Almost Budget Balanced Mechanisms with Scalar Bids for Allocation of a Divisible Good[1]

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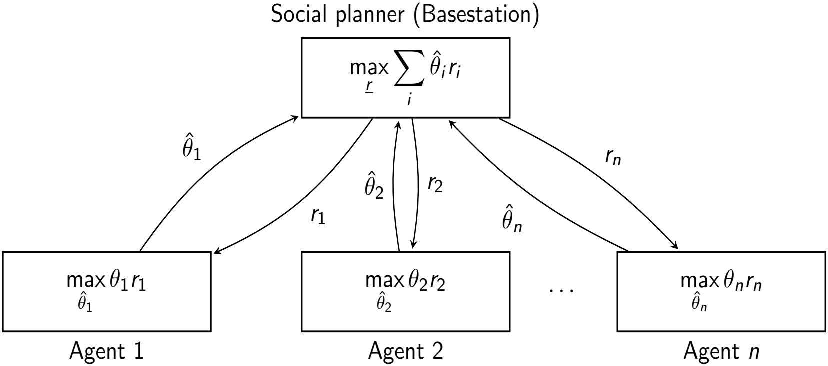
Fifth Workshop on Frontiers of Networks: Theory and Algorithms

ACM Mobihoc 2017

[1] Partly supported by DST

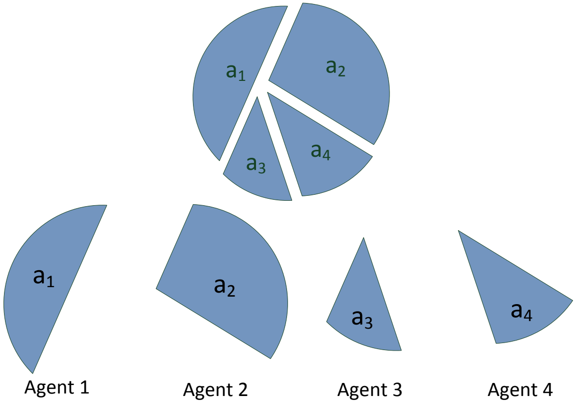
Motivation: Uplink Scheduling Problem

- Agent i's valuation = queue length (θ_i) x instantaneous rate (r_i)
- Valuation function is known to everyone except for θ_i



Agents can report wrong queue lengths

A Divisible Resource

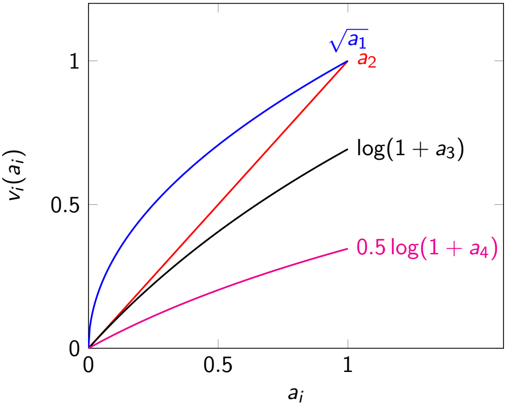


Example

Randomized allocation of a link with capacity C,

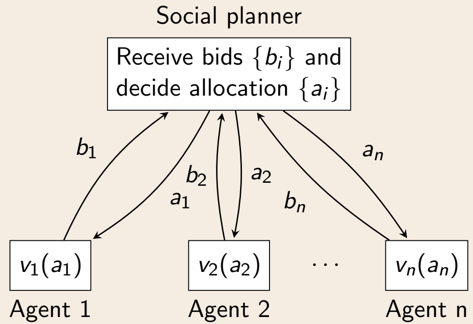
$a_i = Pr[allocation\ to\ agent\ i] * C$

Valuation functions (or Utility functions)



- Valuation functions are agents' private information
- Need to signal the valuation functions to the social planner
- Communication constraints: Restriction to scalar bids
- Strategic agents

Setting and Efficiency

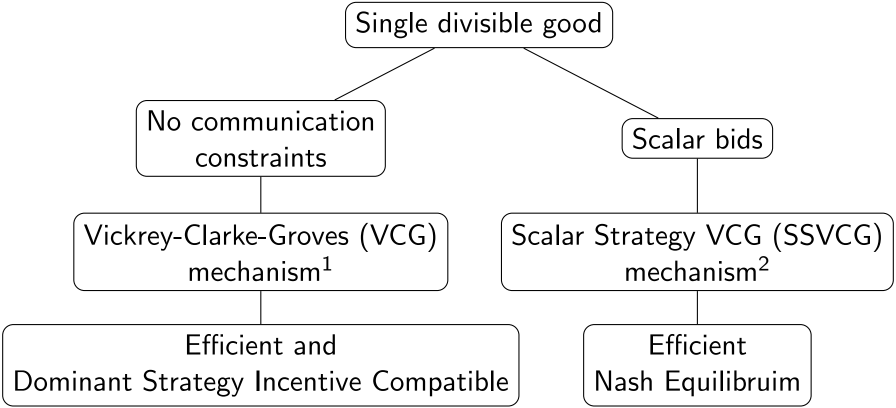


- Agents know the allocation mechanism and are strategic
- Efficiency: Allocate resource such that sum valuation is maximized

