Lecture 19: (23 Sep 2008)

Frequency diversity:

* Narrowband flat fading channels $\Rightarrow$ Delay spread $< \frac{1}{W}$

  $\Rightarrow$ Single-tap filter model.

* Wideband channels ($W >$ coh. bandwidth, $\frac{1}{W} <$ delay spread)

  $\Rightarrow$ Frequency-selective channel

  $\Rightarrow$ Linear time-varying filter with multiple taps.

$$y[m] = \sum_{l} h_l[m]\, x[m-l] + w[m].$$

* If we assume that the channel response has L taps, then the delayed replicas of the signal provide L independent copies of the signal. (Multipath diversity (or) frequency diversity)
* How can we exploit this diversity? In general, these multipath copies can interfere with each other.
* Simple scheme (analogous to repetition coding):
  - Send only one symbol during every delay spread, i.e, every L symbol times.

    $\Rightarrow$ Delayed replicas do not interfere with the original symbols.
  - Assuming $h_l[m]$ are independent & i.d. for diff. m, we get L^{th} order diversity.

– Can we increase the symbol rate, i.e, use more degrees of freedom?

<u>System 1</u>: ML sequence detection to combat ISI (Inter-symbol interference)

– Send a sequence of N uncoded symbols
$$\underline{x} = (x[0] \;\; x[1] \;\; \cdots \;\; x[N-1]).$$

– Received vector $\underline{y} = (y[0] \;\; y[1] \;\; \cdots \;\; y[N+L-2])^T$

– Assume that the channel taps do not vary over this duration.

– Assume h_ℓ are i.i.d. (with equal variance) $\mathcal{CN}$. (Rayleigh model)

$$\underline{y}^T = [h_1 \; h_2 \; \cdots \; h_L] \underbrace{\begin{bmatrix} x[0] & x[1] & \cdots & x[N-1] & 0 & \cdots & 0 \\ 0 & x[0] & x[1] & \cdots & x[N-1] & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & x[0] & x[1] \cdots & x[N-1] \end{bmatrix}}_{X \,:\, L\times(N+L-1) \text{ matrix}} + \underbrace{\underline{w}^T}_{\text{Noise}}$$

<u>Note</u>: N symbols are transmitted in $N+L-1$ symbol times

If $N \gg L$, $\dfrac{N}{N+L-1} \approx 1$ symbol/symbol interval.

[For this, we need $\underset{\text{Coherence time}}{T_c} \gg$ Delay spread. This is a reasonable assumption for typical cellular propagation scenarios.]

- Consider ML detection of $\underline{x}$ from $\underline{y}$.

- This system is equivalent to a MISO system with L transmit antennas and space-time code matrix

$$X = \begin{bmatrix} x[0] & x[1] & \cdots & x[N-1] & 0 & \cdots & \cdots & 0 \\ 0 & x[0] & x[1] & \cdots & \cdots & x[N-1] & 0 \cdots & 0 \\ \vdots & \ddots & & \ddots & & & \ddots & \vdots \\ & & & & & & & 0 \\ 0 & \cdots & 0 & x[0] & x[1] & \cdots & \cdots & x[N-1] \end{bmatrix}$$

$\uparrow$ Transmitted Symbols from antennas during $m=0$ $\quad$ $\uparrow$ $m=L-1$ $\quad$ $\uparrow$ $m=N+L-1$.

- Consider the pairwise error probability of confusing $\underline{x}_A$ with $\underline{x}_B$ (or X_A with X_B).

$$P_r(\underline{x}_A \to \underline{x}_B) \le \prod_{l=1}^{L} \frac{1}{1+\frac{SNR}{4}\lambda_l^2}$$ (from MISO analysis done earlier)

where λ_l^2 are the eigenvalues of $(X_A - X_B)(X_A - X_B)^H$, SNR is the total received SNR per received symbol summed over all paths.

If $(X_A - X_B)$ is rank L, $P_r(\underline{x}_A \to \underline{x}_B) \le K.SNR^{-L}$.

To get full diversity, $X_A - X_B$ should be full rank for each $\underline{x}_A, \underline{x}_B$.

