Lecture 6 : 12/8/08

Rayleigh fading model:

* Assume that there are a large number of statistically independent reflected and scattered paths with random amplitudes in the delay window corresponding to each tap.

$$\text{Phase for } i^{th} \text{ path} = 2\pi f_c \tau_i = 2\pi \frac{d_i}{\lambda} \bmod 2\pi.$$

(d_i = distance travelled by the i^{th} path).

Since $d_i \gg \lambda$, it is reasonable to assume that $2\pi f_c \tau_i = 2\pi \frac{d_i}{\lambda}$ is uniformly distributed between 0 and 2π and that the phases of different paths are independent.

Contribution of i^{th} path in the l^{th} tap gain

$$a_i\left(\frac{m}{W}\right) e^{-j 2\pi f_c \tau_i\left(\frac{m}{W}\right)} \operatorname{sinc}\left[l - \tau_i\left(\frac{m}{W}\right)W\right].$$

This can be modeled as a circularly symmetric random variable.

$h_l[m]$ can be modeled (by central limit theorem) as a circularly symmetric $CN(0, \sigma_l^2)$.

(R. V. X is circularly symmetric if $Xe^{j\phi}$ has the same distribution as X for any ϕ).

Note: Variance of $h_l[m]$ is a function of l, but independent of time m. (Usually used to model small-scale fading alone. We will talk a bit about large-scale fading model later.)

Remarks: (1) $h_l[m] \sim CN(0, \sigma_l^2) \Rightarrow |h_l[m]| \sim$ Rayleigh $\frac{x}{\sigma_l^2} e^{-x^2/2\sigma_l^2}, x \geq 0$

(Proof: Exercise ①)

$$|h_l[m]|^2 \sim \text{exponential } \frac{1}{\sigma_l^2} e^{-x/\sigma_l^2}, \; x \geq 0.$$

Rician model:

A line-of-sight path ("specular" path) that has a known magnitude and a large number of independent paths.

$h_\ell[m]$ for at least one ℓ can be modeled as

$$h_\ell[m] = \underbrace{\sqrt{\frac{K}{K+1}}\,\sigma_\ell e^{j\theta}}_{\text{Specular path with uniform phase } \theta} + \underbrace{\sqrt{\frac{1}{K+1}}\,CN(0, \sigma_\ell^2)}_{\text{Aggregation of large number of reflected and scattered paths}}.$$

K : Ratio of energy in the specular path to
(K-factor) the energy in the scattered paths.

Large $K \Rightarrow$ more deterministic channel.

$|h_\ell[m]| \sim$ Rician distribution.

Tap-gain auto-correlation: (Modeling time correlation of taps)

$\{h_\ell[m]\}$ is modeled as a W.S.S. random process (Not including large-scale fading, only for small-scale fading).

($|h_\ell[m]| \sim$ Rayleigh or Rician in previous section.

$$h_\ell[m] \sim CN(0, \sigma_\ell^2) \text{ or } CN\left(\sqrt{\tfrac{K}{K+1}}\,\sigma_\ell e^{j\theta}, \sigma_\ell^2\right))$$

Time-variations of the channel are characterized by

$$R_\ell[n] = E\left[h_\ell^*[m]\, h_\ell[m+n]\right].$$

Implicitly, we assume

- $h_\ell[m]$ and $h_{\ell'}[m']$ are independent if $\ell \neq \ell'$ $\forall m, m'$.
  (Different ranges of delay contributed to $h_\ell[m]$ for different ℓ. These are from different physical paths in small-scale.)

$\boxed{\text{Lecture 7: 13/8/08}}$

$R_\ell[0]$: Energy of ℓ^{th} tap

$\sum_\ell R_\ell[0]$: Total energy.

Multipath spread $T_d = \frac{1}{W}$ (range of ℓ which contains most of the energy)

Statistical in nature.

Coherence time T_c : Smallest value of n (>0) for which $R_\ell[n]$ is significantly different from $R_\ell[0]$.

Remarks: (1) If we increase the bandwidth we use, the taps are separated by smaller delays ($\frac{1}{W}$ sec.) $\Rightarrow$ fewer actual paths contribute to each tap $\Rightarrow$ Rayleigh model becomes poorer.

(2) Sinc fn. becomes narrower and $R_\ell[0]$ gives a finer grained picture of the amount of power being received in the ℓ^{th} delay window.

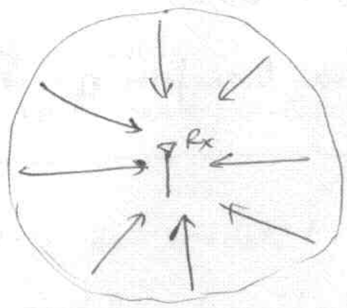
Clarke's model :

- Transmitter is fixed.
- Receiver is moving at speed v.
- Transmitted signal is scattered by stationary objects around the mobile.

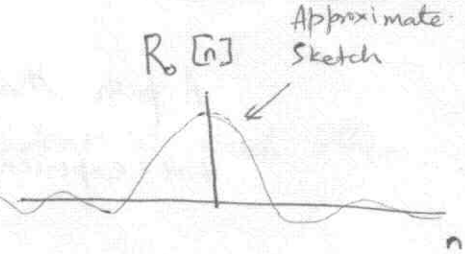
– There are K paths.

i^{th} path arrives at angle $\theta_i = \frac{2\pi i}{K}$ $\quad i = 0, 1, \ldots, K-1$

w.r.t. to the direction of motion.



