Baseband equivalent model. $\boxed{\text{Lecture 4:}}$ (8 Aug 2008)

Usually, we communicate in $\left[f_c - \frac{W}{2}, f_c + \frac{W}{2}\right]$.

Passband multipath fading channel model.
$$y(t) = \sum_i a_i(t)\, x(t-\tau_i(t)).$$

Consider the baseband equivalents for $x(t)$ & $y(t)$.
$$x(t) = \mathrm{Re}\left[x_b(t)\, e^{j2\pi f_c t}\right]$$
$$y(t) = \mathrm{Re}\left[y_b(t)\, e^{j2\pi f_c t}\right]$$

Therefore, we have
$$\mathrm{Re}\left[y_b(t) e^{j2\pi f_c t}\right] = \sum_i a_i(t)\, \mathrm{Re}\left[x_b(t-\tau_i(t))\, e^{j2\pi f_c (t-\tau_i(t))}\right]$$
$$= \mathrm{Re}\left\{\left[\sum_i a_i(t)\, x_b(t-\tau_i(t))\, e^{-j2\pi f_c \tau_i(t)}\right] e^{j2\pi f_c t}\right\}$$

Similarly, one can show
$$\mathrm{Im}\left[y_b(t) e^{j2\pi f_c t}\right] = \mathrm{Im}\left[\left\{\sum_i a_i(t)\, x_b(t-\tau_i(t))\, e^{-j2\pi f_c \tau_i(t)}\right\} e^{j2\pi f_c t}\right]$$

Therefore, the baseband equivalent channel ~~model~~ model is (input-output model)
$$y_b(t) = \sum_i a_i^b(t)\, x_b(t-\tau_i(t))$$
where
$$a_i^b(t) = a_i(t)\, e^{-j2\pi f_c \tau_i(t)}.$$

Baseband equivalent $h_b(\tau, t) = \sum_i a_i^b(t)\, \delta(t-\tau_i(t))$.

Remarks: (1) Baseband output = Sum of the delayed replicas of the baseband input.

Magnitude of the i^{th} term : changes slowly, significant changes occur on the order of seconds or more.

Phase of i^{th} term : changes by $\frac{\pi}{2}$ when the delay changes by $\frac{1}{4f_c}$

(or) path length changes by $\frac{c}{4f_c}$.

If path length is changing at velocity v, time required for phase change of $\frac{\pi}{2}$ is $\frac{c}{4f_c v}$.

For $f_c = 900MHz$, $v = 60 km/h$, single reflecting wall, $\left(D = \frac{f_c v}{c} \right)$ (Doppler shift)

$$\frac{c}{4f_c v} = \frac{\cancel{3} \times \cancel{10^8} \times \cancel{3600}}{\cancel{4} \times \cancel{9} \times \cancel{10^8} \times \cancel{60} \times 10^3} = \underline{\underline{5\,ms}}$$

The phases of both paths in this case are rotating at this rate, but in opposite directions.

Discrete-time baseband model:

* Assume the input is band-limited to W.

$x_b(t)$ is then band-limited to $\frac{W}{2}$.

$x_b(t)$ can be written in terms of its samples as

$$x_b(t) = \sum_n x[n] \operatorname{sinc}(Wt - n),$$

where $x[n] = x_b\left(\frac{n}{W}\right)$ (Sampling rate $= \frac{1}{W}$)

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}.$$

$\{\operatorname{sinc}(Wt-n)\}_n$ is an orthogonal basis that can be used to represent bandlimited signals of bandwidth $\frac{W}{2}$.

* Due to Doppler spread, the bandwidth of the output is generally slightly larger than $\frac{W}{2}$. However, this can be neglected as long as Doppler spread (of the order of 10s to 100s Hz) is small compared to W (100s of kHz, few MHz, or more).

$$y_b(t) = \sum_i a_i^b(t) x_b(t - \tau_i(t))$$

$$= \sum_i a_i^b(t) \sum_n x[n] \operatorname{sinc}(W(t - \tau_i(t)) - n)$$

$$= \sum_n x[n] \sum_i a_i^b(t) \operatorname{sinc}(Wt - W\tau_i(t) - n)$$

Samples of $y_b(t)$ at rate $\frac{1}{W}$.

