR}} \quad (15)$$
can be quantified. From (14), using the approximation $\Gamma_M(x) \simeq \frac{x^M}{M!}$ for very small $x$, we can show that the asymptotic diversity gain of the BF-IC scheme is 1 for imperfect CSIT, i.e.,

$$\text{Diversity Gain } d = \begin{cases} 
1 & \text{for } 0 \leq \rho < 1 \\
M & \text{for } \rho = 1
\end{cases}. \quad (16)$$

This result can be explained intuitively as follows. At very high SNR, the outage probability is dominated by the error in the CSIT rather than channel being in deep fade. However, for USPA, the asymptotic diversity gain is $M$ independent of $\rho$. Therefore, USPA is always better than BF-IC at high SNR. The cross-over SNR $SNR_{cross}(\rho, R, M)$ can be obtained by equating the outage probabilities of the two schemes: (5) and (14). Although there is no closed form expression for cross-over SNR, it can be computed numerically. By comparing the operating SNR with the cross-over SNR, one can switch between BF-IC and USPA.

B. Optimal Spatial Power Allocation (OSPA)

We have seen that neither beamforming nor uniform spatial power allocation is the optimal strategy for any given $\rho$ ($0 < \rho < 1$). We find the optimal spatial power allocation strategy that minimizes the outage probability. Our results show that OSPA allocates a fraction $\lambda$ of the power along the spatial mode corresponding to the imperfect CSIT with the remaining power being equally distributed among the other orthogonal spatial modes.

The overall outage probability is minimized by minimizing $P_{out}(h_{old})$, outage probability given $h_{old}$, for each realization of $h_{old}$. The outage probability for a given $h_{old}$ is given by

$$P_{out}(h_{old}) = \Pr \left( h^H Q h < \frac{e^R - 1}{\mu} \right). \quad (17)$$

Using (2), $P_{out}(h_{old})$ can be simplified as

$$\Pr \left( (\sqrt{\mu} h_{old} + w)^H \left( (\sqrt{\mu} h_{old} + w)^H < \beta \right) \right), \quad (18)$$

where $\beta = \frac{e^R - 1}{\mu}$ and $\mu = \frac{\rho^2}{1 - \rho^2}$. The outage probability given by (18) is equivalent to the outage probability of a MISO channel with a mean feedback of $\sqrt{\mu}h_{old}$, which is minimized without any loss of generality by minimizing over the fraction of the power spent in the direction of the mean feedback [4], [5]. Rest of the power is spent equally in the M-1 orthogonal beams.
Since $Q$ is positive semi-definite, we have the eigenvalue decomposition (EVD) $Q = V\tilde{Q}V^H$, where $\tilde{Q} = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_M\}$ is a diagonal matrix with $\lambda_i \geq 0$ representing the power allocated to the direction indicated by the corresponding column vector of the unitary matrix $V$. It has been shown in [4] that the unitary matrix $V$ that minimizes the outage probability (18) is of the form $V = [\frac{h_{old}}{\sqrt{\gamma}}, v_2, v_3, \ldots, v_M]$, where $\{v_i\}, \ 2 \leq i \leq M$ is an arbitrary set of $(M - 1)$ orthonormal vectors that are orthogonal to $h_{old}$. Hence, we have $d = V^Hh_{old} = [\sqrt{\gamma}, 0, 0, \ldots, 0]^T$ and $g = V^Hw \sim CN(0, I)$. Thus (18) is simplified as

$$P_{out}(h_{old}) = \text{Pr}\left((g + \sqrt{\mu\gamma})^H\tilde{Q}(g + \sqrt{\mu\gamma}) < \beta\right).$$

Let

$$\xi = (g + \sqrt{\mu\gamma})^H\tilde{Q}(g + \sqrt{\mu\gamma}) = g^H\tilde{Q}g + g^H\tilde{Q}\sqrt{\mu\gamma}d + \sqrt{\mu\gamma}d^H\tilde{Q}g + \sqrt{\mu\gamma}d^H\tilde{Q}\sqrt{\mu\gamma}d$$

$$= \sum_{i=1}^{M} \lambda_i |g_i|^2 + 2\lambda_1\sqrt{\mu\gamma}\text{Re}(g_1) + \lambda_1\mu\gamma$$

$$= \lambda_1\{(\text{Re}(g_1) + \sqrt{\mu\gamma})^2 + |\text{Im}(g_1)|^2\} + \sum_{i=2}^{M} \lambda_i |g_i|^2.$$

