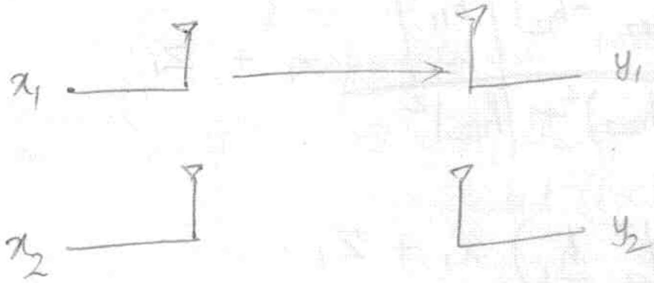
Lecture 18: (22 Sep 2008)

MIMO: 2 x 2 example (contd.)

Scheme	Diversity gain	Degrees of freedom utilized per symbol time
Repetition	4	1/2
Alamouti	4	1
V-BLAST (Spatial Multiplexing)	2	2

For the spatial multiplexing scheme, we used the ML detector. Both symbols (from the 2 tx. antennas) are decoded jointly. This is more complex than detection for the Alamouti scheme (which is a linear receiver to separate the two symbols in one codeword).

Now, we will study V-BLAST with a decorrelator (a sub-optimal detector) and summarize the various MISO, SIMO, MIMO schemes studied so far (What is the equivalent SISO system we get in each case?)



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\underline{y} = H\underline{x} + \underline{w}$$

Suppose we invert the effect of the channel to decouple x_1 and x_2, we get:

$$\underset{\sim}{\tilde{y}} = \underline{x} + H^{-1}\underline{w} = \underline{x} + \underline{\tilde{w}}.$$

$$H^{-1} = \frac{1}{|h_{11}h_{22} - h_{21}h_{12}|}\begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$$

$$\tilde{w}_1 = \frac{h_{22}w_1 - h_{12}w_2}{|h_{11}h_{22} - h_{21}h_{12}|} \sim CN\left(0, \frac{|h_{22}|^2 + |h_{12}|^2}{|h_{11}h_{22} - h_{12}h_{21}|^2} N_0\right)$$

$\tilde{w}_1$ can be written as $\dfrac{\sqrt{|h_{22}|^2 + |h_{12}|^2}}{|h_{11}h_{22} - h_{12}h_{21}|} z_1$ where $z_1 \sim CN(0, N_0)$

$$y_1' = \frac{h_{11}h_{22} - h_{21}h_{12}}{\sqrt{|h_{22}|^2 + |h_{12}|^2}} \tilde{y}_1$$

$$= \frac{h_{11}h_{22} - h_{21}h_{12}}{\sqrt{|h_{22}|^2 + |h_{12}|^2}} x_1 + z_1$$

$$= \frac{\begin{bmatrix} h_{22} & -h_{12} \end{bmatrix}\begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix}}{\sqrt{|h_{22}|^2 + |h_{12}|^2}} x_1 + z_1$$

$$= \left(\underline{\phi}_2^{*}\, \underline{h}_1\right) x_1 + z_1$$

$$\underline{h}_1 = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix}$$

where $$\underline{\phi}_2 = \frac{1}{\sqrt{|h_{22}|^2 + |h_{12}|^2}}\begin{bmatrix} h_{22}^{*} \\ -h_{12}^{*} \end{bmatrix}$$

$$\underline{h}_2 = \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix}$$

$\left(\& \ \underline{\phi}_2^{*}\underline{h}_2 = \underline{0} \right)$ $\quad \underline{\phi}_2^{*} = \begin{bmatrix} h_{22} & -h_{12} \end{bmatrix} / \sqrt{|h_{22}|^2 + |h_{12}|^2}$

Similarly, $y_2' = (\underline{\phi}_1^* \underline{h}_2) x_2 + z_2$ $\qquad H^{-1} = \begin{bmatrix} \underline{\phi}_2^* \\ \underline{\phi}_1^* \end{bmatrix}$

where $$\underline{\phi}_1 = \frac{1}{\sqrt{|h_{11}|^2 + |h_{21}|^2}} \begin{bmatrix} h_{21}^* \\ -h_{11}^* \end{bmatrix}.$$

$(\; \& \;\; \underline{\phi}_1^* \underline{h}_1 = \underline{0} \;)$

The above eqns. for y_1' and y_2' can be interpreted as follows.