$$y[m] = y_b\left(\frac{m}{W}\right) = \sum_n x[n] \sum_i a_i^b\left(\frac{m}{W}\right) \operatorname{sinc}\left(m - n - \tau_i\left(\frac{m}{W}\right)W\right)$$

$$(l = m - n) \qquad = \sum_l x[m-l] \sum_i a_i^b\left(\frac{m}{W}\right) \operatorname{sinc}\left(l - W\tau_i\left(\frac{m}{W}\right)\right)$$

Define $h_l[m] = \sum_i a_i^b\left(\frac{m}{W}\right) \operatorname{sinc}\left(l - W\tau_i\left(\frac{m}{W}\right)\right)$

Now, $y[m] = \sum_l x[m-l]\, h_l[m]$.

$h_l[m]$: l^{th} (complex) filter tap at time m.
(value mainly depends on $a_i^b(t)$ of the paths whose delays $\tau_i(t)$ are close to $\frac{l}{W}$.)

* If $a_i^b(t)$ and $\tau_i(t)$ are time-invariant, $h_\ell[m]$ is independent of m &

$$h_\ell = \sum_i a_i^b \operatorname{sinc}[\ell - \tau_i W].$$

This can be thought of as $\sum_i a_i^b \delta(t-\tau_i)$ convolved with $\operatorname{sinc}(Wt)$ and sampled at $\frac{\ell}{W}$.

*

$$h_\ell[m] = \sum_i a_i^b\left(\frac{m}{W}\right) \operatorname{sinc}\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right).$$

Consider the i^{th} path. It contributes $a_i^b\left(\frac{m}{W}\right)\operatorname{sinc}\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right)$ to $h_\ell[m]$. The significant contribution is from the term where $\operatorname{sinc}\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right)$ is maximum, or $\ell - W\tau_i\left(\frac{m}{W}\right)$ is close to zero. or τ_i is close to $\frac{\ell}{W}$.

$\Rightarrow$ If $\tau_i\left(\frac{m}{W}\right)$ is in the window $\left[\frac{\ell}{W} - \frac{1}{2W}, \frac{\ell}{W} + \frac{1}{2W}\right]$

it contributes most significantly to the ℓ^{th} tap $(h_\ell[m])$ at time m.

* ℓ^{th} tap is the aggregation (mainly) of physical paths with delay in $\left[\frac{\ell}{W} - \frac{1}{2W}, \frac{\ell}{W} + \frac{1}{2W}\right]$.

Additive White Gaussian Noise:

* Include AWGN at the receiver.

$$y(t) = \sum_i a_i(t)\, x(t - \tau_i(t)) + \underset{\substack{\uparrow \\ \text{AWGN, PSD } \frac{N_0}{2}}}{w(t)}.$$

$$\Rightarrow y[m] = \sum_\ell h_\ell[m]\, x[m-\ell] + \underset{\substack{\uparrow \\ \text{Low pass filtered AWGN sampled at } \frac{m}{W}}}{w[m]}.$$

It can be easily verified that $\{w[m]\}$ is a discrete-time white noise process & circularly symmetric.

$\boxed{\text{Lecture 5.}}$ (11 Aug) – Repeat the aggregation of paths in each tap.

Time and Frequency Coherence:

(1) Doppler spread and coherence time.

$$h_\ell[m] = \sum_i a_i^b\left(\frac{m}{W}\right) \text{sinc}\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right)$$

$$= \sum_i a_i\left(\frac{m}{W}\right) e^{-j2\pi f_0 \tau_i\left(\frac{m}{W}\right)} \text{sinc}\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right).$$

* How fast does $h_\ell[m]$ change w.r.t. m?
Consider each term in the summation above.

- Significant changes in a_i occur over periods of seconds or more.
- Significant changes in phase of i^{th} path occur at intervals of $\frac{1}{4D_i}$ where $D_i = f_c \tau_i'(t)$ is the Doppler shift of the path.
- When different paths have different Dopper shifts, magnitude of $h_\ell[m]$ changes significantly. (In reflecting wall, moving antenna case, the Doppler shifts are $+\frac{fv}{c}$ and $-\frac{fv}{c}$).

  These changes in magnitude of $h_\ell[m]$ happen at a time-scale inversely proportional to the Doppler spread D_s:

$$D_s := \max_{i,j} f_c \left|\tau_i'(t) - \tau_j'(t)\right|.$$

  Typical intervals are of the order of 10 ms.