Observe that $\xi$ is symmetric over $\lambda_i$, $i = 2$ to $M$. Hence, there is no reason to prefer any one $\lambda_i$ over others. Therefore, $\lambda_i$'s should be equal for $i = 2$ to $M$. This observation allows the random variable $\xi$ to be expressed in terms of $\lambda_1$ alone, using which the outage probability is determined easily in terms of the CDF of a single random variable. This is not explicitly used in the expressions for outage probability in [4] (see equation (10) in [4]). Further simplification of the outage expression based on this observation is presented below. Denote $\lambda_1$ by $\lambda$ for convenience.

$$\text{Tr}(\tilde{Q}) = 1 \Rightarrow \lambda_i = \frac{1 - \lambda}{M - 1}, \ \text{for} \ i = 2 \ \text{to} \ M$$

$$\Rightarrow \xi = \lambda A + \frac{1 - \lambda}{M - 1} B,$$

where $A = \{\sqrt{2}\text{Re}(g_1) + \sqrt{2\mu\gamma})^2 + |\sqrt{2}\text{Im}(g_1)|^2\}$ is Non-Central Chi-Square distributed with 2 degrees of freedom and non-centrality parameter $\delta = 2\mu\gamma$ and $B = 2\sum_{i=2}^{M} |g_i|^2$ is Central Chi-Square distributed with $2(M - 1)$ degrees of freedom. Observe that $\xi$ depends only on $\gamma = h_{old}^Hh_{old}$. Therefore, we denote the outage probability for a given $\gamma$ and $\lambda$ as $P_{out}(\gamma, \lambda)$, given by

$$P_{out}(\gamma, \lambda) = Pr(\xi < 2\beta) = F_\xi(2\beta),$$

$(23)$
where $F_\xi(.)$ represents the CDF of $\xi$. To complete the solution, it remains only to find the optimal value of $\lambda$ for each $\gamma$. Consider the two extreme cases: $\rho = 0$ and $\rho = 1$.

For $\rho = 0$, $\mu = 0$ and $\xi = 2 \sum_{i=1}^{M} \lambda_i |g_i|^2$ is symmetric over $\lambda_i, i = 1 \text{ to } M$, i.e., all the directions are identical, and hence, equal power is spent in each direction. Therefore, $\lambda_{opt}(\gamma) = \frac{1}{M}$, for $\rho = 0$. As $\rho$ tends to 1, $\mu$ tends to $\infty$. Therefore, the coefficient of $\lambda_1$ becomes large compared to the coefficients of the other $\lambda_i$’s, and hence, it is optimal to spend all the power in that direction. Therefore, $\lambda_{opt}(\gamma) = 1$ for $\rho = 1$.

Consider the case of $0 < \rho < 1$. When $\gamma = 0$, $\delta = 2\mu\gamma = 0$. Following similar analysis as in the case of $\rho = 0$, we arrive at $\lambda_{opt} = \frac{1}{M}$

$$\Rightarrow \lambda_{opt}(\gamma = 0) = \frac{1}{M} \text{ for any } \rho. \quad (24)$$

As $\gamma \to \infty$, $\delta = 2\mu\gamma \to \infty$. Following similar analysis as in the case of $\rho = 1$, we arrive at $\lambda_{opt} = 1$

$$\Rightarrow \lambda_{opt}(\gamma \to \infty) = 1 \text{ for any } \rho. \quad (25)$$

Therefore, for $0 < \rho < 1$, we expect $\lambda_{opt}(\gamma)$ to start from $\frac{1}{M}$ at $\gamma = 0$ and approach 1 as $\gamma$ increases.

Hence, the minimum outage probability for a given $\gamma$ is given by

$$\min_{\lambda} P_{out}(\gamma, \lambda) = P_{out}(\gamma, \lambda_{opt}(\gamma)), \quad (26)$$

where $\lambda_{opt}(\gamma)$ is the solution of

$$\frac{\partial P_{out}(\gamma, \lambda)}{\partial \lambda} = 0 \text{ in the range from } \frac{1}{M} \text{ to } 1. \quad (27)$$

Expressing $P_{out}(\gamma, \lambda)$ as

$$P_{out}(\gamma, \lambda) = \Pr \left( \lambda A + \frac{1 - \lambda}{M - 1} B < 2\beta \right)$$

$$= \int_{0}^{2\beta} f_{A}(a) F_{B} \left( \frac{(2\beta - \lambda a)(M - 1)}{1 - \lambda} \right) da, \quad (28)$$

equation (27) can be simplified as

$$\int_{0}^{2\beta} f_{A}(a) \exp \left( \frac{(M - 1)\lambda a}{2(1 - \lambda)} \right) (2\beta - \lambda a)^{(M-2)}(2\beta - a) da = 0. \quad (29)$$
Although a closed form expression for $\lambda_{\text{opt}}(\gamma)$ does not appear to be available, it can be determined numerically by a one-dimensional numerical search over the range. The overall outage probability can then be determined by averaging $P_{\text{out}}(\gamma, \lambda_{\text{opt}}(\gamma))$ over $\gamma$.

