Optimum Transmission Strategies for the Gaussian Many-to-One Interference Network

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***Abstract*—We study the Gaussian many-to-one interference network which is a special case of general interference network, where only one receiver experiences interference. We allow transmission of messages on *all* the links of the network. This communication model is different from the corresponding many-to-one interference channel. We formulate three transmission strategies for the above network, which involve using Gaussian codebooks and treating interference from a subset of the transmitters as noise. We use sum-rate as the criterion of optimality for evaluating the strategies. For the first two strategies, we characterize the sum-rate capacity under certain channel conditions, while for the other strategy, we derive a sum-rate outer bound and characterize the gap between the outer bound and the achievable sum-rate of the strategy. Finally, we illustrate the regions where the derived channel conditions are satisfied for each strategy.**

keywords: *many-to-one interference network, interference channel, sum capacity.*

I. Introduction

The Interference Network is a multi-terminal communication network introduced by Carleial [1], consisting of M transmitters and N receivers, where each transmitter has an independent message for each of the $2^N - 1$ possible non-empty subsets of the receivers. The multiple access channel (MAC), broadcast channel, interference channel (IC), and X channel are all special cases of the Interference network (IN).

The interference channel has been studied extensively. Although the capacity region of the IC is unknown, several inner and outer bounds for the capacity region and sum-rate capacity have been derived in [2]–[4]. In [5]–[7], sum-rate capacity of the IC is characterized in the low-interference regime: a regime where using Gaussian inputs and treating interference as noise is optimal. This result is extended to the X channel in [8].

The many-to-one interference network is a special case of general interference network, where only one receiver experiences interference. The system model is shown in Fig. 1. We allow transmission of messages on all the links of the network. The communication model assumes that each transmitter *Tx* i, excluding the first has two independent messages, one for its corresponding receiver *Rx* i, and the other to receiver 1. Such a communication scheme has not been studied before.

The many-to-one interference channel is a special case of the many-to-one IN, where each transmitter (*Tx* i) is only interested in communicating with its corresponding receiver (*Rx* i), i.e., each transmitter has only one message. The many-to-one IC is studied in [7,9]–[11]. In [7,9], sum-rate capacity of the many-to-one IC is characterized in the low-interference regime. In [10], the capacity region is characterized to within a constant number of bits. The generalized degrees of freedom of the channel is obtained in [10,11].

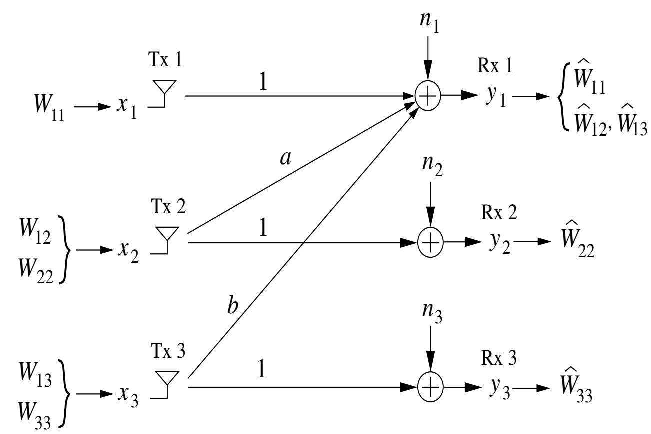

Fig. 1. Many-to-one interference network with 3 transmitters in standard form.

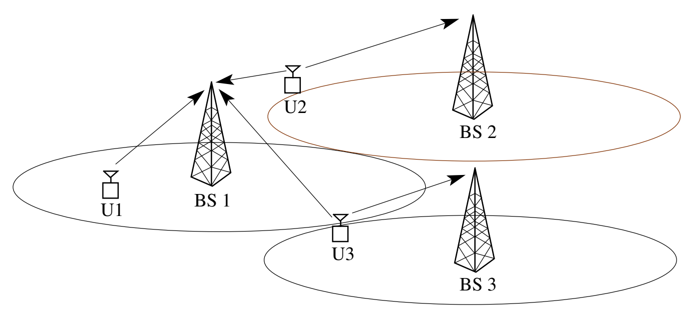

Fig. 2. Illustration of many-to-one interference network in cellular uplink.

We study the more general many-to-one interference network with messages on all the links. Interference networks with messages to all possible receivers could also be used in half-duplex relay networks. See [12] for examples of such networks used in optimization of unicast information flow in multistage decode-and-forward relay networks.

The many-to-one IN can also occur as a communication model both in cellular uplink and downlink as we show below.

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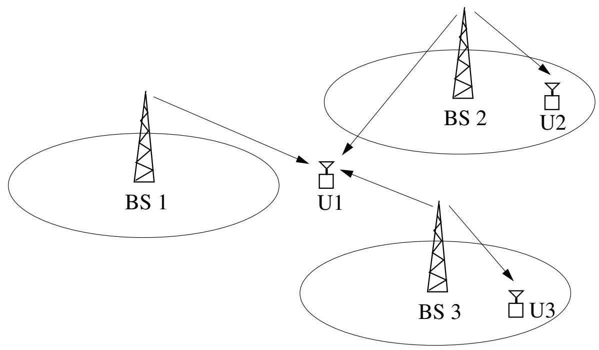

Fig. 3. Applicability of many-to-one interference network in cellular downlink.