- The sinc term: For a path to move from one tap to next, the delay has to change of the order of $\frac{1}{W}$. This is much larger than the change required in the phase (of the order of $\frac{1}{f_c}$).

* Coherence time:

Inversely related to Doppler spread.

$$T_c = \frac{1}{4D_s} \text{ is one possible defn.}$$

For time-scales much smaller than T_c, the channel does not vary much.

Fast vs slow fading:

If $T_c \gg$ coding block duration, slow fading.

$T_c \ll$ $\underbrace{\text{coding block duration}}$, fast fading.

depends on delay constraint of the application.

(2) Delay spread and coherence bandwidth.

$$\text{Delay spread } T_d := \max_{i,j} |\tau_i(t) - \tau_j(t)|.$$

Fn. of t, but we are only interested in order of magnitude of T_d.

If f changes by $\frac{1}{2T_d}$, significant change in phase occurs.

(See example with reflecting wall earlier).

Coherence bandwidth:

Inversely related to Delay spread.

$$W_c = \frac{1}{2T_d} \text{ is one possible defn.}$$

Flat vs. Freq. selective fading:

If $W_c \gg$ input bandwidth, flat fading.

$W_c \ll$ input bandwidth, frequency selective fading.

Remarks:

— If a cell or LAN extends over a few kms or less, path lengths are unlikely to differ by more than 300 to 600 m. $\Rightarrow$ Delays are 1 to 2 μs.

— BW of cellular systems range from 100's of kHz to several MHz. Above multipath spreads are likely to be ~~between~~ within the peaks of 2 to 3 sinc functions or less (2–3 times $\frac{1}{W}$ or less).

$\Rightarrow$ channel filter is usually only a few taps (even accounting for the slow decay of the sinc fn.).

For UWB operating between 3.1 and 10.6 GHz, a few hundred taps are necessary.

— In practice, we need to estimate the taps. Receiver may not be able to explictly have or use information about individual path delays and strengths. Aggregate values of taps and parameters like Doppler spread and delay spread are usually enough to design.

Ex: Key channel parameters and time-scales. (Typical).

Carrier freq. 1 GHz (f_c)

Communication BW (W) 1 MHz.

Distance between tx. & rx (d) 1 km

Velocity of mobile (v) 64 km/hr

Doppler shift for a path ($D = f_c \frac{v}{c}$) 50 Hz

Doppler spread of paths corresponding to a tap (D_s) 100 Hz

Time-scale for change of

* path amplitude ($\frac{d}{v}$) 1 minute
* path phase ($\frac{1}{4D}$) 5 ms
* path to move a tap ($\frac{c}{vW}$) 20 s

Coherence time ($\frac{1}{4D_s}$) 2.5 ms

Delay spread T_d 1 μs

Coherence BW ($1/2T_d$) 500 kHz.

Recap: Continuous-time multipath fading channel model

$$y(t) = \sum_i a_i(t)\, x(t-\tau_i(t)) + w(t).$$

This contains an exact specification of the delay and magnitude of each path.

From this, we derived a discrete-time baseband model:

$$y[m] = \sum_\ell h_\ell[m]\, x[m-\ell] + w[m]$$

where $h_\ell[m]$, a channel tap, contains an aggregate of paths, with the delays smoothed out by the baseband signal bandwidth.

$$h_\ell[m] = \sum_i a_i\left(\frac{m}{W}\right) e^{-j2\pi f_c \tau_i\left(\frac{m}{W}\right)} \operatorname{sinc}\left[\ell - \tau_i\left(\frac{m}{W}\right) W\right].$$

$h_\ell[m]$ contains

- contributions from all paths ($\sum_i$)
- Significant contribution only from paths with delays in $\left[\frac{\ell}{W} - \frac{1}{2W}, \frac{\ell}{W} + \frac{1}{2W}\right]$.

* In practice, $h_\ell[m]$ needs to be measured.

* Above model is a deterministic model based on the specification of location, velocity, propagation environment for the mobile.

* We are interested in a charaterization valid for a range of conditions. We to have an idea of

- how many taps are necessary?
- how do these taps change with time?

~~Also~~ do

We will give a statistical characterization to do this. This may not be exact. However, it helps in analysis and provides insight into system design.