IV. LONG-TERM POWER CONSTRAINT

We relax the short-term power constraint and consider a long-term power constraint i.e. transmitted power can be varied and is allocated dynamically with an average power constraint.

$$\int_{0}^{\infty} f_\Gamma(\gamma)p(\gamma)d\gamma = 1.$$  

(30)

Achieving minimum outage probability involves power allocation in both spatial and temporal domains.

A. Optimal Spatio-Temporal Power Allocation

In this subsection, we outline the approach to determine the optimal spatio-temporal power control that minimizes the outage probability. Let us define a temporal power control policy $p(\gamma)$, such that $Pp(\gamma)$ is the transmit power at the transmitter with the feedback SNR $\gamma$. Now, consider a fixed power allocation policy, for a given $\gamma$ and the corresponding $p(\gamma)$, the problem of minimizing the outage probability can be formulated as

$$\min_{Q} \Pr \left( h^H Q h < \frac{e^R - 1}{Pp(\gamma)} \right).$$  

(31)

As shown in the OSPA subsection, the same can be minimized by minimizing $P_{\text{out}}(\gamma, p(\gamma), \lambda)$, the outage probability given $\gamma$ and the corresponding $p(\gamma)$, over $\lambda$, fraction of the power spent in the direction of the imperfect feedback. Similar to (23), $P_{\text{out}}(\gamma, p(\gamma), \lambda)$ is given by

$$P_{\text{out}}(\gamma, p(\gamma), \lambda) = \Pr \left( \lambda A + \frac{1 - \lambda}{M - 1} B < \frac{2\beta}{p(\gamma)} \right) = F_\xi \left( \frac{2\beta}{p(\gamma)} \right).$$  

(32)

Hence, the minimum outage probability for a given $\gamma$ and $p(\gamma)$ is given by

$$\min_{\lambda} P_{\text{out}}(\gamma, p(\gamma), \lambda) = P_{\text{out}}(\gamma, p(\gamma), \lambda_{\text{opt}}(\gamma, p(\gamma))),$$  

(33)

where $\lambda_{\text{opt}}(\gamma, p(\gamma))$ is the solution of

$$\frac{\partial P_{\text{out}}(\gamma, p(\gamma), \lambda)}{\partial \lambda} = 0 \text{ in the range from } \frac{1}{M} \text{ to } 1.$$  

(34)
It remains to find the optimal temporal power allocation policy \( p(\gamma) \) the minimizes the expected value of (33) over \( \gamma \):

\[
\min_{p(\gamma)} \int_{0}^{\infty} f(\gamma) P_{\text{out}}(\gamma, p(\gamma), \lambda_{\text{opt}}(\gamma, p(\gamma))) d\gamma,
\]

subject to the power constraint (30).

However, finding optimal \( p(\gamma) \) and the corresponding \( \lambda_{\text{opt}}(\gamma, p(\gamma)) \) is computationally intensive, since we do not have closed form expression for \( \lambda_{\text{opt}}(\gamma, p(\gamma)) \). Therefore, based on the intuition from the results for the short-term power constraint, the suboptimal schemes BF-IC with temporal power control and USPA with temporal power control are considered. In both these schemes, we fix a spatial power allocation scheme and determine the corresponding optimal temporal power allocation scheme that minimizes the outage probability.

**B. USPA with Temporal Power Control**

In this subsection, we fix the spatial power allocation scheme to USPA, where the power is distributed equally among the orthogonal independent transmit directions, i.e, \( \lambda = \frac{1}{M} \) or \( Q = \frac{1}{M} \), and determine the corresponding temporal power allocation scheme that minimizes the outage probability numerically. Substituting \( \lambda = \frac{1}{M} \) in (32), the outage probability for a given \( \gamma \) and the corresponding \( p(\gamma) \) is simplified as

\[
P_{\text{out}}(\gamma, p(\gamma)) = \Pr \left( A + B < \frac{2M\beta}{p(\gamma)} \right) = F_{(nc-\chi^2,2M,\delta)} \left( \frac{2M\beta}{p(\gamma)} \right).
\]

Thus, the problem of minimizing the outage probability is formulated as

\[
\min_{p(\gamma)} \int_{0}^{\infty} f(\gamma) F_{(nc-\chi^2,2M,\delta)} \left( \frac{2M\beta}{p(\gamma)} \right) d\gamma,
\]

subject to the power constraint (30)