No.	Strategy
$\mathcal{M}1$	All transmitters transmit to their corresponding receivers and interference at receiver 1 is treated as noise.
$\mathcal{M}2$	A subset of transmitters form a MAC at receiver 1, while interference from other transmitters is treated as noise.
$\mathcal{M}3$	All transmitters form a MAC at receiver 1.

TABLE I
TRANSMISSION STRATEGIES FOR MANY-TO-ONE IN

In cellular uplink, consider the illustration in Fig. 2, where user 1 is within the communication range of base station (BS) 1, whereas users 2 and 3 are at the cell edges of their respective BSs and have the option to either uplink to their respective BSs or to BS 1 if the channel conditions are conducive. In a reverse of the uplink model, in cellular downlink, user 1 is at the cell edge and receives transmission from the nearby BSs along with BS 1, while BS 2 and BS 3 communicate with their respective receivers. This communication system is illustrated in Fig. 3

Allowing messages on the cross links leads to some interesting scenarios. Each transmitter excluding the first, can now make a choice, either transmit to its own corresponding receiver, or transmit to receiver 1, or both. Instead of finding outer and inner bounds to the capacity region of the many-to-one IN, we focus on practical transmission scenarios. We define the transmission strategies for this network in Table I. All strategies involve using Gaussian codebooks and treating interference from a subset of transmitters as noise at receiver 1. Thus, in strategy $\mathcal{M}1$, all transmitters except the first cause interference at receiver 1, while in strategy $\mathcal{M}3$, receiver 1 does not experience any interference

The sum-rate at all the receivers is used as the criterion for optimality. For strategies $\mathcal{M}1$ and $\mathcal{M}2$, we characterize the sum-rate capacity under certain channel conditions, and for strategy $\mathcal{M}3$, we characterize the gap between the achievable sum-rate of the strategy and a sum-rate outer bound.

II. System Model

The many-to-one IN with 3 transmitters is characterized by the following input-output equations written in standard form [1], i.e.,

$$
\begin{aligned}
y_1 &= x_1 + ax_2 + bx_3 + n_1 && (1)\\
y_2 &= x_2 + n_2 && (2)\\
y_3 &= x_3 + n_3, && (3)
\end{aligned}
$$

where x_t is[1] the transmitted symbol by transmitter t, the cross channels from transmitters 2 and 3 to receiver 1 are a and b, respectively, and n_r is the additive complex Gaussian noise at the receivers. The additive noise n_r is a circularly symmetric complex Gaussian (CSCG) random variable with unit variance, i.e., $n_r \sim \mathcal{CN}(0,1)$, $r = 1,2,3$.

As shown in Fig. 1, the 3-transmitter many-to-one IN has five independent messages, W_{11}, W_{12}, W_{13}, W_{22} and W_{33}, where W_{ij} is the message transmitted from transmitter j to receiver i. Transmitter t is subject to a power constraint $\mathbb{E}[|x_t|^2] \leq P_t$.

III. Analysis of Different Strategies for 3-Transmitter Many-to-One Interference Network

We introduce some terminology useful in deriving the results in this section. Let $\mathbf{y}_i^n$ denote the vector of received symbols of length n at receiver i. Let $\mathbf{x}_i^n$ denote the n length vector of transmitted symbols at transmitter i. By Fano's inequality, we have

$$
\begin{aligned}
H(W_{ii}\,|\,\mathbf{y}_i^n) &\leq n\epsilon_n, \quad i = 1,2,3\\
H(W_{1j}\,|\,\mathbf{y}_1^n) &\leq n\epsilon_n, \quad j = 2,3, && (4)
\end{aligned}
$$

where $\epsilon_n \to 0$ as $n \to \infty$.

Before we proceed to analyze the various strategies , we provide a restatement of Lemma 5 in [7], in a form that is easier to apply to many-to-one interference networks. We make use of the following lemma to bound the sum-rate of many-to-one interference network in some cases.

Lemma 1. Let $\mathbf{w}_i^n$ be a complex sequence with average power constraint $\text{trace}(\mathbb{E}(\mathbf{w}_i^n \mathbf{w}_i^{nH})) \leq nP_i$. Let $\mathbf{n}_i^n$ be a complex random vector with components that are distributed as independent $\mathcal{CN}(0,1)$ random variables. Assume that $\mathbf{w}_i^n$ are independent of each other and also independent on $\mathbf{n}_i^n$. Let $w_{iG} \sim \mathcal{CN}(0, P_i)$. For some complex constants c_i, we have,

$$
\begin{aligned}
\sum_{i=1}^{K} h(\mathbf{w}_i^n + \mathbf{n}_i^n) - h(\sum_{i=1}^{K} c_i \mathbf{w}_i^n + \mathbf{n}_1^n) \quad &\leq \\
n\sum_{i=1}^{K} h(w_{iG} + n_i) - nh(\sum_{i=1}^{K} c_i w_{iG} + n_1), && (5)
\end{aligned}
$$

[1]We use the following notation: lowercase letters for scalars, boldface lowercase letters for vectors, and calligraphic letters for sets. $[\cdot]^T$ denotes the transpose operation, $[\cdot]^H$ denotes the Hermitian operation, $\text{trace}(\cdot)$ denotes the trace operation, and $\mathbb{E}\{\cdot\}$ denotes the expectation operation.

when $\sum_{i=1}^{K}|c_i|^2 \le 1$ and equality is achieved if $\mathbf{w}_i^n = \mathbf{w}_{iG}^n$, where $\mathbf{w}_{iG}^n$ denotes a complex random vector with components that are i.i.d $\mathcal{CN}(0, P_i)$.