* $\underline{h}_1$ is the "direction" of the signal from tx. antenna 1.
* $\underline{h}_2$ is the "direction" of the signal from tx. antenna 2.
* $\underline{\phi}_2$ is the "direction" orthogonal to $\underline{h}_2$.
* y_1' is obtained by projecting $\underline{y}$ onto $\underline{\phi}_2$.

  So, we get $\quad y_1' = (\underline{\phi}_2^* \underline{h}_1) x_1 + \cancel{z}_1 \quad$ (Since $\underline{\phi}_2^* \underline{h}_2 = \underline{0}$)

* Similarly, $\underline{\phi}_1$ is the "direction" orthogonal to $\underline{h}_1$

  $\& \;\; y_2' = \underbrace{(\underline{\phi}_1^* \underline{h}_2)}_{} x_2 + z_2. \qquad$ (Since $\underline{\phi}_1^* \underline{h}_1 = \underline{0}$).

  ⇓

  projection of $\underline{h}_2$ onto a unit vector $\underline{\phi}_1$ whose components are independent of the components of $\underline{h}_2$.

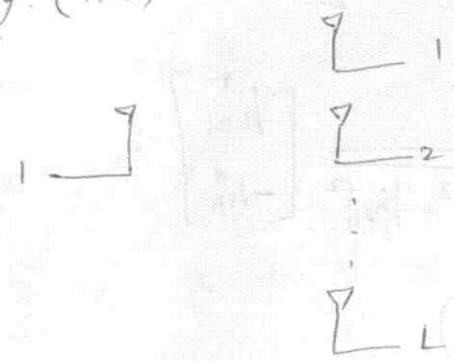
  $\Rightarrow \underline{\phi}_1^* \underline{h}_2$ is like a scalar Rayleigh faded (1×1) channel $\Rightarrow$ Diversity 1.

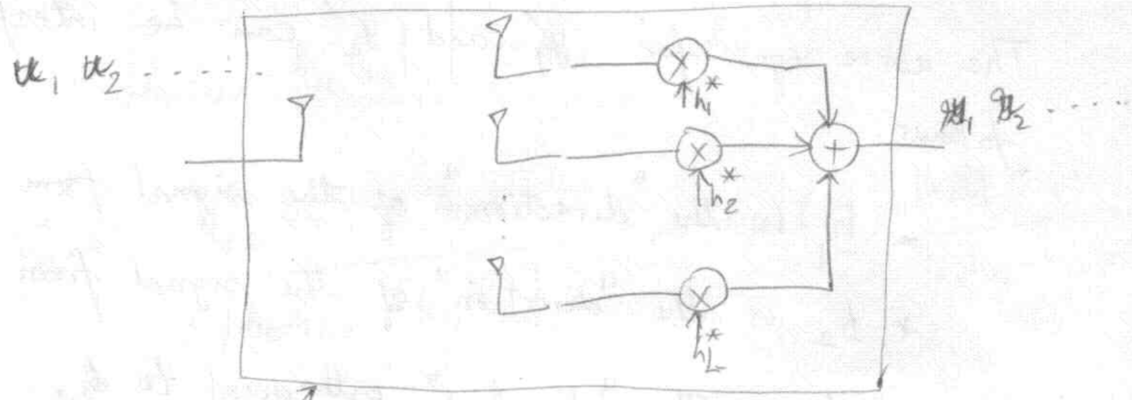
Therefore, the 2 rx. antennas can be used to null the interference. In this case,

Degrees of freedom utilized per symbol time $=$ ~~1~~ 2

Diversity gain $= 1$. $\quad$ (For the ML detector, Div. gain $= 2$)