From calculus of variations [11] (see Theorem 4.2.1 in [11]), if the function \( u \) extremizes \( I(f) = \int_{a}^{b} F(x, f, f') \) among all admissible \( f \), which satisfy the additional restriction that \( K(f) = \int_{a}^{b} G(x, f, f') = K_0 \), then there is a constant \( k \) such that

\[
\frac{d}{dx} \left( \frac{\partial(F + kG)}{\partial f'} \right) - \frac{\partial(F + kG)}{\partial f} = 0.
\]
In the power control problem, $p(\cdot)$ corresponds to $f(\cdot)$ above. Using this, we arrive at the following expression for the temporal control function that minimizes the outage probability with USPA.

$$k_1 = \left( \frac{2M \beta}{p^2(\gamma)} \right) f_{(nc-\chi^2,2M,\delta)} \left( \frac{2M \beta}{p(\gamma)} \right),$$

(38)

where $k_1$ is a constant chosen such that $p(\gamma)$ thus obtained satisfies the power constraint (30) and is non-negative. Since, no closed form expression seems to be available for $p(\gamma)$, it is determined numerically from equations (30) and (38).

C. BF-IC with Temporal Power Control

Here, we fix the spatial power allocation scheme to BF-IC, where the power is spent in only one direction corresponding to the imperfect CSIT, i.e., $\lambda = 1$. Following similar analysis as in the above section,

$$P_{out}(\gamma, p(\gamma)) = \Pr \left( A < \frac{2\beta}{p(\gamma)} \right) = F_{(nc-\chi^2,2,\delta)} \left( \frac{2\beta}{p(\gamma)} \right).$$

(39)

With $\lambda = 1$, (35) is modified to

$$\min_{p(\gamma)} \int_0^\infty f_{\Gamma}(\gamma) F_{(nc-\chi^2,2,\delta)} \left( \frac{2\beta}{p(\gamma)} \right) d\gamma.$$

(40)

Again, using calculus of variations, we arrive at the following expression for the power control function that minimizes the outage probability with BF-IC.

$$k_2 = \left( \frac{2\beta}{p^2(\gamma)} \right) f_{(nc-\chi^2,2,\delta)} \left( \frac{2\beta}{p(\gamma)} \right),$$

(41)

where $k_2$ is a constant, chosen such that $p(\gamma)$ thus obtained satisfies the power constraint (30) and is non-negative. This can be solved numerically as before using equations (30) and (41).

V. Effect of Imperfect CSIR

In the earlier sections, we assumed that the receiver has perfect channel state information (CSI). In practice, the CSI at the receiver is imperfect since it is estimated using training symbols with finite power. Furthermore, the channel estimation process itself requires some bandwidth and
energy resources and this needs to be accounted for. In this section, we incorporate and analyze the effect of imperfect CSIR on the outage performance.

We assume the training and MMSE channel estimation model as in [8]. $M$ training symbols are transmitted at the start of each $T$ symbol block with the $i^{th}$ training symbol being transmitted only from the $i^{th}$ antenna. The MMSE estimate of CSI ($\hat{h}$) is:

$$\hat{h} = \frac{\sqrt{P_t}}{M} \left( \frac{\sqrt{P_t}}{M} h + n \right),$$ (42)

where $P_t$ is the total power used for training, and $n$ is the additive white Gaussian noise vector corresponding to the $M$ training symbols. Let $P_d$ be the power used during data transmission period per symbol. $P_t$ and $P_d$ are related by the equation: $P_t + P_d(T - M) = P_T$. Let $\sigma_E^2$ denote the estimation error variance such that $\text{Cov}(\epsilon) = \sigma_E^2 I_{M \times M}$, where

$$\epsilon = h - \hat{h} = \frac{1}{\sqrt{P_t}} h - \frac{\sqrt{P_t}}{M} n.$$

Since $h$ and $n$ are independent, zero-mean, and have identity covariance matrix, we have

$$\text{Cov}(\epsilon) = \frac{1}{(\sqrt{P_t} + 1)^2} I_{M \times M} + \frac{P_t}{M} I_{M \times M} = \frac{P_t + 1}{(\sqrt{P_t} + 1)^2} I_{M \times M} = \frac{M}{P_t + M} I_{M \times M}$$ (43)