Proof: Let $\mathbf{t}_i^n = c_i(\mathbf{w}_i^n + \mathbf{n}_i^n)$. The left-hand side of (5) can now be written as

$$\sum_{i=1}^{K} h(\mathbf{t}_i^n) - h(\sum_{i=1}^{K}\mathbf{t}_i^n + \tilde{\mathbf{n}}_1^n) + n\sum_{i=1}^{K}\log|c_i|^2,$$

where $\tilde{\mathbf{n}}_1^n$ is a complex random vector with components that are i.i.d $\mathcal{CN}(0, 1-\sum_{i=1}^{K}|c_i|^2)$. The final result follows by applying Lemma 5 in [7], i.e.,

$$\sum_{i=1}^{K} h(\mathbf{t}_i^n) - h(\sum_{i=1}^{K}\mathbf{t}_i^n + \tilde{\mathbf{n}}_1^n) \le n\sum_{i=1}^{K} h(t_{iG}) - nh(\sum_{i=1}^{K} t_{iG} + \tilde{n}_1),$$

where $t_{iG} = c_i(w_{iG} + n_i)$ and equality is achieved if $\mathbf{w}_i^n = \mathbf{w}_{iG}^n$. Since the variance of $\tilde{n}_1$ cannot be negative, we have the condition $\sum_{i=1}^{K}|c_i|^2 \le 1$. □

A. Optimality of Strategy $\mathcal{M}1$

Here, we are interested in a region where the transmitters use Gaussian inputs to communicate with their respective receivers and interference at receiver 1 is treated as noise. This is usually referred to in the interference channel literature as the *low-interference* or the *noisy-interference* regime.

Theorem 1. The sum-rate capacity is achieved by transmitting on the direct channels and treating interference as noise when $|a|^2 + |b|^2 \le 1$.

Proof: If $|b|^2 \le 1$, we have $I(W_{13}\,;\mathbf{y}_3^n) \ge I(W_{13}\,;\mathbf{y}_1^n\,|\,\mathbf{x}_1^n, \mathbf{x}_2^n)$. Therefore,

$$\begin{aligned} H(W_{13}\,|\,\mathbf{y}_3^n) &\le H(W_{13}\,|\,\mathbf{y}_1^n, \mathbf{x}_1^n, \mathbf{x}_2^n) \\ &\overset{(a)}{\le} H(W_{13}\,|\,\mathbf{y}_1^n) \le n\epsilon_n, \end{aligned} \tag{6}$$

where (a) follows since removing conditioning does not reduce the conditional entropy. Thus, we conclude that W_{13} is decodable at receiver 3 when $|b|^2 \le 1$. Note that in this case

$$\begin{aligned} h(\mathbf{x}_3^n\,|\,\mathbf{y}_3^n) &= H(W_{33}\,|\,\mathbf{y}_3^n) + H(W_{13}\,|\,\mathbf{y}_3^n, W_{33}) \\ &\le H(W_{33}\,|\,\mathbf{y}_3^n) + H(W_{13}\,|\,\mathbf{y}_3^n) \\ &\le 2n\epsilon_n, \end{aligned} \tag{7}$$

where (7) follows from (4) and (6).

Similarly, it can be shown that when $|a|^2 \le 1$, W_{12} is decodable at receiver 2 and $h(\mathbf{x}_2^n\,|\,\mathbf{y}_2^n) \le 2n\epsilon_n$. This means that we can set $W_{12} = W_{13} = \phi$ (without loss of sum-rate). Thus, we have shown that the presence of cross messages does not improve the sum-rate when $|a|^2 \le 1$, $|b|^2 \le 1$. Now, assume that $|a|^2 \le 1$ and $|b|^2 \le 1$. The sum-rate can be bounded as follows

$$\begin{aligned} nS &\le H(W_{11}) + H(W_{12}, W_{22}) + H(W_{13}, W_{33}) \\ &= I(\mathbf{x}_1^n;\mathbf{y}_1^n) + I(\mathbf{x}_2^n;\mathbf{y}_2^n) + I(\mathbf{x}_3^n;\mathbf{y}_3^n) + \sum_{i=1}^{3} h(\mathbf{x}_i^n|\mathbf{y}_i^n) \\ &\le h(\mathbf{y}_1^n) - h(a\mathbf{x}_2^n + b\mathbf{x}_3^n + \mathbf{n}_1^n) + h(\mathbf{x}_2^n + \mathbf{n}_2^n) - h(\mathbf{n}_2^n) \\ &\quad + h(\mathbf{x}_3^n + \mathbf{n}_3^n) - h(\mathbf{n}_3^n) + 3\epsilon_n \\ &\overset{(c)}{\le} nh(y_{1G}) + nh(x_{2G} + n_2) + nh(x_{3G} + n_3) \\ &\quad - nh(a x_{2G} + b x_{3G} + n_1) - nh(n_2) - nh(n_3) + 3\epsilon_n \\ &= nI(x_{1G}; y_{1G}) + nI(x_{2G}; y_{2G}) + nI(x_{3G}; y_{3G}) + 3\epsilon_n, \end{aligned}$$

