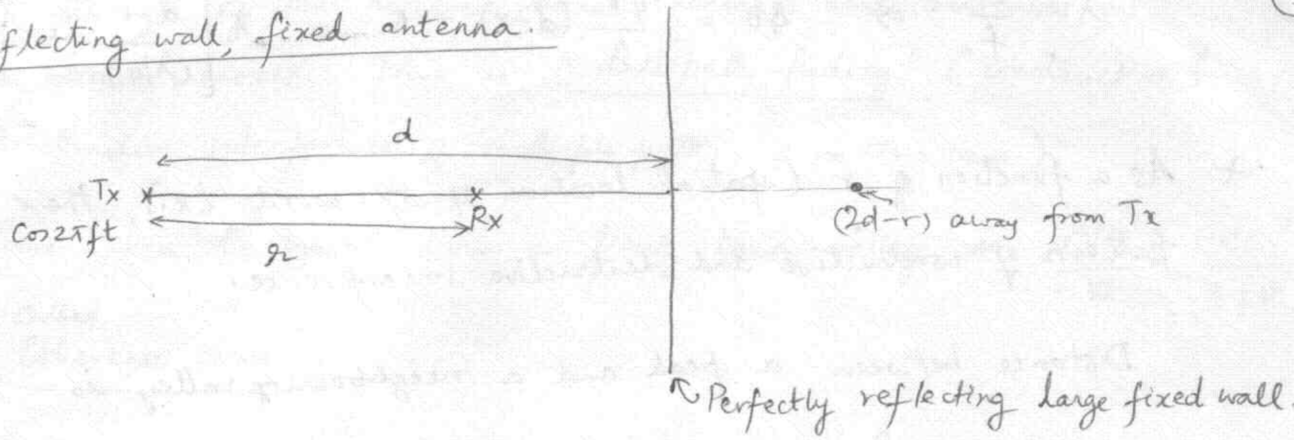
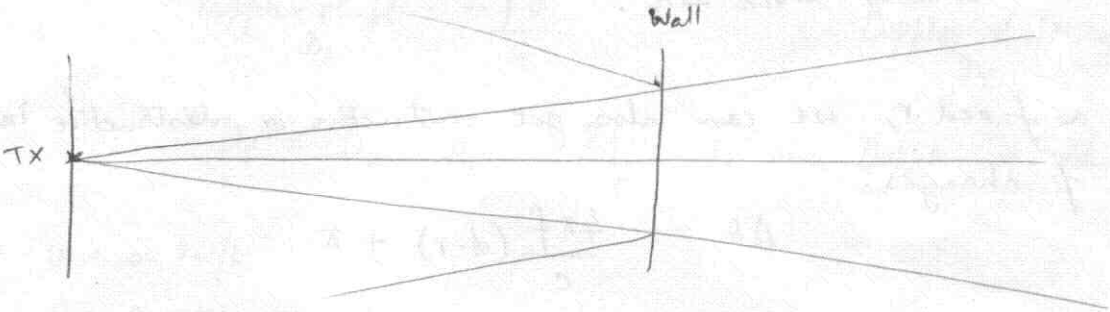
Lecture 2 (4 Aug 2008):

③ Reflecting wall, fixed antenna.



(Assume) Electromagnetic field at the point where the rx antenna will be placed is the sum of the free space field from the transmit antenna and the reflected wave coming from the wall.

(Approximate soln. of Maxwell's eqn. using "ray tracing": above assumption).



* If the wall is very large,

reflected wave at a given point = − (free space wave that would exist on the opposite side of the wall if the wall were not present).