The CSIR is $\hat{h}$. The CSIT, which is a delayed version of the CSIR is $\hat{h}_{old}$, which is the MMSE estimate of $h_{old}$. Using (2) and (42), the correlation between the CSIT ($\hat{h}_{old}$) and CSIR ($\hat{h}$) can be obtained as

$$\text{Cov}(\epsilon) = \frac{1}{(\sqrt{P_t} + 1)^2} I_{M \times M} + \frac{P_t}{M} I_{M \times M} = \frac{P_t + 1}{(\sqrt{P_t} + 1)^2} I_{M \times M} = \frac{M}{P_t + M} I_{M \times M}$$ (43)

$$\Rightarrow \sigma_E^2 = \frac{M}{P_t + M}.$$

Here, we used the fact that estimation error ($n$ in (42)) is white and uncorrelated to the channel ($h$). Let $\rho_e$ denote the correlation coefficient between $\hat{h}$ and $\hat{h}_{old}$. Using $\text{Cov}(\epsilon) = (1 - \sigma_E^2) I_{M \times M} = \frac{P_t}{P_t + M} I_{M \times M}$, $\rho_e$ is given by

$$\rho_e = \frac{P_t}{P_t + M} \rho.$$ (45)

Observe that $\rho_e$ can at most be $\rho$ (for very large training power) and is less than $\rho$ for moderate values of training power. If the training power is very low compared to the noise (i.e., $P_t \rightarrow 0$),
then we have very noisy estimate of the channel and the correlation between estimates will be
zero as the noise in the channel is white.

Given the MMSE estimate of the CSI (\( \hat{h} \)) at the receiver, the mutual information of the BF-IC
scheme after accounting for the training period can be lower bounded using the result in [12]. A
similar mutual information lower bound can be obtained for the USPA scheme using the results
in [9], [8]. This lower bound on the mutual information is given by:

\[
I(x; y|\hat{h}, \hat{h}_{old}) \geq \frac{T - M}{T} \log \left( 1 + \frac{P_d}{1 + \sigma_E^2 P_d} \hat{h}^H Q \hat{h} \right). \tag{46}
\]

Observe that \( Cov(\hat{h}) = (1 - \sigma_E^2)I_{M \times M} \). Hence, scaling it by \( \sqrt{(1 - \sigma_E^2)} \) we obtain \( \hat{h}_{sc} = \frac{1}{\sqrt{(1 - \sigma_E^2)}} \hat{h} \) such that \( Cov(\hat{h}_{sc}) = I_{M \times M} \). Since this scaling does not change the correlation
coefficient, the correlation coefficient between \( \hat{h}_{sc} \) and \( \hat{h}_{old,sc} \) will remain \( \rho_c \). Now, the lower
bound on the mutual information (46) can be written as

\[
I(x; y|\hat{h}_{sc}, \hat{h}_{old}) \geq \frac{T - M}{T} \log \left( 1 + P' h_{sc}^H Q h_{sc} \right),
\]

where \( P' = P_d \frac{1 - \sigma_E^2}{1 + \sigma_E^2 P_d} \).

Substituting the value of \( \sigma_E^2 \) obtained for the training model, we get

\[
P_{0} = \frac{P_d P_t}{P_t + MP_d + M}. \tag{48}
\]

With the above observations, we analyze the loss in performance due to the channel estimation
error at the receiver for USPA and BF-IC.

A. Uniform Spatial Power Allocation with imperfect CSIR

Consider the case where the transmitter does not have CSI, and hence, allocates equal power in
all the M directions i.e. uniform spatial power allocation \( Q = \frac{I_{M \times M}}{M} \). Equation (47) becomes

\[
I(x; y|\hat{h}_{sc}, \hat{h}_{old}) \geq \frac{T - M}{T} \log \left( 1 + P' \frac{\hat{h}_{sc}^H \hat{h}_{sc}}{M} \right). \tag{48}
\]

Clearly, the lower bound is equivalent to system with perfect CSIR, but with different values of
average SNR \( (P') \) and rate \( (R') \). Therefore, the outage probability is upper bounded as follows:

\[
P_{out,USPA}(M, R, P') \leq \Pr \left( \log \left( 1 + P' \frac{\hat{h}_{sc}^H \hat{h}_{sc}}{M} \right) < R \frac{T}{T - M} \right) = \Gamma_M \left( \frac{e^{R'} - 1}{P' / M} \right), \tag{49}
\]

where \( P' = \frac{P_d P_t}{P_t + MP_d + M} \), \( R' = R \frac{T}{T - M} \). \tag{50}
The asymptotic diversity gain of USPA with imperfect CSIR remains $M$. Furthermore, the SNR gap between the perfect and imperfect CSIR cases can be significantly reduced by optimizing the training and data powers. Now, choosing the optimal value of $P_t$ or $P_d$ reduces to maximizing $P'$ with the constraint $P_t + P_d(T - M) = PT$. Expressing $P_t$ and hence $P'$ in terms of $P_d, P, T$ and $M$, we get

$$P' = \frac{P_d(PT - P_d(T - M))}{PT + M - P_d(T - 2M)}. \quad (51)$$

Differentiating w.r.t. $P_d$ and equating to zero results in a quadratic equation in $P_d$ [8].