$$\max_{\{a_i\}} \sum_{i=1}^{n} v_i(a_i)$$

- Question: What should be the allocation and pricing mechanism?

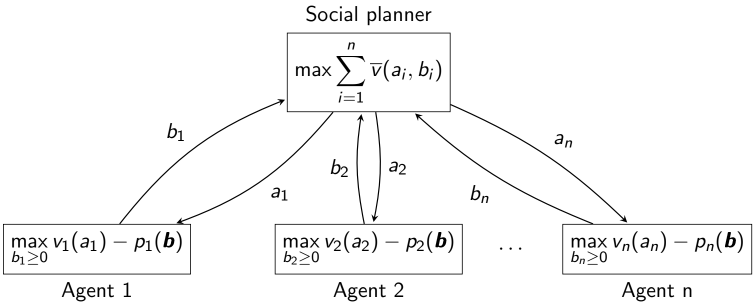
Known Results



[1] Vickrey 1961, Clarke 1971, Groves 1973
[2] Yang & Hajek 2007, Johari & Tsitsiklis 2009

SSVCG mechanism

Scalar-parametrized *surrogate valuation function set*: $\{\overline{v}(\cdot,\theta),\theta \geq 0\}$



Payment imposed on agent i

$$p_i(\boldsymbol{b}) = -\sum_{j \neq i} \overline{v}(a_j^*, b_j) + \sum_{j \neq i} \overline{v}(a_{-i,j}^*, b_j) - r_i(\boldsymbol{b}_{-i})$$

Choice of $r_i(\boldsymbol{b}_{-i})$ arbitrary: A class of mechanisms

Rest of this talk

Problem

Allocation of a single divisible good among strategic agents

- Efficiency
- Scalar bids
- Almost budget balance

Design rebates for the SSVCG setting

Approach

- Formulate rebate design as a convex optimization problem
- Simplification to remove dependence on true valuations
- Solution method to guarantee good approximation

Budget Balance

Budget Balance

Scenarios where revenue maximization is not a consideration

Strong budget balance

Sum of payments $\sum_i p_i(\boldsymbol{b})$ (or) Budget surplus $= 0$

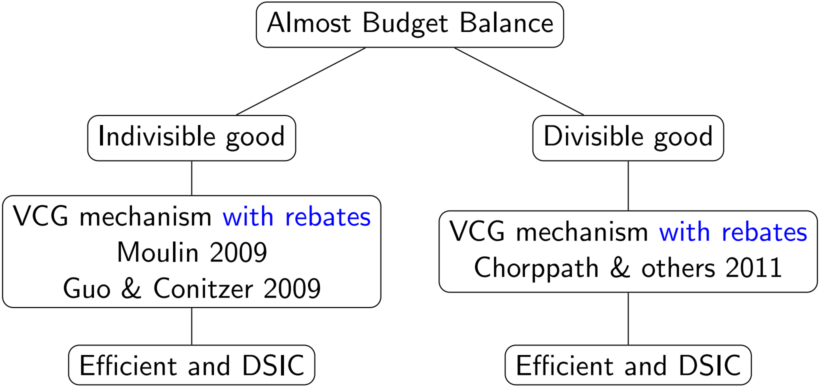
Weak budget balance (or) Feasibility

Budget surplus ≥ 0

- Strong budget balance not possible in our setting*
- Notions of almost budget balance

* Green & Laffont 1977

Known Results: No communication constraints



Two notions of almost budget balance

Choose $r_i(\boldsymbol{b}_{-i})$ to achieve almost budget balance

Formulation of the optimization problem

Scalar bids, divisible case

Choice of objective: Two notions of almost budget balance

Guo & Conitzer

Worst-case fraction of payments retained after rebates

$$\sup_{\boldsymbol{\theta}} 1 - \frac{\sum_{i=1}^{n} r_i(\boldsymbol{\theta}_{-i})}{p_S(\boldsymbol{\theta})}$$

