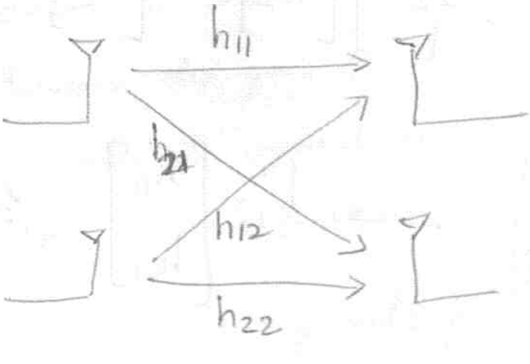
Lecture 17: (10 Sep 2008)

MIMO Channel: (Multiple-Input Multiple-Output channel)

2×2 channel



h_{ij} : channel gain from tx. antenna j to rx. antenna i.
(Rayleigh fading model).

h_{ij} independent

* For the 2×2 channel, there are four independent fading paths between tx. and rx. $\Rightarrow$ Diversity gain = 4 can be achieved.

For $M \times N$ channel, Diversity gain = MN can be achieved.

* Simple scheme to achieve div. gain = 4 : Repetition.

— Transmit the same symbol over the two antennas in two consecutive symbol times (use one antenna at a time).

Time 1 : $y_i[1] = h_{i1} x + w_i[1]$ for $i = 1, 2$.

Time 2 : $y_i[2] = h_{i2} x + w_i[2]$ for $i = 1, 2$.

$$\sum_{i=1}^{2} h_{i1}^* y_i[1] + \sum_{i=1}^{2} h_{i2}^* y_i[2] = \underbrace{\sum_{i=1}^{2}\sum_{j=1}^{2} |h_{ij}|^2 x}_{\text{Div. gain } 4} + \sum_{i=1}^{2} h_{i1}^* w_i[1] + \sum_{i=1}^{2} h_{i2}^* w_i[2]$$

– What about degrees of freedom?

2×2 channel

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} | & | \\ \underline{h}_1 & \underline{h}_2 \\ | & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\underline{h}_i = \begin{bmatrix} h_{1i} \\ h_{2i} \end{bmatrix} \longrightarrow \underline{h}_1 = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} \quad \underline{h}_2 = \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix}$$

* Dimension of the received signal space (space in which received signal lies) is important. This dictates the number of different signals the receiver can reliably distinguish.

– If $\underline{h}_1$ and $\underline{h}_2$ are linearly independent, the received signal space dimension is 2.

$\Rightarrow$ 2 degrees of freedom per transmission (symbol interval).

Recap: Previous examples were 1 degree of freedom per transmission.

SISO: $y = hx + n$. 1 degree of freedom

Time diversity code of block length L: $y_i = h_i x_i + n_i$ for $i = 1, \ldots, L$ — L degrees of freedom over L transmissions. (1 degree of freedom per transmission)

Rx. diversity: $\underline{y} = \underline{h}\, x + \underline{n}$

$\underline{y}$ has L coordinates, but $\underline{h}\, x$ lies along one direction given by $\underline{h}$

$\Rightarrow$ 1 degree of freedom.

Tx. diversity: $y = \underline{h}^H \underline{x} + n$ — 1 degree of freedom

* Degrees of freedom utilized by different schemes:

— Repetition code with div. gain 4:

Transmits one symbol over 2 symbol times.

Uses $\frac{1}{2}$ degree of freedom per symbol time on average.

— Alamouti scheme in the 2×2 channel:

Achieves div. gain 4.

Transmits 2 symbols over 2 symbol times.

Uses 1 degree of freedom

— But we can send 2 symbols per symbol time

(2 degrees of freedom).

* Spatial multiplexing:

* Transmit independent uncoded symbols over the different antennas as well as over different symbol times.

2×2 example:

$$\begin{array}{cccc} t=1 & t=2 & t=3 & \\ x_1 & x_3 & x_5 & \cdots \\ x_2 & x_4 & x_6 & \end{array}$$

Repetition code:

$$\begin{array}{cccc} x_1 & 0 & x_2 & 0 \\ 0 & x_1 & 0 & x_2 \end{array}$$

Alamouti code:

$$\begin{array}{ccccc} x_1 & -x_2^* & x_3 & -x_4^* & \cdots \\ x_2 & x_1^* & x_4 & x_3^* & \end{array}$$

2 symbol streams are multiplexed over the 2 antennas

$\longrightarrow$ Spatial Multiplexing.

This schemes utilizes all the degrees of freedom.

(2 per symbol time).

What about its performance : diversity gain ?

- We have derived a pairwise error prob. bound for confusing codeword X_A with codeword X_B. (for one receive antenna)

$$P_r(X_A \to X_B) \leq \prod_{l=1}^{L} \frac{1}{1 + SNR \frac{\lambda_l^2}{4}}.$$

- This can be extended to the multiple rx antenna case. Suppose there are n_r rx antennas.

$$P_r(X_A \to X_B) \leq \left[\prod_{l=1}^{L} \frac{1}{1 + SNR \frac{\lambda_l^2}{4}} \right]^{n_r}.$$

- In spatial multiplexing, codewords are $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. $N = 1$, $L = 2$.

$$P_r(\underline{x}_A \to \underline{x}_B) \leq \left[\frac{1}{1 + SNR \frac{\|\underline{x}_A - \underline{x}_B\|^2}{4}} \right]^2$$

Only one λ_l is non-zero and it is equal to $\|\underline{x}_A - \underline{x}_B\|^2$.

At high SNR

$$P_r(\underline{x}_A \to \underline{x}_B) \leq \frac{16}{SNR^2 \|\underline{x}_A - \underline{x}_B\|^4}.$$

Diversity gain = 2.

Note: * There is no coding across transmit antennas.

No transmit diversity is exploited.

* Diversity comes from the receive antennas.

* $\|\underline{x}_A - \underline{x}_B\|^4$ is analogous to $\det\left[(X_A - X_B)(X_A - X_B)^H\right]$ discussed earlier.

* For the same bits/transmission,

		Div. gain	DOF used	Coding gain
2×2	Alamouti scheme	4	1	?
	Vs			
	Spatial multiplexing	2	2	?

Since more DOF are used in spatial multiplexing better packing of bits should be possible. At low SNR, this is important. At high SNR, the scheme with better diversity gain will be better in terms of error prob.

Spatial Mux.: $\max_{i \neq j} \Pr(\underline{x}_i \to \underline{x}_j) \leq 4\, SNR^{-2}$ (BPSK 2 b/s/Hz & (av. tr. energy per symbol time normalized to 1))

Alamouti: $\max_{i \neq j} \Pr(\underline{x}_i \to \underline{x}_j) \leq 10000\, SNR^{-4}$ (2 b/s/Hz & av. tr. energy = 1 per symbol time 4-PAM)

* Multiple antennas can be used to achieve
  (i) diversity gain
  (ii) additional degrees of freedom (multiplexing)