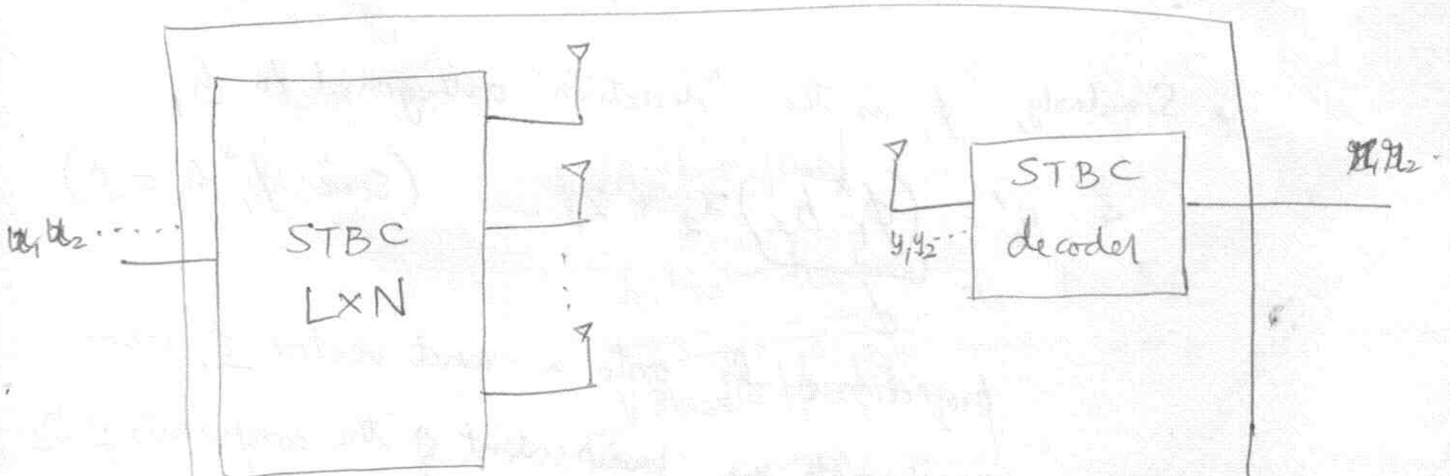
Recap Receive Diversity: (1xL)





Equivalent SISO channel obtained by MRC.

Transmit diversity $L\times 1$



Equivalent SISO channel obtained with STBC encoder/decoder.