Moulin

Worst-case ratio of sum of payments to sum of valuations

$$\sup_{\boldsymbol{\theta}} \frac{p_S(\boldsymbol{\theta}) - \sum_{i=1}^{n} r_i(\boldsymbol{\theta}_{-i})}{\sigma(\boldsymbol{\theta})} = \sup_{\boldsymbol{\theta}} \frac{p_S(\boldsymbol{\theta})}{\sigma(\boldsymbol{\theta})} \left(1 - \frac{\sum_{i=1}^{n} r_i(\boldsymbol{\theta}_{-i})}{p_S(\boldsymbol{\theta})}\right)$$

$p_S(\boldsymbol{\theta})$ = Sum of payments under zero rebates
$\sigma(\boldsymbol{\theta})$ = Optimal sum of valuations

We use an adaptation of the Moulin notion to optimize the rebates

Constraints on the choice of rebates

(F) Feasibility: Sum of net payments ≥ 0

(VP) Voluntary Participation: $v_i(a_i^*) - p_i(\boldsymbol{b}) \geq 0$ for each agent i (or)

$$r_i(\boldsymbol{b}_{-i}) \geq q_i(\boldsymbol{b}_{-i}),$$

where $q_i(\boldsymbol{b}_{-i})$ is the negative of the utility under zero rebates

- (VP) constraint depends on true valuation functions
- Are there nontrivial rebate functions that satisfy (VP) and (F) constraints?

Some Design Choices

- Surrogate valuation functions of the form $\overline{v}(a, \theta) = \theta U(a)$
- Deterministic and anonymous rebates
  - Information available to planner is symmetric to permutation of agent labels
- Linear rebates

$$r_i(\boldsymbol{b}_{-i}) = c_0 + c_1(\boldsymbol{b}_{-i})_{[1]} + \cdots + c_{n-1}(\boldsymbol{b}_{-i})_{[n-1]},$$

where $(\boldsymbol{b}_{-i})_{[j]}$ is the j^{th} largest entry of $\boldsymbol{b}_{-i}$

- Restrict bids to come from $\hat{\Theta} = \{\boldsymbol{b} \in \mathbb{R}_+^n | b_1 \geq b_2 \geq \ldots \geq b_n \geq 0\}$
  - Each $\boldsymbol{b}$ is a Nash equilibrium for some valuation profiles or in the closure
  - Objective depends only on the ordered bids
  - No dependence on true valuations

Optimization problem

$$r_i(\boldsymbol{b}_{-i}) = c_0 + c_1 b_1 + \ldots + c_{i-1} b_{i-1} + c_i b_{i+1} + \ldots + c_{n-1} b_n.$$

$$\min_{\boldsymbol{c}} \sup_{\boldsymbol{\theta} \in \hat{\Theta}} \left[\frac{p_S(\boldsymbol{\theta}) - \sum_{i=1}^{n} r_i(\boldsymbol{\theta}_{-i})}{\sigma_S(\boldsymbol{\theta})} \right]$$

subject to

(F) $$nc_0 + \sum_{i=1}^{n-1} c_i (i\theta_{i+1} + (n-i)\theta_i) \leq p_S(\boldsymbol{\theta}), \ \forall \boldsymbol{\theta} \in \hat{\Theta}$$

(VP) $$c_0 + \sum_{j=1}^{i-1} c_j \theta_j + \sum_{j=i}^{n-1} c_j \theta_{j+1} \geq q_i(\boldsymbol{\theta}), \ \forall \boldsymbol{\theta} \in \hat{\Theta}, \forall i.$$

(VP) constraint still involves true valuation functions

Simplification of constraints and a reformulation

Simplification of (VP) constraints

Constraints (F) and (VP) together imply that $c_0 = c_1 = 0$

Let $c_0 = c_1 = 0$. Then, the (VP) constraint is equivalent to

$$\sum_{i=2}^{k} c_i \geq 0, \text{ for } k = 2, 3, \ldots, n-1.$$

Proof using:

- Appropriate choice of $\boldsymbol{\theta}$
- Nash equilibrium property
- Some technical assumptions on the true and surrogate valuations

Min-max problem as a generalized linear program

Introduce auxiliary variable t

$$\min_{\boldsymbol{c},t} \; t$$

subject to

$$\text{(F)} \quad \sum_{i=2}^{n-1} c_i(i\theta_{i+1} + (n-i)\theta_i) \le p_S(\boldsymbol{\theta}), \; \forall \boldsymbol{\theta} \in \hat{\Theta}$$

$$\text{(VP)} \sum_{i=2}^{k} c_i \ge 0, \; k = 2, 3, \ldots, n-1,$$

$$\text{(W)} \quad \sum_{i=2}^{n-1} c_i(i\theta_{i+1} + (n-i)\theta_i) + t\sigma_S(\boldsymbol{\theta}) \ge p_S(\boldsymbol{\theta}), \forall \boldsymbol{\theta} \in \hat{\Theta}.$$

(W) captures the constraint associated with the worst-case objective.

