Coverage and Rate in Cellular Networks with Multi-User Spatial Multiplexing

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***Abstract*—In this paper we consider a multi-user spatial multiplexing (SM) cellular network, where N_t streams are transmitted to N_t users in the cell. Specifically, we obtain the coverage and rate expressions for a system employing zero-forcing (ZF) receiver. Compared to single stream transmission (SST), it is interesting to see that SM degrades the rate for a notable percentage of users. For the case of two and four receiver antennas, the increase in mean rate of SM is modest compared to single stream transmission (SST) while SST provides a gain over SM for cell edge users.**

I. Introduction

Multiple-input multiple-output (MIMO) communications are now an integral part of current cellular standards. MIMO is a mature technology and there is an extensive body of literature for various MIMO techniques [1], [2]. In this paper, we consider open-loop multi-user spatial multiplexing technique for cellular downlink. We analyse the coverage and average ergodic rate with a linear zero-forcing receiver *in the presence of interference from other cells*.

Spatial multiplexing has been extensively studied in the presence of additive Gaussian noise [3], [4], [5]. Fewer results exist that characterize the performance of SM with external interference [6]. However, interference is a performance limiting factor in current cellular networks, and hence it is important to study the performance of SM in the presence of co-channel interference. In this paper, we model the locations of the base stations (BSs) by a spatial Poisson point process (PPP) and consider distance dependent inter-cell interference. The PPP model was used for base station location modelling in [7] to analyse the coverage in cellular networks with one antenna.

PPP model was also used in [8], [9] and [10] to analyse spatial multiplexing with ZF in MIMO ad hoc networks. In ad hoc networks, an interferer can be arbitrarily close (much closer than the intended transmitter) to the receiver in consideration. This results in interference that is heavy-tailed. On the other hand, in a cellular network the user usually connects to the closest BS and hence the distance to the nearest interferer is greater than the distance to the serving BS. This leads to a more tamed interference distribution compared to ad hoc networks. Because of this the insights obtained in this paper differ from [8], [9], [10].

In this paper, we extend the framework in [7] to a cellular MIMO network. We derive general expressions for coverage probability, and ergodic rate in a cellular downlink with SM and a ZF receiver. We observe that single-stream transmission provides a higher rate compared to SM with increasing transmit antenna for cell edge users.

II. System Model

We model the locations of the base stations (BSs) by a spatial Poisson point process [11] Φ of density λ. The merits and demerits of this model for BS locations have been extensively discussed in [7]. We assume a nearest BS connectivity model, where in a mobile tries to establish a connection with its closest BS. This results in a Voronoi tessellation of the plane corresponding to the BS locations, where the service area of a BS is the Voronoi cell associated with it.

We assume that the BSs are equipped with N_t antenna and the users (UE) are equipped with N_r antenna. In this paper we focus on downlink and hence the N_t at the BSs are used for transmission and the N_r antenna at the UE are used for reception. For convenience, we assume $N_r = nN_t$ with $n \geq 1$. We assume that all the BSs transmit with equal power which for convenience we set to unity. Hence each transmit antenna uses a power of $1/N_t$.

We assume the standard pathloss model $\ell(x) = \|x\|^{-\alpha}$, $\alpha > 2$. Independent Rayleigh fading with unit mean is assumed between any pair of antenna. We focus on the downlink performance and hence without loss of generality, we consider and analyse the performance of a typical mobile user located at the origin. The $N_r \times 1$ fading vector between the q-th antenna of the BS $x \in \Phi$ and the typical mobile at the origin is denoted by $\mathbf{h}_{x,q}$. We assume $\mathbf{h}_{x,q} \sim \mathcal{CN}(\mathbf{0}_{N_r \times 1}, \mathbf{I}_{N_r})$.

We consider the case where each BS uses its N_t antenna to serve *independent* data streams to N_t users in its cell[1]. Let $\hat{o} \in \Phi$ denote the BS that is closest to the mobile user at the origin. We assume that the UE at the origin is interested in decoding the k-th stream transmitted by its associated BS $\hat{o}$. Focusing on the k-th stream transmitted by $\hat{o}$, the received

[1] We make the assumption that every cell has at least N_t users. This is true with high probability when there are large number of users which is normally the case.

