EE5040: Adaptive Signal Processing
Problem Set 3: Linear least-mean-squares estimation

1. (Sayed II.13, Correlated component) Assume that a zero-mean random variable \( X \) consists of two components, \( X = X_c + Z \), and that only \( X_c \) is correlated with the observation vector \( Y \). Show that the linear least-mean-squares estimator of \( X \) given \( Y \) is simply the linear least-mean-squares estimator of \( X_c \) given \( Y \).

2. (Sayed II.8, Weighted error cost) Show that the linear least-mean-squares estimator of \( X \) given \( Y \), given by \( \hat{X} = K_0 Y \) where \( K_0 \) is any solution to the linear system of equations \( K_0 R_Y = R_{XY} \), also minimizes \( E[\hat{X}^H W \hat{X}] \) for any \( W \geq 0 \).

3. (Sayed II.5, Minimum of a quadratic form) Consider the quadratic cost function \( J(x) = (x - c)^H A (x - c) \) where \( A \) is a Hermitian nonnegative-definite matrix and \( x \) and \( c \) are column vectors. Argue that the minimum value of \( J(x) \) is zero and it is achieved at \( x = c + d \) for any \( d \) satisfying \( Ad = 0 \).