"Generalized" LP because the above LP has a *continuum* of linear constraints parametrized by $\boldsymbol{\theta} \in \hat{\Theta}$

Can we replace $\hat{\Theta}$ with a compact set?

Prove and use:

- Monotonicity of VCG payments:
  For fixed $\boldsymbol{\theta}_{-i}$, the map $\theta_i \mapsto p_S(\theta_i, \boldsymbol{\theta}_{-i})$ is increasing.
- Scaling property of VCG payments:
  For fixed $\boldsymbol{\theta}$, the map $\lambda \mapsto p_S(\lambda\boldsymbol{\theta})/\lambda$ is decreasing.

For the (W) constraint, $\hat{\Theta}$ can be replaced by $\Theta = \{\boldsymbol{\theta} \in \hat{\Theta} : 1 = \theta_1\}$

For the (F) constraint, $\hat{\Theta}$ can be replaced by $\{\boldsymbol{\theta} \in \hat{\Theta} : 1 = \theta_1 = \theta_2\}$

Helps in the guarantee for constraint sampling

Constraint sampling with deterministic guarantee

Sample constraints using an ϵ-cover of Θ:

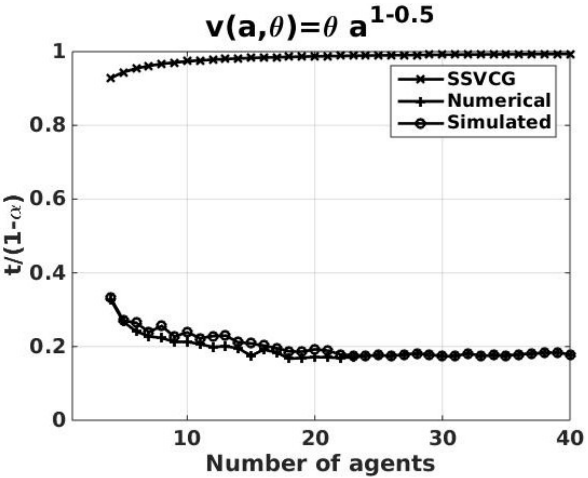
$$|\text{Value of the Sampled LP - Value of the Generalized LP}| \leq K\epsilon$$

under some conditions

- Proof for a general uncertain convex program (UCP)
- Problem here is a special case

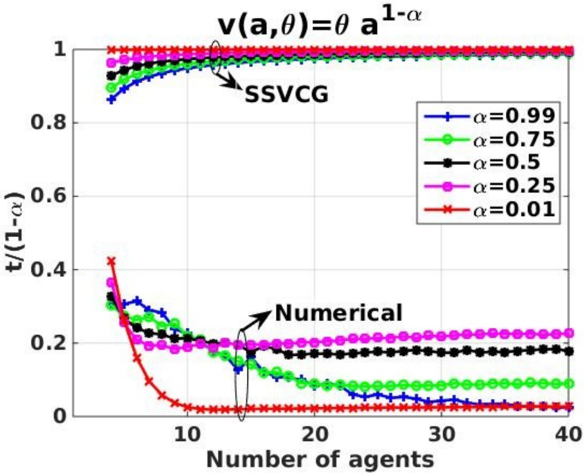
Numerical Results

Worst-case objective vs. Number of agents



Significant reduction in budget surplus with linear rebates

Worst-case objective vs. Number of agents



Significant reduction in budget surplus with linear rebates

Summary

Problem

Allocation of a single divisible good among strategic agents

- Efficiency
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Contributions

- Rebate design as a convex optimization problem
- Simplification to remove dependence on true valuations
- Solution method to guarantee good approximation
- Numerical results to show significant reduction in budget surplus

Open Questions

- Almost budget balance criterion in Guo & Conitzer 2009
- Network setting in Johari & Tsitsiklis 2009
- Optimality of linear rebates
- Relaxation of the anonymous rebates constraint

Thank you

http://www.ee.iitm.ac.in/∼skrishna/

D. Thirumulanathan, H. Vinay, S. Bhashyam, R. Sundaresan, "Almost Budget Balanced Mechanisms with Scalar Bids For Allocation of a Divisible Good", European Journal of Operational Research, Volume 262, Issue 3, 1 November 2017, Pages 1196-1207, ISSN 0377-2217.