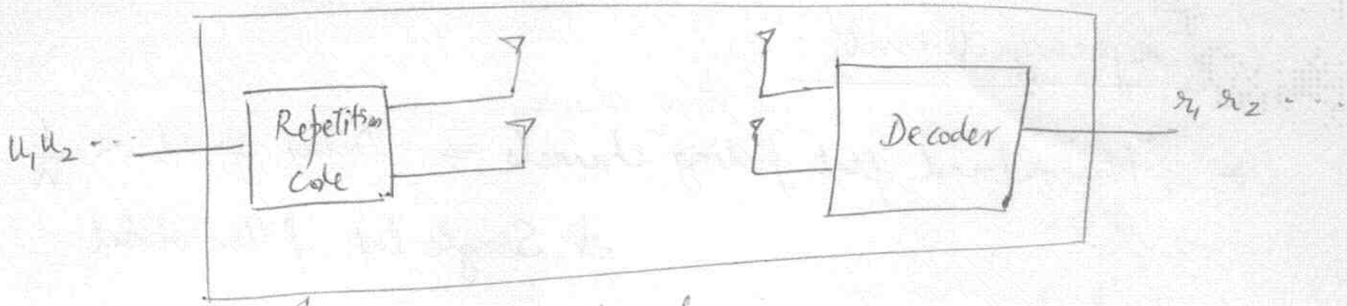
Eg: Alamouti encoder for 2×1 Decoder

$$\begin{bmatrix} x_1 & -x_2^* \\ x_2 & +x_1^* \end{bmatrix}$$

$$r_1 = h_1^* y_1 + h_2 y_2^*$$

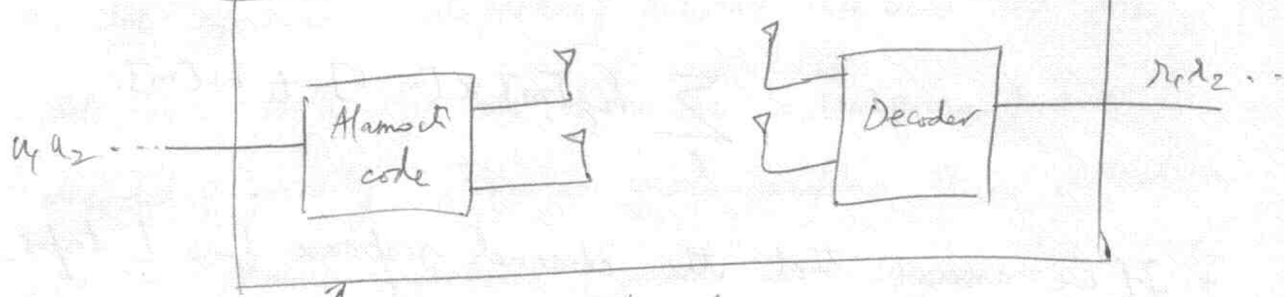
$$r_2 = h_2^* y_1 - h_1 y_2^*$$

2x2 MIMO

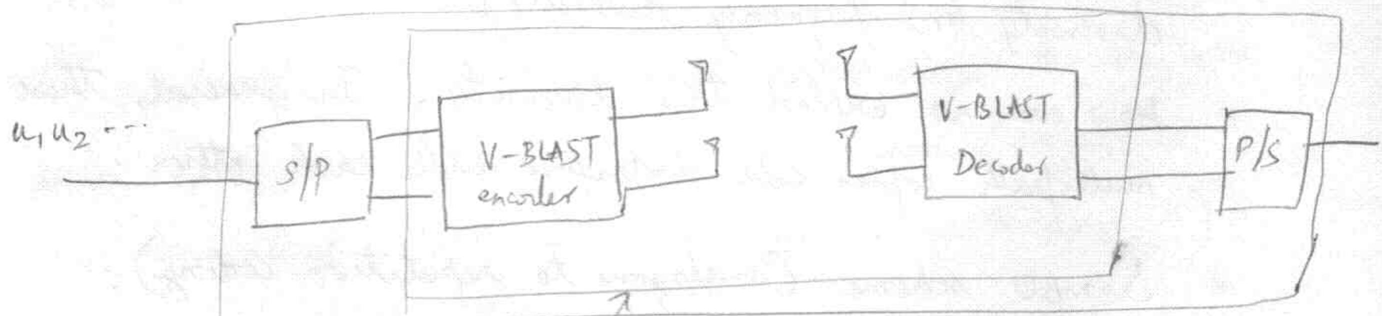


Equivalent SISO channel
1/2 symbol per symbol time

Diversity of 4



Equivalent SISO channel
1 symbol per symbol time
Diversity of 4



Equivalent SISO channel

(all symbols go thru' channel with div. gain 2)

→ 2 symbols in parallel, both achieve div. gain 2. (ML decoder)

The equivalent SISO channel view will be important to understand whether these tx. & rx. schemes are capacity lossless or not.

Lecture 19: (23 Sep 2008)

Frequency diversity:

* Narrowband flat fading channels $\Rightarrow$ Delay spread $< \frac{1}{W}$

  $\Rightarrow$ Single-tap filter model.

* Wideband channels ($W >$ coh. bandwidth, $\frac{1}{W} <$ delay spread)

  $\Rightarrow$ Frequency-selective channel

  $\Rightarrow$ Linear time-varying filter with multiple taps.

$$y[m] = \sum_{\ell} h_{\ell}[m]\, x[m-\ell] + w[m].$$

* If we assume that the channel response has L taps, then the delayed replicas of the signal provide L independent copies of the signal. (Multipath diversity (or) frequency diversity).

* How can we exploit this diversity? In general, these multipath copies can interfere with each other.

* Simple scheme (analogous to repetition coding):

  - Send only one symbol during every delay spread, i.e, every L symbol times.

    $\Rightarrow$ Delayed replicas do not interfere with the original symbols.

  - Assuming $h_{\ell}[m]$ are independent & i.d. for diff. m, we get L^{th} order diversity.