- We can also relate prob. of symbol error to this pairwise error probability.

$$\underset{\text{Given } \underline{x}_A \text{ is transmitted}}{Pr(x[m] \text{ being in error})} \leq \sum_{\underline{x}_B : \underline{x}_B[m] \neq \underline{x}_A[m]} Pr(\underline{x}_A \to \underline{x}_B)$$

For each such $\underline{x}_B$, we need $X_A - X_B$ to be full rank.

Suppose $\underline{x}_A$ & $\underline{x}_B$ differ in the m^{th} position only.

$$X_A - X_B = \begin{bmatrix} 0 & \cdots & 0 & x_A[m]-x_B[m] & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & 0 & x_A[m]-x_B[m] & 0 & \cdots & 0 \\ \vdots & & & & & \ddots & & \\ 0 & \cdots & & & & 0 & x_A[m]-x_B[m] & 0 \end{bmatrix}$$

By observation, $X_A - X_B$ is full rank (each row is a shifted version of 1^{st} row).

If $\underline{x}_A$ & $\underline{x}_B$ differ in more positions, we will still have full rank (similarly shifted rows).

$\Rightarrow$ $\boxed{\text{Uncoded transmission (with ISI) combined with ML sequence detection achieves full diversity of } L.}$

Number of symbols transmitted per symbol time ≈ 1

(Can be made 1 : continuous transmission, Viterbi algo. for MLSD with a decoding delay $\approx$ 5-6 times L).

(See book for more details on Viterbi algorithm).

The above system is referred to as a "single carrier" system in t

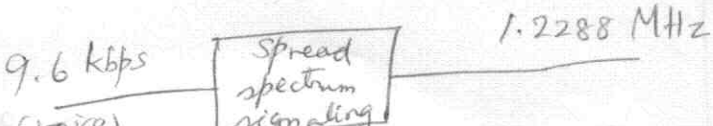
textbook.

$\boxed{\text{Lecture 20:}}$ (24 Sep 2008)

<u>System 2</u>: Direct-sequence spread spectrum.

- Using a large bandwidth provides frequency diversity.
- In a spread spectrum system, the transmission bandwidth is much larger than the symbol rate.

<u>Eg</u>: IS-95



$$\frac{1.2288 \times 10^6}{9.6 \times 10^3} = 128 \triangleq \text{Processing gain (or) Spreading factor}$$

- Not a very efficient use of bandwidth for a single link. (In a cellular system based on spread-spectrum, several links share the same bandwidth to improve the spectral efficiency)

- Other aspects of spread-spectrum signalling:

  * Power spread across large bandwidth
    - $\Rightarrow$ Good anti-jamming properties
    - $\Rightarrow$ Power level close to noise floor

  * ISI typically negligible

  * In a cellular scenario with many links, interference is not negligible.

Suppose we send one symbol every n samples.

Received signal: $$y[m] = \sum_{\ell=0}^{L-1} h_\ell[m]\, x[m-\ell] + w[m]$$

(Tx. signal) $\underbrace{x[0]\ x[1] \cdots x[n-1]}_{1^{st}\ \text{symbol}},\ \underbrace{x[n]\ x[n+1] \cdots x[2n-1]}_{2^{nd}\ \text{symbol}}\ x[2n] \cdots$

Channel assumptions: ① No. of taps $= L$.

② Channel does not vary much over n durations, i.e.,

$$\frac{n}{W} \ll T_c \text{ (coherence time)}$$

Delay spread T_d.

(Reasonable assumption for typical cellular scenarios).

(Rx. signal): $h_0 * (x[0]\ x[1] \cdots x[n-1]\ x[n] \cdots)$

$+ h_1 * (0\ x[0]\ x[1] \cdots)$

$+ h_2 * (0\ \ 0\ \ x[0]\ x[1] \cdots)$

$+ \cdots$

$+ h_{L-1} * (\underbrace{0\ \ 0\ \cdots\ 0}_{L-1\ \text{zeros}}\ x[0]\ x[1] \cdots)$

Denote $\underline{x}_i = (x[in]\ x[in+1] \cdots x[in+n-1])$.