where $x_{iG} \sim \mathcal{CN}(0, P_i)$, y_{iG} denotes y_i with $x_j = x_{jG}$, $\forall\, i, j$, and in (c), we have used Lemma 1 to bound the term $h(\mathbf{x}_2^n + \mathbf{n}_2^n) + h(\mathbf{x}_3^n + \mathbf{n}_3^n) - h(a\mathbf{x}_2^n + b\mathbf{x}_3^n + \mathbf{n}_1^n)$, under the condition $|a|^2 + |b|^2 \le 1$. As $n \to \infty$, $\epsilon_n \to 0$, we have

$$S \le \log\left(1 + \frac{P_1}{1 + |a|^2P_2 + |b|^2P_3}\right) + \sum_{i=2}^{3}\log(1 + P_i).$$

□

Remark 1. Theorem 1 was proved for the many-to-one interference channel in [7, Theorem 4] using genie aided bounding techniques.

B. Optimality of Strategy $\mathcal{M}2$

We ask the following question: Are there channel conditions such that the sum-rate capacity is achieved by a two-user MAC at receiver 1 formed by transmitter 1 and either transmitter 2 or transmitter 3, while the interference from the other transmitter is treated as noise? Observe that the other transmitter forms a point-to-point channel and is a source of interference for the two-user MAC. We characterize the sum-rate capacity in the following theorem.

Theorem 2. The sum-rate capacity is achieved by the two-user MAC formed by transmitter 1 and either transmitter 2 or transmitter 3 to receiver 1, for the following two sub-regions of the many-to-one interference region, respectively.

(i) $|a| \ge \dfrac{1 + |b|^2P_3}{\sqrt{1 - |b|^2}}, \quad |b| \le 1$

(ii) $|b| \ge \dfrac{1 + |a|^2P_2}{\sqrt{1 - |a|^2}}, \quad |a| \le 1.$

Proof: We prove statement (i) below. This represents the case where transmitters 1 and 2 form a MAC at receiver 1 while interference from receiver 3 is treated as noise. The proof for the second statement follows along similar lines.

We use genie-aided bounding techniques to derive the optimality of strategy $\mathcal{M}2$. Specifically, we use the concept of *useful genie* and *smart genie* introduced in [7] to obtain the sum-rate capacity for strategy $\mathcal{M}2$. Let a genie provide the following side information to receiver 1:

$$s_1 = x_1 + a\,x_2 + \eta\,z_1, \tag{8}$$

where $z_1 \sim \mathcal{CN}(0, 1)$ and η is a positive real number. We allow z_1 to be correlated to n_1 with correlation coefficient ρ.

A genie is said to be useful if it results in a genie-aided channel whose sum-rate capacity is achieved by Gaussian inputs, i.e., the sum-rate capacity of the genie-aided channel equals $I(x_{1G},\, x_{2G}\,;\, y_{1G}, s_{1G}) + I(x_{3G}\,;\, y_{3G})$, where $x_{iG} \sim \mathcal{CN}(0, P_i)$, y_{iG}, s_{1G} are y_i and s_1 with $x_j = x_{jG}, \forall\, i, j$.

Lemma 2. (Useful Genie) The sum-rate capacity of the genie-aided channel with side information (8) given to receiver 1 is achieved by using Gaussian inputs and by treating interference as noise at receiver 1, if the following conditions hold:

$$\eta^2 \;\le\; |a|^2, \qquad |b|^2 \;\le\; 1 - |\rho|^2, \tag{9}$$

and the sum-rate of the genie-aided channel is bounded as

$$S \;\;\le\;\; I(x_{1G},\, x_{2G}\,;\, y_{1G}, s_{1G}) + I(x_{3G}\,;\, y_{3G}). \tag{10}$$

Proof: The sum-rate of the genie-aided channel can be bounded as

$$\begin{aligned} nS \;&\le\; H(W_{11}, W_{12}, W_{22}) + H(W_{13}, W_{33}) \\ &=\; I(W_{11}, W_{12}, W_{22}\,;\, \mathbf{y}_1^n,\, \mathbf{s}_1^n) + H(W_{11}\,|\,\mathbf{y}_1^n,\, \mathbf{s}_1^n) \\ &\quad + H(W_{12}\,|\,\mathbf{y}_1^n,\, \mathbf{s}_1^n, \mathbf{x}_1^n) + H(W_{22}\,|\,\mathbf{y}_1^n,\, \mathbf{s}_1^n, \mathbf{x}_1^n, W_{12}) \\ &\quad + I(W_{13}, W_{33}; \mathbf{y}_3^n) + H(W_{13}|\mathbf{y}_3^n) + H(W_{33}|\mathbf{y}_3^n, W_{13}) \\ &\overset{(a)}{\le}\; I(\mathbf{x}_1^n,\, \mathbf{x}_2^n\,;\, \mathbf{y}_1^n,\, \mathbf{s}_1^n) + h(\mathbf{x}_1^n\,|\,\mathbf{y}_1^n) + H(W_{12}\,|\,\mathbf{y}_1^n) \\ &\quad + H(W_{22}\,|\,\mathbf{s}_1^n,\, \mathbf{x}_1^n) + I(\mathbf{x}_3^n\,;\, \mathbf{y}_3^n) + H(W_{13}\,|\,\mathbf{y}_3^n) \\ &\quad + H(W_{33}\,|\,\mathbf{y}_3^n), \end{aligned} \tag{11}$$

where (a) follows from the fact that removing conditioning cannot reduce the conditional differential entropy.