$N_r \times 1$ signal vector at the typical mobile user is

$$\mathbf{y}_k = \frac{a_{\hat{o},k}}{\sqrt{r^\alpha}}\mathbf{h}_{\hat{o},k} + \frac{1}{\sqrt{r^\alpha}}\sum_{q=1,q\neq k}^{N_t}\mathbf{h}_{\hat{o},q}a_{\hat{o},q} + \mathbf{I}(\Phi) + \mathbf{w}, \quad (1)$$

where

$$\mathbf{I}(\Phi) = \sum_{x\in\Phi\setminus\hat{o}}\frac{1}{\sqrt{\|x\|^\alpha}}\sum_{q=1}^{N_t}\mathbf{h}_{x,q}a_{x,q},$$

denotes the interference from other BSs. The symbol transmitted from the the q-th antenna of the base station $x \in \Phi$ is denoted by $a_{x,q}$ and $\mathbb{E}[|a_{x,q}|^2] = 1/N_t$. The additive white Gaussian noise is given by $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}_{N_r\times 1}, \sigma^2\mathbf{I}_{N_r})$. The distance between the typical mobile user at the origin and its associated (closest) BS is denoted by $r = \|\hat{o}\|$. Observe that r is a random variable since the BS locations are random.

We now compute the post-processing $\mathtt{SINR}$ with a zero-forcing receiver. Each UE has $N_r = nN_t$, $n \geq 1$ receive antenna. Hence the receive antenna can be used to cancel the self-interference caused by the $N_t - 1$ streams and $n-1$ other interfering BSs. Technically, some of the $(n-1)N_t$ receive antenna can be used for diversity enhancement. However, in this paper we assume the $(n-1)N_t$ receive antenna are entirely used to cancel interference from other users. This requires the receiver to have some capability to estimate the channel of the closest $n-1$ interferers.

The receive filter $\mathbf{v}$ for the typical user at the origin is chosen orthogonal to the channel vectors of the transmitters that need to be cancelled out. We assume that the $n-1$ interferers closest to the UE are cancelled. Since the typical UE is interested in the stream k, $\mathbf{v}$ is chosen as a unit norm vector orthogonal to the following vectors:

$$\mathbf{h}_{\hat{o},q} : q = 1, 2, .., k-1, k+1, .., N_t,$$
$$\mathbf{h}_{x,q} : x \in \{x_1, x_2, ..., x_{n-1}\},\ q = 1, 2, .., N_t,$$

where $\{x_1, x_2, ..., x_{n-1}\}$ are the $(n-1)$ BSs closest to the typical UE in consideration excluding $\hat{o}$. Hence at the receiver,

$$\mathbf{v}^\dagger\mathbf{y}_k = \frac{a_{\hat{o},k}}{\sqrt{r^\alpha}}\mathbf{v}^\dagger\mathbf{h}_{\hat{o},k} + \sum_{q=1,q\neq k}^{N_t}\frac{a_{\hat{o},q}}{\sqrt{r^\alpha}}\mathbf{v}^\dagger\mathbf{h}_{\hat{o},q} + \mathbf{v}^\dagger\mathbf{I}(\Phi) + \mathbf{v}^\dagger\mathbf{w}.$$

Since $\mathbf{v}^\dagger$ is designed to null the closest $n-1$ interferers, $\mathbf{v}^\dagger\mathbf{I}(\Phi) = \mathbf{v}^\dagger\mathbf{I}(\hat{\Phi})$ where $\hat{\Phi} = \Phi \setminus \{x_1, ..., x_{n-1}\}$. So we have

$$\tilde{y}_k = \frac{a_{\hat{o},k}}{\sqrt{r^\alpha}}\mathbf{v}^\dagger\mathbf{h}_{\hat{o},k} + \mathbf{v}^\dagger\mathbf{I}(\hat{\Phi}) + \mathbf{v}^\dagger\mathbf{w}.$$

Let $S \triangleq |\mathbf{v}^\dagger\mathbf{h}_{\hat{o},k}|^2$ and $H_{x,q} \triangleq |\mathbf{v}^\dagger\mathbf{h}_{x,q}|^2$. It can be shown that S and $H_{x,q}$ are i.i.d. exponential random variables. Hence the post processing zero-forcing signal-to-interference-noise ratio ($\mathtt{SINR}$) is

$$\mathtt{SINR} = \frac{Sr^{-\alpha}}{N_t\sigma^2 + \underbrace{\sum_{x\in\hat{\Phi}}\|x\|^{-\alpha}\sum_{q=1}^{N_t}H_{x,q}}_{\hat{\mathbf{I}}(\hat{\Phi})}}. \quad (2)$$

III. Coverage

In this section, we analyze the coverage using the ZF receiver described above. A mobile user is said to be in coverage if the received $\mathtt{SINR}$ is greater than the threshold needed to establish the connection. The probability of coverage is denoted by $\mathtt{P}_c(T, \alpha)$ and is given by

$$\mathtt{P}_c(T, \alpha) \triangleq \mathbb{P}[\mathtt{SINR} > T]. \quad (3)$$

From the above expression we see that coverage probability is the CCDF of the $\mathtt{SINR}$. The ZF receiver is designed such that it can cancel interference from $(n-1)$ BSs apart from the same cell interference. So we first compute the distribution of distance between the typical user and the $(n-1)^{th}$ BS which will be used in the analysis later.

