Lecture 14: 27/8/2008

Antenna diversity: (or spatial diversity).

- Obtained by placing multiple antennas at the transmitter and/or the receiver.
- Channels between different pairs of antennas are independent if
  - * antennas are spaced sufficiently apart

    This spacing depends on
    - scattering environment
    - carrier frequency
- More about MIMO channel modelling in Chapter 7. For now, we will assume that independent channels can be achieved.

We will discuss the following:

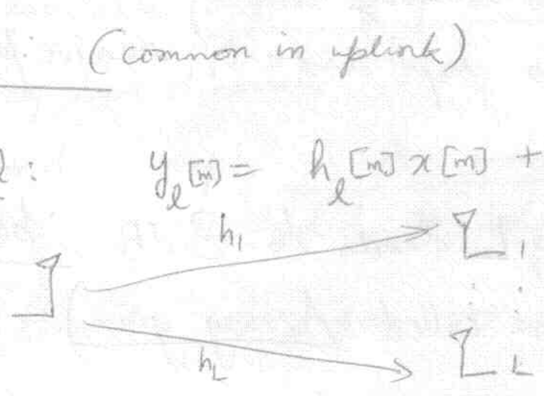
(1) Receive diversity [Single Input Multiple Output channel (SIMO)]

(2) Transmit diversity [Multiple Input Single Output channel (MISO)]

(3) 2x2 MIMO [2x2 Multiple Input Multiple Output channel (MIMO) Two Input. Two Output]

Receive diversity: (common in uplink)

Model: $y_\ell[m] = h_\ell[m]\, x[m] + w_\ell[m] \quad, \ell = 1, \ldots, L$



$w_\ell[m] \sim \mathcal{CN}(0, N_0)$ independent across antennas.

$h_\ell[m] \sim$ independent Rayleigh fading

Diversity gain $= L$. (Similar to Time Diversity analysis)

$x[i]$ is detected based on $y_1[i], y_2[i], \ldots, y_L[i]$.

$\hat{x}[i]$ can be detected from $\underline{h}^H \underline{y}$ as before.

BPSK : $(\pm a)$

$$P_r(\text{error} / \underline{h}) = Q\left(\sqrt{2\|\underline{h}\|^2 \text{SNR}}\right) \quad \text{where } \text{SNR} = \frac{a^2}{N_0}$$

Received SNR conditioned on the channel $\underline{h}$ is

$$\|\underline{h}\|^2 \text{SNR}.$$

This can be thought of as consisting of two parts

$$\left(\frac{1}{L}\|\underline{h}\|^2\right)\left(L\,\text{SNR}\right)$$

* By adding multiple receive antennas, received power increases (linearly with L). This is reflected in L SNR.

(In the time-diversity repetition coding, received power increases with more transmissions only because transmit power increases linearly. If total transmit power per symbol is kept constant, receive power will not increase.)

$\Rightarrow$ Doubling L leads to 3 dB power gain.

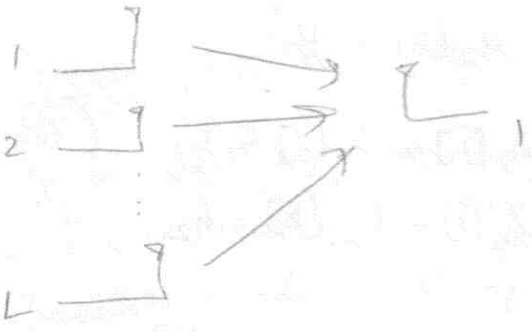
(Also called Array gain)

* $\frac{1}{L}\|\mathbf{h}\|^2$ represents the averaging over multiple independent paths.

$$\frac{1}{L}\|\mathbf{h}\|^2 = \frac{1}{L}\sum_{l=1}^{L}|h_l|^2$$

As L increases, due to law of large numbers, $\frac{1}{L}\|\mathbf{h}\|^2$ converges to 1 and Pr(deep fade) reduces. This gives us the exponent L in the Pr(error) or the Diversity gain.