We bound the term $H(W_{22}\,|\,\mathbf{s}_1^n,\, \mathbf{x}_1^n)$. If $\eta^2 \le |a|^2$, then we have $I(W_{22}\,;\, \mathbf{s}_1^n\,|\,\mathbf{x}_1^n) \ge I(W_{22}\,;\, \mathbf{y}_2^n)$. Thus,

$$\begin{aligned} H(W_{22}\,|\,\mathbf{s}_1^n,\, \mathbf{x}_1^n) \;&\le\; H(W_{22}\,|\,\mathbf{y}_2^n) \\ &\le\; n\epsilon_n. \end{aligned} \tag{12}$$

Note that the term $H(W_{13}\,|\,\mathbf{y}_3^n)$ is bounded as (6) when $|b|^2 \le 1$. Using (4), (6), and (12) in (11), we have

$$\begin{aligned} nS \;&\le\; I(\mathbf{x}_1^n, \mathbf{x}_2^n\,;\, \mathbf{y}_1^n,\, \mathbf{s}_1^n) + I(\mathbf{x}_3^n\,;\, \mathbf{y}_3^n) + 5n\epsilon_n \\ &=\; I(\mathbf{x}_1^n, \mathbf{x}_2^n; \mathbf{s}_1^n) + I(\mathbf{x}_1^n, \mathbf{x}_2^n; \mathbf{y}_1^n\,|\,\mathbf{s}_1^n) + I(\mathbf{x}_3^n; \mathbf{y}_3^n) + 5n\epsilon_n \\ &=\; h(\mathbf{s}_1^n) - h(\mathbf{s}_1^n\,|\,\mathbf{x}_1^n, \mathbf{x}_2^n) + h(\mathbf{y}_1^n\,|\,\mathbf{s}_1^n) \\ &\quad - h(\mathbf{y}_1^n\,|\,\mathbf{s}_1^n, \mathbf{x}_1^n, \mathbf{x}_2^n) + h(\mathbf{y}_3^n) - h(\mathbf{y}_3^n\,|\,\mathbf{x}_3^n) + 5n\epsilon_n \\ &=\; h(\mathbf{s}_1^n) - h(\eta\,\mathbf{z}_1^n) + h(\mathbf{y}_1^n\,|\,\mathbf{s}_1^n) - h(b\,\mathbf{x}_3^n + \mathbf{n}_1^n\,|\,\mathbf{z}_1^n) \\ &\quad + h(\mathbf{y}_3^n) - h(\mathbf{n}_3^n) + 5n\epsilon_n \\ &\overset{(b)}{\le}\; nh(s_{1G}) - nh(\eta\, z_1) + nh(y_{1G})\,|\,s_{1G}) \\ &\quad - h(b\,\mathbf{x}_3^n + \tilde{\mathbf{n}}_1^n) + h(\mathbf{x}_3^n + \mathbf{n}_3^n) - nh(n_3) + 5n\epsilon_n \\ &\overset{(c)}{\le}\; nh(s_{1G}) - nh(\eta\, z_1) + nh(y_{1G})\,|\,s_{1G}) \\ &\quad + nh(x_{3G} + n_3) - nh(b\,x_{3G} + \tilde{n}_1) - nh(n_3) + 5n\epsilon_n \\ &=\; n\,I(x_{1G}, x_{2G}\,;\, y_{1G},\, s_{1G}) + n\,I(x_{3G}\,;\, y_{3G}) + 5n\epsilon_n, \end{aligned}$$

where $\tilde{n_1} \sim \mathcal{CN}(0, 1-|\rho|^2)$, (b) follows since Gaussian inputs maximize differential entropy for a given covariance constraint and from the application of Lemma 1 and Lemma 6 in [7], (c) follows from applying Lemma 1 in [6] (which is a special case of the extremal inequality considered in [13]) to the term $h(\mathbf{x}_3^n + \mathbf{n}_3^n) - h(b\,\mathbf{x}_3^n + \tilde{\mathbf{n}}_1^n)$, and using the condition $|b|^2 \le 1 - |\rho|^2$. □

Next, we show that the genie is smart. A smart genie is one which does not improve the sum-rate when Gaussian inputs are used, i.e., $I(x_{1G},\, x_{2G}\,;\, y_{1G}, s_{1G}) = I(x_{1G},\, x_{2G}\,;\, y_{1G})$.