– K is assumed to be large.



$$y(t) = \sum_{i=0}^{K-1} \underbrace{a_{\theta_i}} \, x(t - \underbrace{\tau_{\theta_i}(t)})$$

$$a_{\theta_i} = \frac{a}{\sqrt{K}} \text{ for all } i. \quad \left(\begin{array}{l}\text{Uniform power in each path} \\ \text{\& isotropic antenna gain}\end{array}\right)$$

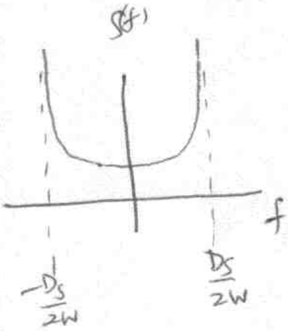
– Suppose $W \ll$ Coh BW. $\quad y[m] = h_0[m]\, x[m] + w[m]$ (Single tap model).

– $R_0[n]$ can be shown to be $2a^2\pi J_0\left(\pi \frac{D_s}{W}\right)$ ← Auto-correlation function.

$E[h_0[m+n]\, h_0^*[m]]$

where $J_0(x) = \frac{1}{\pi}\int_0^{\pi} e^{jx\cos\theta}\, d\theta$, $\quad D_s = \frac{2 f_c v}{c}$.

↳ Zeroth-order Bessel function of the first kind



$$S(f) = \begin{cases} \dfrac{4a^2 W}{D_s\sqrt{1 - \left(\frac{2fW}{D_s}\right)^2}} & -\frac{D_s}{2W} \le f \le \frac{D_s}{2W} \\ 0 & \text{else.} \end{cases}$$ ← Doppler spectrum

– If we define coherence time T_c to be the value of $\frac{n}{W}$ such that $R_0[n] = 0.05 R_0[0]$, then $T_c = \frac{J_0^{-1}(0.05)}{\pi D_s}$. (This is inversely proportional to D_s).

– Note: * $S(f)$ is zero for f beyond the maximum Doppler shift.

* $S(f)df$ has the physical interpretation of the received power along paths that have Doppler shifts in the range $[f, f+df]$.

Steps to derive $R_0[n]$ and $S(f)$: (Exercise ②)

* First derive $S(f)$. ~~Exer~~

* Show that inverse transform of $S(f)$ is $R_0[n]$.

1) Deriving $S(f)$) :

A path that arrives at angle θ w.r.t. direction of motion will experience a Doppler shift of $\dfrac{v\cos\theta}{\lambda}$

Instantaneous frequency of received path at angle θ

$$f(\theta) = \frac{v}{\lambda}\cos\theta + f_c = f_m\cos\theta + f_c .$$

$D_s = 2f_m$.

$$f(\theta) = \frac{D_s}{2}\cos\theta + f_c .$$

Note : $f(\theta) = f(-\theta)$.

$$df = -\sin\theta \, d\theta \left(\frac{D_s}{2}\right).$$

$$\theta = \cos^{-1}\left[\frac{f-f_c}{D_s/2}\right].$$

$$\Rightarrow \sin\theta = \sqrt{1-\left(\frac{f-f_c}{D_s/2}\right)^2}$$

Calculate : ① Received power from paths that have Doppler shifts in $[f, f+df]$.

② Received power from paths ~~with between~~ angles in $[\theta, \theta+d\theta]$.

Relate ① and ②.

Inverse transform of $S(f)$.

- Make a change of variables $f = \frac{D_s}{2W} \cos\theta$.

Modeling a channel with multiple taps $\{h_\ell[m]\}$:

- Each tap is modeled using the Clarke's flat fading model described above.
- Taps are assumed to be independent of each other.

Statistical models for large-scale fading:

* We saw statistical models for small-scale fading
  - Rayleigh fading $h_\ell[m] \sim CN(0, \sigma_\ell^2)$.
  - Rician fading

  So far, power of $h_\ell[m]$ does not depend on m.

  In practice, it could be a slowly varying fn. of m.

  ($\sigma_\ell^2[m]$ could be used here where it is a slowly varying fn. of m).

* Now, we look at large-scale fading models.

* Free space propagation.
  - Average received power decreases as r^{-2}.
* 2-ray ground reflection model
  - Average received power decreases as r^{-4}.

Simple generalization for average path loss:

$$\overline{PL}(r) \propto \left(\frac{r}{r_0}\right)^n$$ Average path loss at distance r from tx.

in dB $$\overline{PL}_{(dB)} = \overline{PL}_{(r_0)} + 10\, n \log\left(\frac{r}{r_0}\right).$$

n : path loss exponent.

r_0 : Reference distance

Log-Normal Shadowing :

* Above model is too simple. At the same distance r from tx., we may not see the same path loss because obstacles could be different at different locations.

* A model based on measurements:

$$PL_{(dB)}(r) = \overline{PL}_{(dB)}(r) + X_{\sigma\,(dB)}.$$

$$X_{\sigma\,(dB)} \sim N(0, \sigma^2).$$

Due to "shadowing" by surrounding environment
(Log of shadowing factor is normal)
in this model.

More sophisticated models : (for specific scenarios)

Examples : - Longley-Rice model
- Okamura model
- Hata model.
- Extensions of Hata model.