Transmit diversity: (common in downlink)



Model:

$$y[m] = \sum_{l=1}^{L} \underbrace{h_l[m]}_{\mathcal{CN}(0,1)} x_l[m] + \underbrace{w[m]}_{\mathcal{CN}(0,N_0)}$$

* Simple method to get a diversity gain of L.
  - Transmit the same symbol over the L different antennas during L symbol times. At any time, only one antenna is used. (Repetition code, does not use all available degrees of freedom).
  - Any time diversity code of block length L can be used: simply use one antenna at a time and transmit the coded symbols successively over the diff. antennas one antenna at a time (Provides coding gain over the repetition code)

* One can also design codes specifically for the transmit diversity system. These code are referred to as "space-time codes". Codewords span across antennas and time.

* Simple space-time codes. (More MIMO in chaps. 7–10).

Alamonti scheme:

Consider $L = 2$.

$$y[m] = h_1[m]\, x_1[m] + h_2[m]\, x_2[m] + w[m].$$

* Transmits symbols u_1 and u_2 over 2 symbol times.

$$x_1[1] = u_1 \qquad x_1[2] = -u_2^*$$

$$x_2[1] = u_2 \qquad x_2[2] = u_1^*.$$

$\boxed{\text{Lecture 15}}$ (3/9/2008)

* If we assume $h_1[1] = h_1[2] = h_1$ (Coh. time >> 2 symbols)

$$h_2[1] = h_2[2] = h_2,$$

$$\begin{bmatrix} y[1] & y[2] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix} + \begin{bmatrix} w[1] & w[2] \end{bmatrix}$$

Re-write as:

$$\begin{bmatrix} y[1] \\ y^*[2] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} w[1] \\ w^*[2] \end{bmatrix}.$$

H: Columns are orthogonal

$\Rightarrow$ detection of u_1, u_2 decomposes into 2 separate scalar problems.

$$f(\underline{y}/\underline{u}, \underline{h}) \sim CN(H\underline{u}, N_0 I)$$

$$\|\underline{y} - H\underline{u}\|^2 = (\underline{y} - H\underline{u})^H (\underline{y} - H\underline{u})$$

$$= \underline{y}^H \underline{y} - \underline{y}^H H \underline{u} - \underline{u}^H H^H \underline{y} + \underline{u}^H H^H H \underline{u}$$

$H^H \underline{y}$ is sufficient for detection.

$$H^H \underline{y} = H^H H \underline{u} + H^H \underline{w}$$

(H is orthogonal) $\left(H^H H = (|h_1|^2 + |h_2|^2) I \right)$

$$\underline{r} = H^H \underline{y} = \underline{u}\left(|h_1|^2 + |h_2|^2\right) + \underline{w}'$$

$$\underline{w}' \sim CN\left(\underline{0}, N_0 (|h_1|^2 + |h_2|^2) I\right)$$

$$r_1 = (|h_1|^2 + |h_2|^2)\, u_1 + w_1$$

$$= \|\underline{h}\|^2 u_1 + w_1 \leftarrow$$

$$r_2 = \|\underline{h}\|^2 u_2 + w_2 . \leftarrow$$

} Diversity 2.

* In the repetition code, one symbol is transmitted over 2 symbol intervals. Here, 2 symbols are transmitted over 2 symbol intervals.

* To maintain the same transmit power in both cases, each symbol is transmitted with half the power in each interval for the Alamouti scheme.

* Alamouti scheme works for any constellation.
Suppose, we use BPSK. → 2 bits over 2 symbol times
For the same rate, repetition code should use 4-PAM.
To achieve same d_{min}, 4-PAM needs 5 times the energy as BPSK
For the same transmit power, factor of 2 more power is used on each symbol in repetition coding

Overall, repetition requires 2·5 times more power than Alam[outi]
scheme ($\approx$ 4 dB).

* Repetition uses only one $\boxed{\text{degree of freedom}}$ of the channel over the 2 symbol durations
  → along the dimension $[h_1 \; h_2]$.