* $$E_r(f,t) = \frac{\alpha \cos 2\pi f\left(t - \frac{r}{c}\right)}{r} - \frac{\alpha \cos 2\pi f\left(t - \frac{(2d-r)}{c}\right)}{2d-r}$$

(Superposition of 2 waves, both at freq. f, but with diff. phases)

* Phase of second term − Phase of first term $= \left(\frac{2\pi f(2d-r)}{c} + \pi\right) - \left(\frac{2\pi f r}{c}\right)$

$$= \frac{2\pi f(2d-2r)}{c} + \pi = \frac{4\pi f}{c}(d-r) + \pi$$

Phase difference $\Delta\theta = \frac{4\pi f}{c}(d-r) + \pi$

If $\Delta\theta = 2\pi n$, the two waves add constructively. (n integer).

$\Delta\theta = \pi(2n+1)$, the two waves add destructively.

$$\lambda = \frac{c}{f} \Rightarrow \quad \Delta\theta = \frac{4\pi}{\lambda}(d-r) + \pi \quad = \pi\left[\frac{d-r}{(\lambda/4)} + 1\right].$$

* As a function of r (spatial location of rx. w.r.t. tx.), there is a pattern of constructive and destructive interference.

  Distance between a peak and a neighbouring valley is

  $$\Delta x_c = \frac{\lambda}{4} \qquad \left(\text{If } (d-r) \text{ increases by } \frac{\lambda}{4},\ \Delta\theta \text{ increases by } \pi\right)$$

  Δx_c : coherence distance.

  The received signal does not change appreciably between distances much smaller than Δx_c.

* For a fixed r, we can also get constructive or destructive interference if f changes.

  $$\Delta\theta = \frac{4\pi f}{c}(d-r) + \pi$$

  if f changes by $\dfrac{1}{4\left[\frac{(d-r)}{c}\right]}$, we change from a peak to a valley.

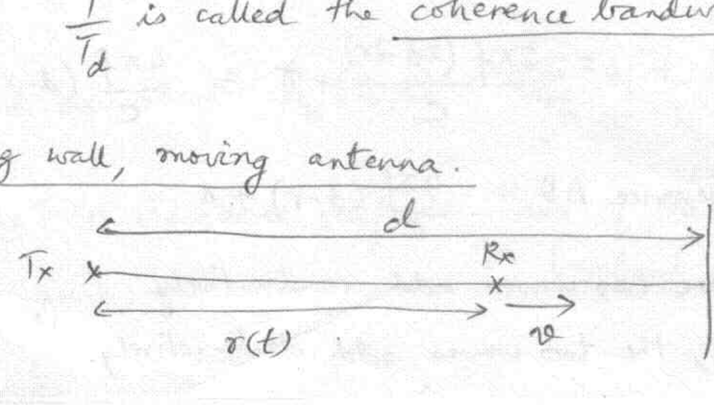
  $$\frac{1}{2}\,\frac{1}{\left[\frac{2d-r}{c} - \frac{r}{c}\right]}$$

  $\left(\frac{2d-r}{c} - \frac{r}{c}\right)$ is the delay spread T_d : difference between the propagation delays between the 2 paths.

  If f changes by an amount much smaller than $\frac{1}{T_d}$, then the signal does not change much.

  $\frac{1}{T_d}$ is called the coherence bandwidth.

(4) Reflecting wall, moving antenna.



* The strength of the received signal increases and decreases as the receiver moves. This is "multipath fading" (constructive & destructive interference of multiple paths, 2 in this case).

* Time taken to travel from a peak to a valley is $\frac{\lambda/4}{v} = \frac{c}{4fv}$.
(Called Coherence time)

*
$$E_r(f,t) = \frac{\alpha \cos 2\pi f\left(\left(1-\frac{v}{c}\right)t - \frac{r_0}{c}\right)}{r_0 + vt} - \frac{\alpha' \cos 2\pi f\left(\left(1+\frac{v}{c}\right)t + \frac{r_0 - 2d}{c}\right)}{2d - r_0 - vt}.$$

$\downarrow$ Sinusoid at $f\left(1-\frac{v}{c}\right)$

Doppler shift $D_1 = -\frac{fv}{c}$

$\downarrow$ Sinusoid at $f\left(1+\frac{v}{c}\right)$

Doppler shift $D_2 = +\frac{fv}{c}$.

$$D_s = D_2 - D_1 = \frac{2fv}{c}$$ is the Doppler spread.

