Lecture 31: (22 Oct 2008)

What are the fundamental limits on performance in communication systems?

- Mathematical theory introduced by Shannon (1948) (now called "Information Theory")

A fundamental measure of performance that can be achieved for a given communication channel is <u>Capacity</u>.

Capacity: The maximum rate of communication for which arbitrarily small probability of error can be achieved.

In wireless communications, we are interested in fading channels (flat fading & frequency selective fading) for point-to-point communication.

The results for these channels are developed from basic results for AWGN channels and time-invariant ISI channels.

<u>Review</u>:

<u>Capacity of the AWGN channel</u>:

Discrete-time real AWGN channel:

$$y[m] = x[m] + w[m].$$

AWGN $w[m] \sim N(0, \sigma^2)$

Average Transmit Power constraint P: $E[x^2[m]] \leq P$.

$$\text{Capacity } C = \frac{1}{2} \log_2\left(1 + \frac{P}{\sigma^2}\right) \text{ bits/transmission}$$

bits/channel use.

(See Section 5.1.2 and Appendix B.5)

Heuristic argument based on packing spheres.

Rigorous proof in Information theory books: ① Achievability proof for $R < C$

② Converse for $R > C$.

Practical codes that achieve performance close to capacity have been constructed for this AWGN channel.

Eg: If $\frac{P}{\sigma^2} = 1$ (0 dB), $C = \frac{1}{2}$ bits/transmission.

LDPC code, block length = 8000 can get BER of 10^{-4} at ≈ 0.1 dB.

→ Lower BER can be achieved at 0.1 dB by increasing block length. (or) 10^{-4} can be achieved at < 0.1 dB by increasing block length.

→ Over n uses of the channel, we send one of 2^{nR} messages to achieve rate R.

Continuous-time AWGN channel:

Bandpass channel with BW W Hz

Transmit power constraint $\bar{P}$

AWGN with PSD $\frac{N_0}{2}$

$$\Big\downarrow \text{ Passband–baseband conversion } + \text{ Sampling at } W \text{ complex samples/sec}$$

Complex discrete-time AWGN channel

$$y[m] = x[m] + w[m]$$

$$w[m] \sim \mathcal{CN}(0, N_0) \quad \text{i.i.d.}$$

circularly symmetric

Transmit power constraint per sample $\bar{P}\left(\frac{1}{W}\right)$

$$\Big\downarrow \text{ 2 real channels}$$

AWGN discrete-time with

$$\sigma^2 = \frac{N_0}{2}$$

$$P = \frac{\bar{P}}{2W} \Rightarrow \log\left(1 + \frac{\bar{P}}{N_0 W}\right) \text{ bits/complex sample}$$

$$\Rightarrow C_{awgn}(\bar{P}, W) = W \log\left(1 + \frac{\bar{P}}{N_0 W}\right) \text{ bits/sec.}$$

(W samples/sec)

or

$$C_{awgn} = \log\left(1 + \frac{\bar{P}}{N_0 W}\right) \text{ bits/s/Hz} \quad \text{(spectral efficiency)}$$

$\triangleq$ SNR