

Lecture 31: (22 Oct 2008)

What are the fundamental limits on performance in communication systems?

- Mathematical theory introduced by Shannon (1948)
(now called "Information Theory")

A fundamental measure of performance ^{that can be achieved} for a given communication channel is Capacity.

Capacity: The maximum rate of communication for which arbitrarily small probability of error can be achieved.

In wireless communications, we are interested in fading channels (flat fading & frequency selective fading) for point-to-point communication.

The results for these channels are developed from basic results for AWGN channels and time-invariant ISI channels.

Review:

Capacity of ^{the} AWGN channel:

Discrete-time real AWGN channel:

$$y[m] = x[m] + w[m].$$

$$\text{AWGN } w[m] \sim \mathcal{N}(0, \sigma^2)$$

$$\text{Average Transmit Power constraint } P: E[x^2[m]] \leq P.$$

$$\text{Capacity } C = \frac{1}{2} \log_2 \left(1 + \frac{P}{\sigma^2} \right) \begin{array}{l} \text{bits/transmission} \\ \text{bits/channel use.} \end{array}$$

(See Section 5.1.2 and Appendix B.5)

↙
Heuristic argument
based on packing spheres.

Rigorous proof in Information theory books: ① Achievability proof for $R < C$
② Converse for $R > C$.

Practical codes that achieve performance close to capacity have been constructed for this AWGN channel.

Eg.: If $\frac{P}{\sigma^2} = 1$ (0 dB), $C = \frac{1}{2}$ bits/transmission.

LDPC code, block length = 8000 can get
BER of 10^{-4} at ≈ 0.1 dB.

→ Lower BER can be achieved at 0.1 dB by increasing block length. (or) 10^{-4} can be achieved at < 0.1 dB by increasing block length.

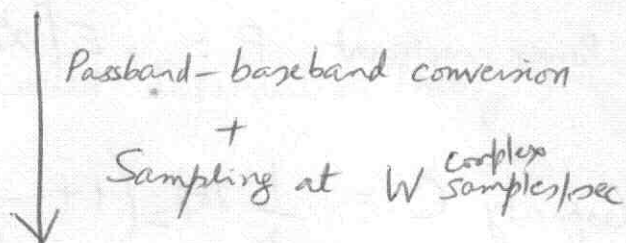
→ Over n uses of the channel, we send 2^{nR} messages to achieve rate R .

Continuous-time AWGN channel:

Bandpass channel with BW W Hz

Transmit power constraint \bar{P}

AWGN with PSD $\frac{N_0}{2}$



Complex discrete-time AWGN channel

$$y[m] = x[m] + w[m]$$

$$w[m] \sim \mathcal{CN}(0, N_0) \quad \text{i.i.d.}$$

↑
circularly symmetric

Transmit power constraint per sample $\bar{P} \left(\frac{1}{W} \right)$

↓ Z real channels

AWGN discrete-time with

$$\sigma^2 = \frac{N_0}{2}$$

$$P = \frac{\bar{P}}{2W} \Rightarrow \log \left(1 + \frac{\bar{P}}{N_0 W} \right) \text{ bits/complex sample}$$

$$\Rightarrow C_{\text{awgn}}(\bar{P}, W) = W \log \left(1 + \frac{\bar{P}}{N_0 W} \right) \text{ bits/sec.}$$

(W samples/sec)

or

$$C_{\text{awgn}} = \log \left(1 + \underbrace{\frac{\bar{P}}{N_0 W}}_{\triangleq \text{SNR}} \right) \text{ bits/s/Hz} \quad (\text{spectral efficiency})$$