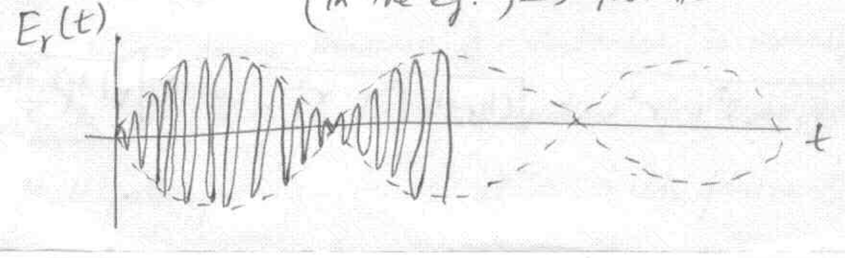
* Eg: $v = 60$ km/h
$f = 900$ MHz
$$D_s = 2 \times 900 \times 10^6 \times \frac{60 \times 10^3}{3600} \times \frac{1}{3 \times 10^8} \text{ Hz} = 100 \text{ Hz}.$$

* Visualization of the effect of Doppler spread.
- Let the rx. be closer to the wall. $\Rightarrow r_0 + vt \approx 2d - r_0 - vt$
$(r_0 + vt \approx d)$

$$E_r(f,t) \approx \frac{2\alpha \sin 2\pi f\left[\frac{vt}{c} + \frac{r_0 - d}{c}\right] \sin 2\pi f\left(t - \frac{d}{c}\right)}{r_0 + vt}.$$

Product of 2 sinusoids at freq. f and $\frac{fv}{c}$.

(in the eg.) $\rightarrow$ 900 MHz ; 50 Hz



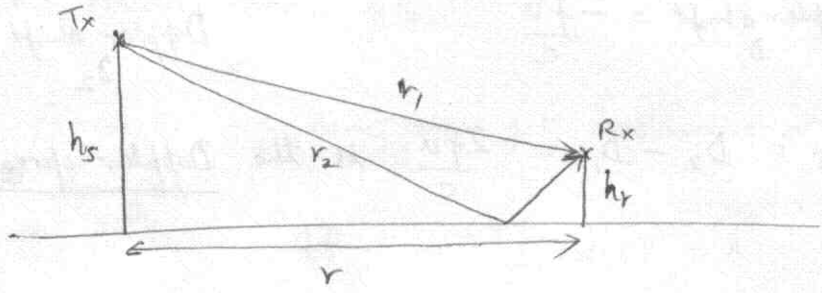
* Peaks become zeros every $\simeq 5$ ms.

When the difference in the length of the 2 paths changes by $\frac{\lambda}{4}$, there is a significant change in phase difference and overall received amplitude. This happens at short time-scales – order of milliseconds.

The effect of the time-varying attenuation in the denominator is seen only over durations of the order of seconds or minutes.

For modulation/detection, attenuation due to the denominator is not important.

(5) <u>Reflection from a ground plane.</u> $\boxed{\text{Lecture 3: (5 Aug 2008)}}$