Lemma 3. (Smart Genie) If Gaussian inputs are used, and interference is treated as noise, then under the following condition,

$$\eta\rho = 1 \;+\; |b|^2 P_3, \tag{13}$$

the genie does not increase the sum rate, i.e.,

$$I(x_{1G},\, x_{2G}\,;\, y_{1G},\, s_{1G}) = I(x_{1G},\, x_{2G}\,;\, y_{1G}). \tag{14}$$

Proof: Note that

$$\begin{aligned} &I(x_{1G},\, x_{2G}\,;\, y_{1G}, s_{1G}) \\ &\qquad=\;\; I(x_{1G},\, x_{2G}\,;\, y_{1G}) + I(x_{1G},\, x_{2G}\,;\, s_{1G}\,|\,y_{1G}). \end{aligned}$$

The second term on the right hand side can be expanded as

$$I(x_{1G}\,;\, s_{1G}\,|\,y_{1G}) + I(x_{2G}\,;\, s_{1G}\,|\,y_{1G},\, x_{1G}).$$

Consider

$$\begin{aligned} &I(x_{1G}; s_{1G}\,|\,y_{1G}) \\ &\;= I(x_{1G}\,;\, x_{1G} + a\,x_{2G} + \eta z_1\,|\,x_{1G} + a\,x_{2G} + b\,x_{3G} + n_1). \end{aligned}$$

From Lemma 8 in [7], if x, n, z are Gaussian with x being independent of the two zero-mean random variables n, z, then $I(x\,;\, x+z\,|\,x+n) = 0$, iff $\mathbb{E}(z\,\bar{n}) = \mathbb{E}(|n|^2)$, where $\bar{n}$ denotes the complex conjugate of n. Thus, $I(x_{1G}; s_{1G}\,|\,y_{1G})$ becomes zero if $|a|^2 P_2 + \eta\,\rho = 1 + |a|^2 P_2 + |b|^2 P_3$ which reduces to (13). Now, consider

$$\begin{aligned} &I(x_{2G}\,;\, s_{1G}\,|\,y_{1G},\, x_{1G}) \\ &\qquad\qquad= I(x_{2G}\,;\, a\,x_{2G} + \eta\, z_1\,|\,a\,x_{2G} + b\,x_{3G} + n_1) \\ &\qquad\qquad\overset{(d)}{=}\; 0. \end{aligned}$$

where (d) follows from [7, Lemma 8] and (13). □

Combining conditions (9) and (13), we have

$$|a| \;\;\ge\;\; \frac{1 + |b|^2 P_3}{|\rho|} \quad ; \quad |b| \;\le\; \sqrt{1 - |\rho|^2} \tag{15}$$

As $|\rho|$ varies from 0 ro 1, $|b|$ varies from 1 to 0. For a fixed value of b, we have the constraint $|\rho| \le \sqrt{1 - |b|^2}$. Note that choosing $\rho = \sqrt{1 - |b|^2}$ results in the best bound for a. Thus, (15) can be rewritten as statement (i) in the Theorem. □

C. Optimality of Strategy $\mathcal{M}3$

In strategy $\mathcal{M}3$, all transmitters form a MAC at receiver 1. We derive a sum-rate outer bound to the many-to-one IN and characterize the gap between the outer bound and the achievable sum-rate of strategy $\mathcal{M}3$.

Theorem 3. When all transmitters transmit to receiver 1, if

$$|a| \;\ge\; \frac{1 + |b|^2 P_3}{|\rho|} \quad \text{and} \quad |b| \;\ge\; 1, \tag{16}$$

then the gap between the sum-rate outer bound and the sum-rate of strategy $\mathcal{M}3$ is given by

$$\log\left(\frac{1-(1+|b|^2P_3)^{-1}|\rho|^2}{1-|\rho|^2}\right), \quad (17)$$

where ρ denotes a constant with $|\rho| \in [0,1]$.

Proof: We use genie-aided techniques to derive the sum-rate outer bound. Let a genie provide the side information given in (8) to receiver 1. We prove below that the genie is useful.

Lemma 4. (Useful Genie) The sum-rate capacity of the genie-aided channel with side information (8) given to receiver 1 is achieved by using Gaussian inputs when all transmitters transmit to receiver 1, if the following condition holds:

$$\eta^2 \;\le\; |a|^2, \qquad |b|^2 \;\ge\; 1, \quad (18)$$

and the sum-rate of the genie-aided channel is bounded as

$$S \;\le\; I(x_{1G},\, x_{2G},\, x_{3G}\,;\, y_{1G}, s_{1G}). \quad (19)$$

Proof: The sum-rate S of the genie-aided channel is bounded as

$$\begin{aligned} nS &\le H(W_{11}, W_{12}, W_{13}, W_{22}, W_{33}) \\ &= I(\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{x}_3^n; \mathbf{y}_1^n, \mathbf{s}_1^n) + H(W_{11}\,|\,\mathbf{y}_1^n, \mathbf{s}_1^n) \\ &\quad + H(W_{12}\,|\,\mathbf{y}_1^n, \mathbf{s}_1^n, \mathbf{x}_1^n) + H(W_{22}|\mathbf{y}_1^n, \mathbf{s}_1^n, \mathbf{x}_1^n, W_{12}) \\ &\quad + H(W_{13}|\mathbf{y}_1^n, \mathbf{s}_1^n, \mathbf{x}_1^n, \mathbf{x}_2^n) \\ &\quad + H(W_{33}|\mathbf{y}_1^n, \mathbf{s}_1^n, \mathbf{x}_1^n, \mathbf{x}_2^n, W_{13}) \\ &\le I(\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{x}_3^n; \mathbf{y}_1^n, \mathbf{s}_1^n) + H(W_{11}|\mathbf{y}_1^n) + H(W_{12}|\mathbf{y}_1^n) \\ &\quad + H(W_{22}|\mathbf{s}_1^n, \mathbf{x}_1^n) + H(W_{13}|\mathbf{y}_1^n) \\ &\quad + H(W_{33}|\mathbf{y}_1^n, \mathbf{x}_1^n, \mathbf{x}_2^n). \end{aligned} \quad (20)$$