$$(T - M)(T - 2M)(P_d^2) - 2(T - M)(PT + M)(P_d) + PT(PT + M) = 0. \quad (52)$$

### B. BF-IC with imperfect CSIR

As in the case of perfect CSIR, the transmitter does beamforming to the imperfect CSIT i.e., $\hat{h}_{old}$. Therefore, the transmit covariance matrix is $Q = \hat{h}_{old}^H \hat{h}_{old} = \hat{h}_{old,sc}^H \hat{h}_{old,sc}^H \hat{h}_{old,sc}$, where $\hat{h}_{old,sc}$ is a scaled version of $\hat{h}_{old}$ with identity covariance matrix. The correlation coefficient between $\hat{h}_{sc}$ and $\hat{h}_{old,sc}$ is equal to $\rho_e$, and hence, the lower bound on the system with imperfect CSIR is equivalent to the system with perfect CSIR with the parameters: average SNR ($P'$) and rate ($R'$) given by (50) and $\rho_e$. This immediately results in an upper bound on the outage probability. Following the simplifications as in Section III-A and the appendix, we get

$$P_{out,BF-IC}(M, R', P', \rho) \leq \frac{1}{(1 + \mu')^{M-1}} \sum_{i=0}^{M-1} \binom{M - 1}{i} (\mu')^i \Gamma(i+1) \left( \frac{e^{R'} - 1}{P'} \right), \quad (53)$$

where $\mu' = \frac{\rho^2}{1 - \rho^2}$, and $P'$, $R'$ are given by equation (50). If there is no feedback delay, the estimation error for the MMSE channel estimate reduces as the SNR increases, and $\rho_e \to 1$. Therefore, the asymptotic diversity gain would be $M$. However, in the presence of feedback delay, $\rho_e < 1$ and the asymptotic diversity gain of BF-IC with imperfect CSIR is 1.

### VI. RESULTS & OBSERVATIONS

In this section, the results are mainly presented for a $2 \times 1$ MISO channel. Similar results can be also be obtained for any MISO system (For example, a $4 \times 1$ system is chosen in Figure 7). The rate of transmission (R) is chosen to be 2 nats/s/Hz throughout this section. Fig. 1 shows the performance of USPA (5) and BF-IC (14) for different values of feedback delay captured by $\rho$. 

16
BF-IC is better at lower SNRs and worse at high SNRs when compared to USPA for any $\rho < 1$. This behavior is explained with asymptotic diversity gain at high SNR (16). Fig. 2 shows the diversity gain of USPA and BF-IC for different number of transmit antennas ($M$) for $\rho = 0.999$. USPA does not require feedback and has a diversity gain of $M$, where as at high SNR, the outage probability with BF-IC is more dominated by the error in CSIT than the channel being in deep fade. Thus, the diversity gain of BF-IC scheme is equal to 1, for any non zero delay in the feedback and any number of transmit antennas. Hence, USPA outperforms BF-IC at high SNR for all values of $\rho < 1$. Cross-over SNR is defined as the SNR after which USPA outperforms BF-IC. In Fig. 3, the cross-over SNR is plotted as a function of $\rho$ by equating (5) and (14). It can be seen that the cross-over SNR is a monotonically increasing function of $\rho$. Thus, the higher the value of $\rho$, i.e., the better the quality of the feedback, the wider the range of SNR over which BF-IC results in lower outage probability than USPA.

Fig. 4 shows $\lambda_{opt}(\gamma)$, the fraction of power spent in the direction of imperfect CSIT, as a function $\gamma$ for the OSPA scheme. Observe that $\lambda_{opt}(\gamma)$ is larger for higher values of $\rho$ implying that when the quality of feedback is higher, more power is spent in the direction of feedback. Fig. 5 compares the outage probability of BF-IC and USPA with OSPA for $\rho = 0.9$. We observe that (a) OSPA provides negligible gain in performance, (b) OSPA is computationally complex as it requires the transmitter to compute the optimal value of $\lambda$ for each value of feedback SNR and adapt the power in the spatial modes correspondingly, and (c) OSPA requires an estimate of $\rho$ to determine $\lambda_{opt}(\gamma)$ and any mismatch between the estimated value and the actual value will hurt the performance. On the other hand, USPA and BF-IC do not require any estimate of $\rho$ and are very simple. Therefore, we suggest switching between BF-IC and USPA by comparing the operating average SNR with the cross-over SNR. For a given average SNR, it is also possible to choose between USPA and BF-IC based on the instantaneous feedback SNR $\gamma$ (instead of switching based on the average SNR irrespective of $\gamma$). Equations (5) and (13) are the outage probabilities of USPA and BF-IC for a given $\gamma$. For each $\gamma$, this strategy chooses the best of USPA and BF-IC. Therefore, this would perform better than switching based on average SNR. However, we know that switching based on average SNR is already very close to the performance of OSPA. Therefore, the possible improvement due to switching based on instantaneous SNR
(instead of average SNR) is very small.