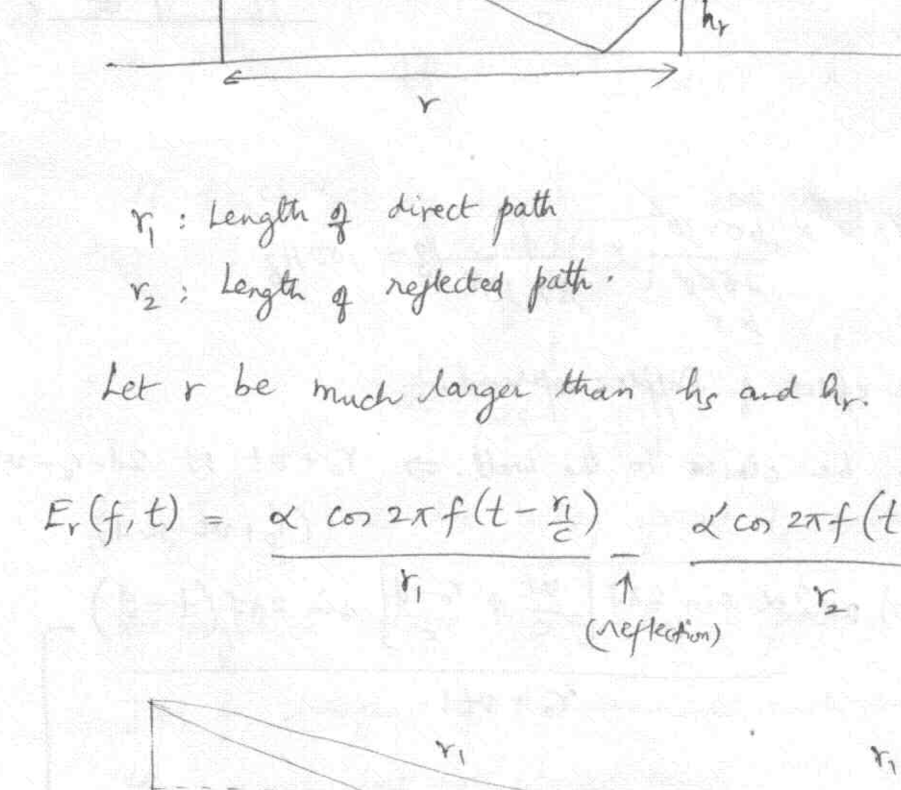


r_1 : Length of direct path

r_2 : Length of reflected path.

Let r be much larger than h_s and h_r.

$$E_r(f,t) = \frac{\alpha \cos 2\pi f\left(t - \frac{r_1}{c}\right)}{r_1} \underset{\substack{\uparrow \\ \text{(reflection)}}}{-} \frac{\alpha \cos 2\pi f\left(t - \frac{r_2}{c}\right)}{r_2}$$



$$r_1 = \sqrt{(h_s - h_r)^2 + r^2}$$

$$r_2 = \sqrt{(h_s + h_r)^2 + r^2}$$

$$r_2 - r_1 = \sqrt{(h_s+h_r)^2 + r^2} - \sqrt{(h_s-h_r)^2 + r^2} = r\left[\sqrt{1+\left(\frac{h_s+h_r}{r}\right)^2} - \sqrt{1+\left(\frac{h_s-h_r}{r}\right)^2}\right]$$

For small x, $\sqrt{1+x} \approx 1 + \frac{x}{2}$

$$r_2 - r_1 \approx r\left[1 + \frac{(h_s+h_r)^2}{2r^2} - 1 - \frac{(h_s-h_r)^2}{2r^2}\right]$$

$$= \frac{4h_sh_r}{2r} = \frac{2h_sh_r}{r}.$$

As r increases, $r_2 - r_1$ goes to zero as r^{-1} and becomes small compared to $\lambda = \frac{c}{f}$.

$$* \; E_r(f,t) \approx \frac{\alpha}{r_1}\left[\cos 2\pi f\left(t - \frac{r_1}{c}\right) - \cos 2\pi f\left(t - \frac{r_2}{c}\right)\right] \qquad (r_1 \approx r_2).$$

$$= -\frac{2\alpha}{r_1} \sin 2\pi f\left(t - \frac{r_1+r_2}{2c}\right) \sin 2\pi f\left(\frac{r_2-r_1}{2c}\right)$$

$$(\text{large } r) \approx -\frac{2\alpha}{r_1} \sin 2\pi f\left(t - \frac{r_1+r_2}{2c}\right) 2\pi f\left(\frac{r_2-r_1}{2c}\right)$$

($\sin\theta \approx \theta$ for small θ)
True if $r_2 - r_1 \ll \frac{c}{\pi f}$
Happens as r increases.