$\Rightarrow$ Tx: $(\underline{x}_0\ \underline{x}_1\ \underline{x}_2 \cdots)$

Rx: $h_0 * (\underline{x}_0\ \underline{x}_1\ \underline{x}_2 \cdots)$

$+ h_1 * (0\ \underline{x}_0\ \underline{x}_1 \cdots) + \cdots + h_{L-1} * (\underbrace{0\ 0 \cdots 0}_{L-1\ \text{zeros}}\ \underline{x}_0\ \underline{x}_1 \cdots)$

Let x_i be generated as $b_i \underline{u}$, where b_i is the bit I
$\underline{u}$ is the spreadin
(or) spreading

(Rx. Signal)

$$h_0 * \left(b_0 \underline{u} \quad b_1 \underline{u} \quad b_2 \underline{u} \quad b_3 \underline{u} \quad \cdots \right)$$

$$+ h_1 * \left(0 \quad b_0 \underline{u} \quad b_1 \underline{u} \quad b_2 \underline{u} \quad \cdots \right)$$

$$+ \cdots$$

$$+ h_{L-1} * \left(\underbrace{0 \; 0 \; 0 \cdots 0}_{L-1 \text{ zeros}} \quad b_0 \underline{u} \quad b_1 \underline{u} \quad b_2 \underline{u} \quad \cdots \right)$$

$\boxed{\text{Lecture 21:}}$ (26 Sep 2008)

Look at the first $n+L-1$ samples to detect b_0 (Ignoring ISI).

Define $$\underline{u}^{(l)} = \Big(\underbrace{0 \; 0 \cdots 0}_{l \text{ zeros}} \; \underbrace{u[0] \; u[1] \cdots u[n-1]}_{n \text{ samples}} \; \underbrace{0 \cdots 0}_{L-l-1 \text{ zeros.}} \Big)^T$$

$$\underline{y}_0 = \left(y[0] \quad y[1] \quad \cdots \quad y[n+L-2] \right)^T$$

$$= h_0 \, b_0 \, \underline{u}^{(0)} + h_0 \, b_1 \left(\underline{0} \, u[0] \cdots u[L-1] \right)$$

$$+ h_1 \, b_0 \, \underline{u}^{(1)} + h_1 \, b_1 \left(\underline{0} \, u[0] \cdots u[L-2] \right)$$

$$+ \cdots \qquad + \cdots$$

$$+ h_{L-1} \, b_0 \, \underline{u}^{(L-1)} \quad \cancel{+ h_{L-1} b_1 (\ldots}$$

$$+ \underline{n}_0$$

$$\underline{u}^{(0)T} \underline{y}_0 \approx h_0 b_0 + w_0 \qquad \text{where } w_i = \underline{u}^{(i)T} \underline{n}_0$$

$$\underline{u}^{(1)T} \underline{y}_0 \approx h_1 b_0 + w_1 \qquad \underline{u}^{(i)T} \underline{u}^{(j)} \approx 0$$

$$\vdots \qquad \underline{u}^{(i)T} \underline{u}^{(i)} = 1$$

$$\underline{u}^{(L-1)T} \underline{y}_0 \approx h_{L-1} b_0 + w_{L-1} \qquad \underline{u}^{(i)T} \left(\underline{0} \text{ (Part of } \underline{u}) \right) \approx 0.$$