Fig. 5 also shows the performance of BF-IC and USPA with the corresponding optimal temporal power control. As in the case of the short-term power constraint, temporal power control with BF-IC is better for low SNR and temporal power control with USPA is better at high SNR. The trends in the results are similar in both cases and the cross-over SNR is slightly lower with temporal power control.

Fig. 6 compares the outage probability of BF-IC and USPA for perfect CSIR with BF-IC and USPA for imperfect CSIR for $\rho = 0.9$ and $M = 2$. $T$ is chosen to be 100. Outage probability for two cases: (a) Preamble power same as data power ($P_d = P_t$) and (b) optimized preamble power (52) is considered. Note that optimal power chosen for USPA is used as it is for BF-IC. The results suggest that the loss in performance due to the channel estimation error at the receiver is not significant for both USPA and BF-IC, if the power is chosen according to (52). Finally, Figure 7 shows a similar result (as in Figure 6) for a $4 \times 1$ MISO system.

VII. SUMMARY

The problem of minimum outage transmission for a MISO system with $M$ transmit antennas with delayed feedback under both short-term and long-term power constraints is considered. The delay in the feedback is captured by $\rho$, the correlation coefficient between delayed CSIT and perfect CSIR. For a short-term power constraint, we derive an analytic expression for the outage probability of beamforming using imperfect CSIT, where the power is spent in only one direction corresponding to the imperfect CSI available with the transmitter and compare it with that of USPA, where the power is distributed equally among the $M$ orthogonal and independent transmit directions. We also determine the optimal transmit strategy, i.e., OSPA, that minimizes the outage probability numerically. OSPA involves allocating a fraction of the power in the direction of the imperfect CSIT and the rest of the power is equally distributed among the $M - 1$ orthogonal and independent transmit directions. Results show that, for any $\rho < 1$, BF-IC is better at low SNR and worse at high SNR when compared to USPA. Furthermore, the asymptotic diversity gain for BF-IC is equal to 1 for any $\rho < 1$, independent of the number of transmit antennas. BF-IC is close to optimal at low SNR, while USPA is close to optimal at high SNR, i.e., OSPA does not improve the outage probability significantly compared to switching between BF-IC and USPA depending
on the average SNR. The cross-over SNR can be determined numerically by equating the outage probabilities of BF-IC and USPA schemes. For a long-term power constraint, where the transmit power is varied with time based on the feedback SNR, we formulate the problem to determine the optimal spatial and temporal power allocation scheme that minimizes the outage probability. Due to the difficulty in solving this problem, and motivated by the low penalty in the short-term power constraint case, we resort to the following suboptimal schemes: BF-IC with temporal power control and USPA with temporal power control. We determine the outage probabilities of both these schemes by first determining a condition that the power control function should satisfy using a result from the calculus of variations, and then solving for the power control function numerically. Again, we observe that BF-IC is better at low SNR, while USPA is better at high SNR. Finally, under the short term power constraint, we extend our analysis to delayed feedback with imperfect CSIR. The performance loss due to imperfect CSIR is shown to be minimal if the training power is chosen appropriately.

APPENDIX

DERIVATION OF EQUATION (14)

\[
F_{(nc-\chi^2,2M,\delta)}(y) = \sum_{k=0}^{\infty} \left( \frac{\delta}{2} \right)^k \frac{e^{-\frac{y}{2}}}{k!} F_{\chi^2,2M+2k}(y),
\]

(54)

where \(F_{\chi^2,2M+2k}(\cdot)\) is the cdf of a central \(\chi^2\) random variable with \(2M + 2k\) degrees of freedom. Using (54), (13) and substituting \(\delta = 2\mu\gamma\), \(P_{\text{out BF-IC}}(M, R, P, \rho)\) is simplified as

\[
P_{\text{out BF-IC}}(M, R, P, \rho) = \int_{0}^{\infty} f_{\Gamma}(\gamma) \sum_{k=0}^{\infty} \left( \frac{\mu\gamma}{k!} e^{-\mu\gamma} F_{\chi^2,2+2k}(2\beta) \right) d\gamma
\]

\[
= \sum_{k=0}^{\infty} \frac{\mu^k}{k!} F_{\chi^2,2+2k}(2\beta) \int_{0}^{\infty} f_{\Gamma}(\gamma) \gamma^k e^{-\mu\gamma} d\gamma
\]

\[
= \sum_{k=0}^{\infty} \frac{\mu^k}{k!} \int_{0}^{\beta} x^k e^{-x} \frac{(M + k - 1)!}{(M - 1)!(1 + \mu)^{(M+k)}} dx
\]

\[
= \frac{1}{(1 + \mu)^M} \int_{0}^{\beta} x^k g(x) dx,
\]