Alamouti scheme uses 2 $\boxed{\text{degrees of freedom}}$ available in the channel over the 2 symbol durations
→ along dimensions $[h_1 \; h_2^*]$
and $[h_2 \; -h_1^*]$.

$\boxed{\text{Lecture 16:}}$ (9 Sep 2008)

Space-time code design:

* In order to design a good code to exploit time diversity, we had to maximize the minimum product distance between codewords.
* We can show a similar criterion for space-time codes.

- A codeword in a space-time code is a matrix

X_i : $L \times N$ matrix
L antennas, N time intervals (block length = N).

A space-time code is specified by a set of codeword matrices $\{X_i\}$.

Eg: Alamouti scheme
X_i is of the form $\begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix}$. $L=2$ & $N=2$

Repetition scheme
X_i is of the form $\begin{bmatrix} u & 0 \\ 0 & u \end{bmatrix}$

– Assume that the channel remains the same for N time intervals

$$[y[1] \ \ldots \ y[N]] = [h_1 \ h_2 \ \ldots \ h_L] \begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix} + [w[1] \ldots w[N]]$$

$$\underline{y}^T = \underline{h}^H X + \underline{w}^T$$

$$\underline{h} = \begin{bmatrix} h_1^* \\ h_2^* \\ \vdots \\ h_L^* \end{bmatrix}$$

Normalize the codewords such that the average energy per symbol time is 1 $\Rightarrow SNR = \frac{1}{N_0}$.

• Pairwise error probability $P_r(X_A \to X_B)$

$$P_r(X_A \to X_B \,|\, \underline{h}) \le Q\left(\frac{\|\underline{h}^H(X_A - X_B)\|}{2\sqrt{N_0/2}}\right)$$

$$P_r(X_A \to X_B) \le E\left[Q\left(\frac{\|\underline{h}^H(X_A - X_B)\|}{2\sqrt{N_0/2}}\right)\right]$$

$$= E\left[Q\left(\sqrt{\frac{SNR\ \underline{h}^H(X_A - X_B)(X_A - X_B)^H \underline{h}}{2}}\right)\right]$$

$(X_A - X_B)(X_A - X_B)^H$ is Hermitian (A complex square matrix X is Hermitian if $X = X^H$)

and can be diagonalized as

$U \Lambda U^H$, where U is unitary.

$\Lambda = \text{diag}(\lambda_1^2, \lambda_2^2, \ldots \lambda_L^2)$ where λ_ℓ are the singular values of $X_A - X_B$.

$$\Pr(X_A \to X_B) \leq E\left[Q\left(\sqrt{\frac{SNR \sum_{l=1}^{L} |\tilde{h}_l|^2 \lambda_l^2}{2}} \right) \right]$$

where $\underline{\tilde{h}} = U^H \underline{h}$.

$\underline{\tilde{h}}$ has the same distribution as $\underline{h}$.

$|\tilde{h}_l|^2$ is exponential.

$$\Rightarrow \Pr(X_A \to X_B) \leq \prod_{l=1}^{L} \frac{1}{1 + SNR \frac{\lambda_l^2}{4}}$$

— If all λ_l^2 are > 0 for all codeword differences $(X_A - X_B)$, maximal diversity gain of L is achieved. This is the <u>rank criterion</u>.

Num. of positive eigen values = rank of $(X_A - X_B) \leq N$

Therefore, full diversity is possible only for $N \geq L$.

— If all $\lambda_l^2 > 0$, then

$$\Pr(X_A \to X_B) \leq \frac{4^L}{SNR^L \underbrace{\prod_{l=1}^{L} \lambda_l^2}_{\text{determinant of } (X_A - X_B)(X_A - X_B)^H}} \quad \text{(at high SNR)}$$

Therefore, the coding gain is determined by the minimum of the determinant $\det[(X_A - X_B)(X_A - X_B)^H]$ over all codeword pairs X_A, X_B.

Therefore, we should maximize the minimum determinant.

This is the <u>determinant criterion</u>.