Correlation with $\underline{u}^{(\ell)}$ gives

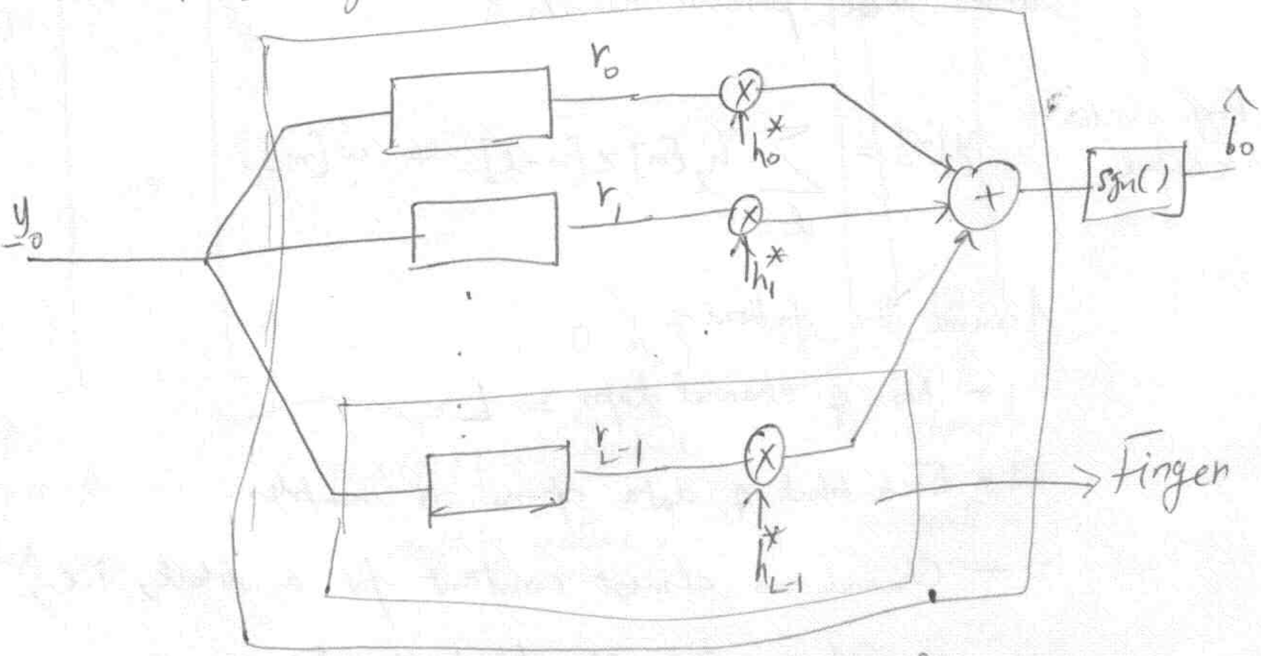
$$h_\ell b_0 \underline{u}^{(\ell)T} \underline{u}^{(\ell)} + (\text{Terms from } b_0 \text{ in other taps})$$
$$+ (\text{Terms from } \underbrace{\text{other bits}}_{b_1, b_{-1}} \text{ in other taps})$$
$$+ (\text{noise})$$

$$r_0 = \underline{u}^{(0)T} \underline{y}_0 \approx h_0 b_0 + n_0$$
$$r_1 = \underline{u}^{(1)T} \underline{y}_0 \approx h_1 b_0 + n_1$$
$$\vdots$$
$$r_{L-1} \approx h_{L-1} b_0 + n_1$$

Suppose $b_0 = \pm 1$ (BPSK),

$$\hat{b}_0 = \operatorname{sgn}\left(\sum_{\ell=0}^{L-1} h_\ell^* r_\ell\right)$$

$\Rightarrow$ Div. gain $= L$.



RAKE receiver / Matched filter

Summary :

* Div. gain L (freq. diversity) exploited by spread-spectrum signaling

* ISI is negligible if spreading factor is high

⇒ RAKE receiver (MF) is sufficient to get div. gain

* Interference would be high when several spread-spectrum signals are added. Power control & multiuser detection may be necessary.

— Discussion in class on m-sequences & pseudo-random sequences.

$\boxed{\text{Lecture } 22:}$ (29 Sep 2008)

System 3: OFDM

- Converts the frequency-selective ISI channel into a set of parallel flat fading channels with no ISI.

- Frequency diversity is achieved by coding + interleaving across these parallel channels.

Freq-selective channel.

$$y[m] = \sum_{\ell} h_\ell[m] x[m-\ell] + w[m]$$

Assume the following

* No. of channel taps $= L$.