(55)

where

\[
g(x) = \sum_{k=0}^{\infty} \frac{\binom{M+k-1}{k}}{k!} \left( \frac{\mu x}{1 + \mu} \right)^k.
\]

(56)
LEMMA. For any $m, n > 0$,
\[
\binom{m+n}{m} = \binom{m+n}{n} = \sum_{i=0}^{\min(m,n)} \binom{m}{i} \binom{n}{i}.
\] (57)

Proof: Since the above lemma is symmetric in $m$ and $n$, we can assume that $m < n$ without any loss of generality. Observe that L.H.S is the number of ways to chose $m$ objects out of $m + n$, the same can be done by separating the $m + n$ objects in to 2 sets with sizes $m$ and $n$ and choosing $i$ objects from the first set and choosing $n - i$ objects from the second set and varying $i$ from 0 to $m$. So,
\[
\binom{m+n}{m} = \sum_{i=0}^{m} \binom{m}{i} \binom{n}{n-i} = \sum_{i=0}^{\min(m,n)} \binom{m}{i} \binom{n}{i}.
\] (58)

Using the above lemma, $g(x)$ can be simplified as
\[
g(x) = \sum_{k=0}^{\infty} \sum_{i=0}^{\min(k,M-1)} \binom{M-1}{i} \binom{k}{i} \frac{1}{i!} \left( \frac{x}{1+\mu} \right)^k
\]
\[
= \sum_{i=0}^{M-1} \binom{M-1}{i} \frac{x^i}{i!} (1+\mu)^{-1} \sum_{(k-i)=0}^{\infty} \frac{1}{(k-i)!} \left( \frac{x}{1+\mu} \right)^{k-i}
\]
\[
= e^{\left( \frac{x}{1+\mu} \right)} \sum_{i=0}^{M-1} \binom{M-1}{i} \left( \frac{x}{1+\mu} \right)^i.
\] (59)

After substituting (59) in (55), we get
\[
P_{\text{out BF-IC}}(M, R, P, \rho) = \frac{1}{(1+\mu)^M} \sum_{i=0}^{M-1} \binom{M-1}{i} \mu^i \int_0^\beta e^{-\left( \frac{x}{1+\mu} \right)} \left( \frac{x}{1+\mu} \right)^i dx
\]
\[
= \frac{1}{(1+\mu)^M} \sum_{i=0}^{M-1} \binom{M-1}{i} \mu^i \Gamma(i+1) \left( \frac{e^R - 1}{P} \right).
\] (60)

REFERENCES


Fig. 1. Outage probabilities for Beamforming using imperfect CSIT (BF-IC) for various values of $\rho$, and uniform spatial power allocation (USPA) for $M = 2$ and $R = 2$ nats/s/Hz. Cross-over SNR is the SNR at which USPA and BF-IC have the same outage probability.
Fig. 2. Outage probability with BF-IC for various values of $M$ for $\rho = 0.999$ and beamforming for $\rho = 1$ and $R = 2 \text{nats/s/Hz}$.
Fig. 3. Cross-over SNR $SNR_{cross}$ as a function of $\rho$ for $M = 2$ and $R = 2$ nats/s/Hz.
Fig. 4. Fraction of the power in the direction of imperfect CSIT $\lambda_{\text{opt}}(\gamma)$ for different values of $\rho$ and $P$; $M = 2$ and $R = 2$ nats/s/Hz.
Fig. 5. Outage Probabilities for uniform spatial power allocation (USPA) and beamforming using imperfect CSIT (BF-IC) with and without temporal power control, and optimal spatial power allocation (OSPA) for $\rho = 0.9$; $M = 2$ and $R = 2$ nats/s/Hz.
Fig. 6. Outage probabilities for Beamforming using imperfect CSIT (BF-IC) for $\rho = 0.9$, and uniform spatial power allocation (USPA) for $M = 2$ and $R = 2 \text{ nats/s/Hz}$. 
Fig. 7. Outage probabilities for Beamforming using imperfect CSIT (BF-IC) for $\rho = 0.9$, and uniform spatial power allocation (USPA) for $M = 4$ and $R = 2$ nats/s/Hz.