We bound the term $H(W_{33}\,|\,\mathbf{y}_1^n, \mathbf{x}_1^n, \mathbf{x}_2^n)$. If $|b|^2 \ge 1$, then $I(W_{33}; \mathbf{y}_1^n\,|\,\mathbf{x}_1^n, \mathbf{x}_2^n) \ge I(W_{33}\,;\,\mathbf{y}_3^n)$. Therefore,

$$\begin{aligned} H(W_{33}\,|\,\mathbf{y}_1^n,\, \mathbf{x}_1^n, \mathbf{x}_2^n) &\le H(W_{33}\,|\,\mathbf{y}_3^n) \\ &\le n\epsilon_n. \end{aligned} \quad (21)$$

Note that the term $H(W_{22}|\mathbf{s}_1^n, \mathbf{x}_1^n)$ is again bounded as in (12) if $\eta^2 \le |a|^2$.

Using (4), (12) and (21) in (20), we have

$$\begin{aligned} nS &\le I(\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{x}_3^n; \mathbf{y}_1^n, \mathbf{s}_1^n) + 5n\epsilon_n \\ &= I(\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{x}_3^n; \mathbf{y}_1^n) + I(\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{x}_3^n; \mathbf{s}_1^n\,|\,\mathbf{y}_1^n) + 5n\epsilon_n \\ &\overset{(a)}{\le} nI(x_{1G}, x_{2G}, x_{3G}; y_{1G}) + h(\mathbf{s}_1^n\,|\,\mathbf{y}_1^n) \\ &\quad - h(\mathbf{s}_1^n\,|\,\mathbf{y}_1^n, \mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{x}_3^n) + 5\epsilon_n \\ &\overset{(b)}{\le} nI(x_{1G}, x_{2G}, x_{3G}; y_{1G}) + nh(s_{1G}\,|\,y_{1G}) \\ &\quad - nh(\eta z_1\,|\,n_1) + 5\epsilon_n \\ &= nI(x_{1G},\, x_{2G},\, x_{3G}\,;\, y_{1G}, s_{1G}) + 5\epsilon_n, \end{aligned}$$

where (a) follows from the optimality of Gaussian inputs for Gaussian MAC, (b) follows from Lemma 1 in [7]. Here, y_{1G} denotes y_1 with x_i being Gaussian distributed, i.e., $y_{1G} = x_{1G} + ax_{2G} + bx_{3G} + n_1$. As $n \to \infty$, $\epsilon_n \to 0$ and we get the desired bound. □

Strat.	Channel conditions	Gap from Outer-bound
$\mathcal{M}1$	$\|a\|^2 + \|b\|^2 \le 1$	0
$\mathcal{M}2$	(i) $\|a\| \ge \frac{1+\|b\|^2P_3}{\sqrt{1-\|b\|^2}},\ \|b\| \le 1$	0
	(ii) $\|b\| \ge \frac{1+\|a\|^2P_2}{\sqrt{1-\|a\|^2}},\ \|a\| \le 1$	0
$\mathcal{M}3$	(i) $\|a\| \ge \frac{1+\|b\|^2P_3}{\|\rho\|},\ \|b\| \ge 1$	$\log\left[\frac{1-\frac{\|\rho\|^2}{1+\|b\|^2P_3}}{1-\|\rho\|^2}\right]$
	(ii) $\|b\| \ge \frac{1+\|a\|^2P_2}{\|\rho\|},\ \|a\| \ge 1$	$\log\left[\frac{1-\frac{\|\rho\|^2}{1+\|a\|^2P_2}}{1-\|\rho\|^2}\right]$

TABLE II
SUMMARY OF RESULTS FOR MANY-TO-ONE INTERFERENCE NETWORK

Unlike in the case of strategy $\mathcal{M}2$, here the genie does in fact increase the sum-rate and hence is not smart. However, we can choose the parameters ρ and η to get a good sum-rate outer bound as follows. Consider

$$\begin{aligned} &I(x_{1G},\, x_{2G},\, x_{3G}\,;\, y_{1G}, s_{1G}) \\ &\quad = I(x_{1G}, x_{2G}, x_{3G}; y_{1G}) + I(x_{1G}, x_{2G}, x_{3G}; s_{1G}\,|\,y_{1G}). \end{aligned}$$

The second term on the right hand side can be expanded as

$$I(x_{1G},\, x_{2G}; s_{1G}\,|\,y_{1G}) + I(x_{3G}\,;\, s_{1G}\,|\,y_{1G}, x_{1G},\, x_{2G}). \quad (22)$$