A. Distance to the serving BS and $(n-1)$-th BS.

Recall that r denotes the distance to the serving (nearest) BS. We have

$$\mathbb{F}_r(r_0) = \mathbb{P}[r > r_0] = \mathbb{P}[B(o, r_0) \text{ is empty }], = e^{-\lambda\pi r_0^2},$$

where $B(o, r)$ represents a ball of radius r around the origin. Hence the nearest neighbour PDF is

$$f_r(r) = e^{-\lambda\pi r^2}2\pi\lambda r. \quad (4)$$

We now compute the distance to the $(n-1)$-th closest BS conditioned on the distance to the nearest BS r. Let R denote the distance to the $n-1$-th BS. Hence the event $R \leq R_0$ equals the event that there are at least $n-1$ base stations in the region between two concentric circles of radius r and R centred at origin. Hence

$$F_{R|r}(R_0 \mid r_0) = \mathbb{P}[R \leq R_0 | r = r_0] = \sum_{k=n-1}^{\infty} e^{-\pi\lambda(R_0^2 - r_0^2)}\frac{[\lambda\pi(R_0^2 - r_0^2)]^k}{k!},\ R_0 > r_0.$$

Hence the conditional PDF is

$$f_{R|r}(R|r) = \frac{\mathrm{d}}{\mathrm{d}R}F_{R|r}(R|r) = \frac{2\pi\lambda R}{(n-2)!}e^{-\pi\lambda(R^2-r^2)}\left(\pi\lambda(R^2 - r^2)\right)^{n-2}.$$

B. Coverage Probability

We first provide the main result which deals with the coverage probability for a general N_t, $n > 1$ and α.

Theorem 1. *The probability of coverage with ZF receiver is given by*

$$\mathtt{P}_c(T, \alpha) = \int_0^\infty\int_r^\infty e^{-TN_t\sigma^2 r^\alpha}\mathcal{L}_{\mathbf{I}_R}(Tr^\alpha)f_{R|r}(R|r)f_r(r)\mathrm{d}R\mathrm{d}r,$$

where $\mathcal{L}_{\mathbf{I}_R}(s)$ the conditional Laplace transform of the interference and is given in (5).

Proof: The proof closely follows the main Theorem in [7]. We only highlight the steps that differ significantly. We have

$$\mathtt{P}_c(T, \alpha) = \int_0^\infty\int_r^\infty e^{-TN_t\sigma^2 r^\alpha}\mathcal{L}_R(Tr^\alpha)f_{R|r}(R|r)f_r(r)\mathrm{d}R\mathrm{d}r,$$

where $\mathcal{L}_{\mathbf{I}_R}$ is the Laplace transform of the interference conditioned on R.

$$\mathcal{L}_{\mathbf{I}_R}(s) = \mathbb{E}\left[e^{-s\hat{\mathbf{I}}(\hat{\Phi})}\right] = \mathbb{E}\exp\left(-s\sum_{x\in\hat{\Phi}}\|x\|^{-\alpha}\sum_{q=1}^{N_t}H_{x,q}\right).$$

Since $H_{x,q}$ are i.i.d exponential, their sum $\sum_{q=1}^{N_t}H_{x,q}$ is gamma distributed. Using the Laplace transform of the gamma distribution,

$$\begin{aligned}
&\mathcal{L}_{\mathbf{I}_R}(s)\\
&=\mathbb{E}\prod_{x\in\hat{\Phi}}\mathbb{E}\exp\left(-s\|x\|^{-\alpha}\sum_{q=1}^{N_t}H_{x,q}\right),\\
&=\mathbb{E}\prod_{x\in\hat{\Phi}}\frac{1}{(1+s\|x\|^{-\alpha})^{N_t}},\\
&\stackrel{(a)}{=}\exp\left(-\lambda 2\pi\int_R^\infty\left(1-\frac{1}{(1+sx^{-\alpha})^{N_t}}\right)x\mathrm{d}x\right),\\
&=\exp\left(-\lambda\pi R^2\left({}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};-R^{-\alpha}s\right)-1\right)\right).
\end{aligned} \tag{5}$$

where (a) follows from the probability generating functional (PGFL) of the PPP [11]. ${}_2F_1(a,b,c,z)$ is the standard hypergeometric function[2]. ■

C. Special case: Interference limited $\sigma^2=0$.

In this section we focus on the coverage probability for particular values of n, N_t and α in the absence of noise. We begin with the $n=1$ case.