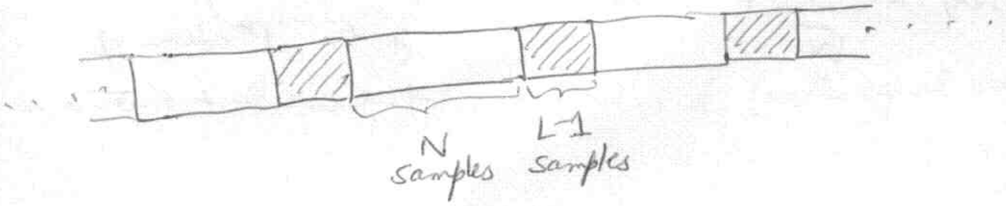
* Each block of data spans N samples.

* Channel is almost constant for a block, i.e., Coherence time $\gg$ block duration.

$$y[m] = \sum_{l=0}^{L-1} h_l \, x[m-l] + w[m].$$

* A gap of $L-1$ samples between successive blocks of data can be used to eliminate interference between blocks.

  (The simple scheme discussed earlier: Send one symbol every L samples is a trivial special case of this $N=1$. Here, we send data of length N with gaps in between)



* In order to be able to diagonalize the channel matrix corresponding to each block that is transmitted, a "cyclic prefix" is transmitted before each block ($L-1$ length cyclic prefix).

$$\underbrace{\begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ \vdots \\ y[N-1] \end{bmatrix}}_{\text{channel output for a block}} = \underbrace{\begin{bmatrix} h_{L-1} & \cdots & h_0 & 0 & \cdots & & 0 \\ 0 & \ddots & & \ddots & & & \vdots \\ \vdots & & \ddots & & \ddots & & 0 \\ 0 & \cdots & & 0 & h_{L-1} & \cdots & h_1 \; h_0 \end{bmatrix}}_{N \times (N+L-1) \text{ channel matrix (Toeplitz matrix)}} \underbrace{\begin{bmatrix} d[-L+1] \\ \vdots \\ d[0] \\ \vdots \\ d[N-1] \end{bmatrix}}_{\text{channel input for block + previous } L-1 \text{ samples}} + \begin{bmatrix} w[0] \\ \vdots \\ \vdots \\ w[N-1] \end{bmatrix}$$

We want to diagonalize the channel matrix to get parallel channels. However, we want to do this without knowing the channel.

Fact: A circulant matrix can be diagonalized without knowing the entries of the matrix.

Show $\begin{bmatrix} \text{DFT} \\ \text{matrix} \end{bmatrix}_{N\times N} \begin{bmatrix} \text{Circulant} \\ \text{matrix} \end{bmatrix}_{N\times N} = \begin{bmatrix} \text{Diagonal} \\ \text{matrix} \end{bmatrix}_{N\times N} \begin{bmatrix} \text{DFT} \\ \text{matrix} \end{bmatrix}_{N\times N}$.

F

$(n,k)^{th}$ entry is $\frac{1}{\sqrt{N}} e^{-j\frac{2\pi nk}{N}}$.

Diagonal entries are the DFT coefficients of the 1st column of the circulant matrix.

If we use a cyclic prefix, i.e.,

$$d[-L+1] = d[N-L+1]$$
$$\vdots \qquad \vdots$$
$$d[-1] = d[N-1]$$

then, we have

$$\begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ y[N-1] \end{bmatrix}_{N\times 1} = \underbrace{\begin{bmatrix} h_0 & 0 & \cdots & 0 & h_{L-1} & \cdots & h_1 \\ h_1 & h_0 & & & & \ddots & \vdots \\ \vdots & & \ddots & & & & h_{L-1} \\ h_{L-1} & & & \ddots & & & 0 \\ 0 & \ddots & & & \ddots & & \vdots \\ \vdots & & \ddots & & & \ddots & 0 \\ 0 & \cdots & 0 & h_{L-1} & \cdots & h_1 & h_0 \end{bmatrix}}_{N\times N} \underbrace{\begin{bmatrix} d[0] \\ \vdots \\ d[N-1] \end{bmatrix}}_{N\times 1} + \underbrace{\begin{bmatrix} w[0] \\ \vdots \\ w[N-1] \end{bmatrix}}_{N\times 1}$$

(We ignore output samples at the receiver corresponding to the cyclic prefix indices)