In the proof of Lemma 3, we showed that by choosing $\eta\,\rho = 1 + |b|^2P_3$, we can make $I(x_{1G},\, x_{2G}; s_{1G}\,|\,y_{1G}) = 0$. Now, consider

$$\begin{aligned} I(x_{3G}; s_{1G}\,|\,y_{1G},\, x_{1G}, x_{2G}) &= I(x_{3G}; \eta\, z_1\,|\,b\, x_{3G} + n_1) \\ &= h(\eta\, z_1\,|\,b\, x_{3G} + n_1) - h(\eta\, z_1\,|\,n_1) \\ &\overset{(c)}{=} h(\eta\, z_1\,|\,b\, x_{3G} + n_1) - h(\eta\, \tilde{z}_1) \\ &= \log\left(\frac{\eta^2(1+|b|^2P_3) - \eta^2|\rho|^2}{(1+|b|^2P_3)\eta^2(1-|\rho|^2)}\right) \\ &= \log\left(\frac{1-(1+|b|^2P_3)^{-1}|\rho|^2}{1-|\rho|^2}\right), \end{aligned} \quad (23)$$

where $\tilde{z}_1 \sim \mathcal{CN}(0, 1-|\rho|^2)$ and (c) follows from [7, Lemma 6]. Note that (23) represents the gap between the sum-rate outer bound and the sum-rate of strategy $\mathcal{M}3$. Combining condition (18) with $\eta\,\rho = 1 + |b|^2P_3$, we get (16). □

Due to the underlying symmetry in the MAC at receiver 1, a result corresponding to Theorem 3 with the channel coefficients a, b and power levels P_2, P_3 interchanged is also true and further can be proved along similar lines. The results of this section are succinctly summarized in Table II.

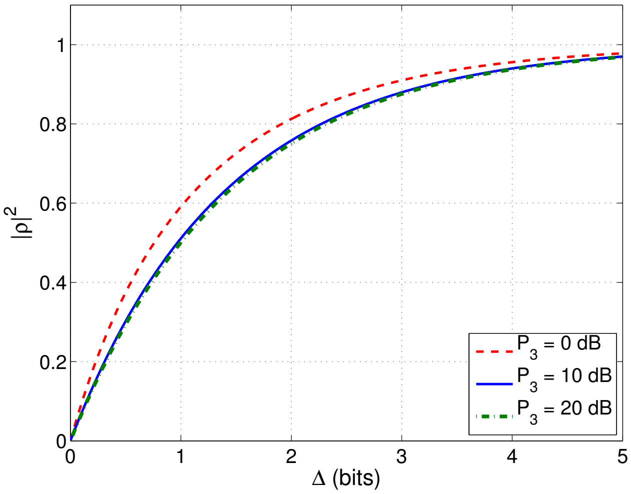

Fig. 4. Variation of $|\rho|^2$ as a function of the gap Δ in bits. $|b| = 1.5$.

IV. Numerical results

In this section, we illustrate the regions where the derived channel conditions are satisfied for each strategy. First, we numerically analyze the sum-rate outer bound for the optimality of strategy $\mathcal{M}3$, given in Theorem 3.

Let the gap between the sum-rate outer bound and the achievable sum-rate of strategy $\mathcal{M}3$ given in (23) be denoted by Δ. Using (23) and solving for ρ in terms of Δ, we get

$$|\rho|^2 \quad \leq \quad \frac{2^{\Delta} - 1}{2^{\Delta} - 1/(1 + |b|^2 P_3)} \tag{24}$$

In Fig. 4, we plot $|\rho|^2$ as a function of Δ for different values of P_3 for fixed value of $|b| = 1.5$. It can be observed that $|\rho|^2$ is a monotonically increasing function of Δ. Thus, to obtain a lower gap from the outer bound, a lower value of $|\rho|^2$ must be chosen. This in turn makes the sub region in (16) smaller. This relationship is explored further is the next figure.

In Fig. 5, we plot the sub region in (16) for the sum-rate optimality of strategy $\mathcal{M}3$ as a graph in the $|a| - |b|$ plane for various values of Δ, along with the sub-regions in Table II for strategies $\mathcal{M}1$ and $\mathcal{M}2$. We assume $P_1 = P_2 = P_3 = 0$ dB. As mentioned above, the sub region in (16) shrinks for increasing values of Δ.

V. Conclusions

We considered the Gaussian many-to-one interference network with 3 transmitters with messages on all the links. We formulated different transmission strategies and obtained sufficient channel conditions under which the strategies were either optimal or within a gap from an outer bound. In the process, sum-rate capacity was characterized in some sub-regions of the many-to-one interference network.

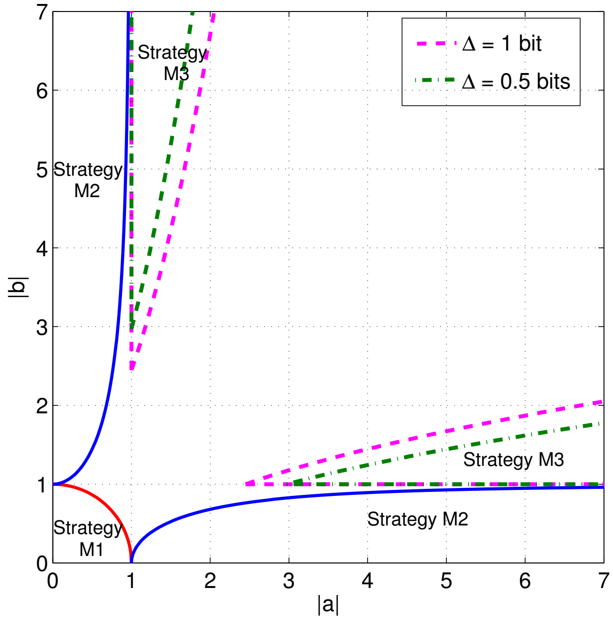

Fig. 5. A plot of the channel conditions in Table II for the three strategies. $P_1 = P_2 = P_3 = 0$ dB.

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