$E_r(f,t)$ goes down as r^{-2} $\left(\frac{1}{r_1} \text{ \& } \frac{1}{r} \text{ in } (r_2-r_1)\right)$

Power goes down as r^{-4}.

(In free space, power went down as r^{-2}).

\+ Power decay with distance (large-scale)

Free space: Power $\propto r^{-2}$

With obstacles: Power decays faster than r^{-2}.

Above 2-ray example: r^{-4}.

Exact approach difficult without detailed modeling of obstacles.

Simple decay models are used in practice.

This power decay because of obstacles is usually referred to as shadowing. Shadow fades last for multiple seconds or minutes.

Multipath fading occurs at a much faster time scale.

Moving antenna, multiple reflectors :

* Ray tracing is not simple.
* Linearity still holds.
  - Received waveform is a sum of responses from different paths.
* Another type of reflection : Scattering (e.g. reflections from rough objects)
  - Received waveform is better modeled as an integral (rather than a sum) over paths with infinitesimally small differences in their lengths.
*

Input-Output model of the wireless channel :

* So far, we focused on the response to sinusoidal inputs $\phi(t) = \cos 2\pi f t$. The received signal can be written as

$$\sum_i a_i(f,t)\,\phi(t-\tau_i(f,t)),$$

where $a_i(f,t)$: overall attenuation on path i.

$\tau_i(f,t)$: Propagation delay on path i.

Overall attenuation : Product of attenuation factors due to
- ant. pattern of tx.
- ant. pattern of rx.
- nature of the reflectors
- distance

* Assume $a_i(f,t)$ and $\tau_i(f,t)$ are independent of f.

Then, using superposition, we can generalize this to

$$(\text{output})\; y(t) = \sum_i a_i(t)\, x(t-\tau_i(t)) \quad \text{where } x(t) \text{ is an arbitrary input.} \qquad (A)$$

Remarks: (1) In practice, attenuations and propagation delays are slowly varying functions of frequency due to:

- Frequency-dependent antenna gains
- Time-varying path lengths

If we are interested in narrowband communication (narrow bandwidth compared to center frequency) this can be a reasonable assumption.

(2) Only individual path attenuations and delays are assumed to be independent of f. The overall response is still a fn. of frequency since different paths have different delays ("small-scale" fading discussed in example 4 above).

Here
$$a_1(t) = \frac{|\alpha|}{r_0 + vt} \; ; \; \tau_1(t) = \frac{r_0 + vt}{c} - \frac{\angle\phi_1}{2\pi f}$$

$$a_2(t) = \frac{|\alpha|}{2d - r_0 - vt} \; ; \; \tau_2(t) = \frac{2d - r_0 - vt}{c} - \frac{\angle\phi_2}{2\pi f}.$$

($\angle\phi_i$ is to account for phase changes at the tx., reflector, rx.)
For this example, $\phi_1 = 0$, $\phi_2 = \pi$.

* Eqn. A is a linear model for the channel. This can also be written as

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t)\, x(t-\tau)\, d\tau$$

where $h(\tau, t) = \sum_i a_i(t)\, \delta(\tau - \tau_i(t))$.

$h(\tau, t)$: response at time t due to an impulse input at $t-\tau$.

Thus, the channel can be thought of as a linear time-varying filter.

* For a slowly varying channel, we can interpret

$$H(f; t) = \int_{-\infty}^{\infty} h(t, \tau)\, e^{-j2\pi f\tau}\, d\tau = \sum_i a_i(t)\, e^{-j2\pi f \tau_i(t)}$$

as the frequency response at time t. This is useful if the time-scale at which the channel varies is much longer than the delay spread.

"Underspread" channels