1) Case $n=1$: When $n=1$, $N_t=N_r$ and hence only the self interference can be cancelled. In this case $R=r$ and the integration with respect to R will not be necessary. We have

$$\mathcal{L}_R(Tr^\alpha)=e^{-\lambda\pi r^2\left({}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};-T\right)-1\right)}.$$

Substituting for $f_r(r)$, the coverage probability reduces to

$$\begin{aligned}
\mathtt{P}_c(T,\alpha)&=\int_0^\infty e^{-\lambda\pi r^2\left({}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};-T\right)\right)}2\pi r\lambda\mathrm{d}r,\\
&={}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};-T\right)^{-1}.
\end{aligned}$$

The coverage probability can be further simplified when $\alpha=4$ and the coverage results are provided in Table I. When noise is neglected, we observe that the coverage probability does not depend on the density of BSs. The coverage probability is plotted[3]. for different N_t in Figure 1 as a function of the $\mathtt{SINR}$ threshold T.

[2] ${}_2F_1(a,b,c,z)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1\frac{t^{b-1}(1-t)^{c-b-1}}{(1-tz)^a}\mathrm{d}t.$

[3] Figure 1 and Figure 2 were generated in about 5 seconds each in Mathematica on a standard Dell desktop. This is a very short time compared to the time taken if the curves were to be obtained by Monte-Carlo simulation of the entire system.

N_t	Coverage probability $P_c(T,4)$
2	$2\left(\frac{T}{T+1}+3\sqrt{T}\tan^{-1}\left(\sqrt{T}\right)\right)^{-1}$
3	$4\left(\frac{T(7T+9)}{(T+1)^2}+15\sqrt{T}\tan^{-1}\left(\sqrt{T}\right)\right)^{-1}$
4	$2\left(\frac{T(T(57T+136)+87)}{24(T+1)^3}+\frac{35}{8}\sqrt{T}\tan^{-1}\left(\sqrt{T}\right)\right)^{-1}$

TABLE I: Coverage probability for $\alpha=4$ and $\sigma^2=0$ for different N_t. Since $n=1$, we have $N_r=N_t$.

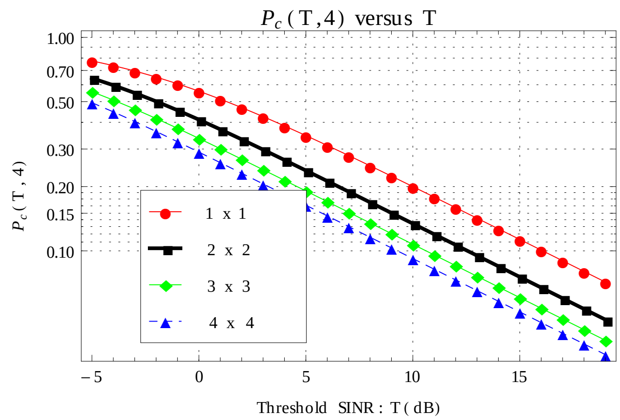

Fig. 1: Coverage probability versus T for 1×1, 2×2, 3×3, 4×4 antenna configurations with $\sigma^2=0$ and $\alpha=4$.

2) Case $n\geq 2$: Here $N_r=nN_t$, so the interference from $n-1$ BSs can be cancelled. Setting $\sigma^2=0$ and substituting for $f_r(r)$ and $f_{R|r}(R|r)$ in Theorem 1, the coverage is

$$\begin{aligned}
\mathtt{P}_c(T,\alpha)=&\int_0^\infty\int_r^\infty e^{-\lambda\pi R^2\left({}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};-T\frac{r^\alpha}{R^\alpha}\right)\right)}\\
&\cdot\frac{4(\pi\lambda)^n}{(n-2)!}(R^2-r^2)^{n-2}Rr\mathrm{d}R\mathrm{d}r.
\end{aligned}$$

Using the transformation $R/r\to\beta$ and $r\to t$ (which implies $\beta>1$), using the Jacobian for change of variables we obtain

$$\begin{aligned}
\mathtt{P}_c(T,\alpha)=&\frac{4(\pi\lambda)^n}{(n-2)!}\int_0^\infty\int_1^\infty e^{-\lambda\pi t^2\beta^2\left({}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};-T\beta^{-\alpha}\right)\right)}\\
&\cdot t^{2n-1}\beta(\beta^2-1)^{n-2}\mathrm{d}\beta\mathrm{d}t.
\end{aligned}$$

Exchanging the integrals and integrating with respect to t, the probability of coverage is

$$\mathtt{P}_c(T,\alpha)=\int_1^\infty\frac{2(n-1)\beta^{1-2n}(\beta^2-1)^{n-2}}{{}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};-T\beta^{-\alpha}\right)^n}\mathrm{d}\beta. \tag{6}$$

We see that the coverage probability can be evaluated using a single integral. Let $\beta=R/r$ denote the ratio of the distance of the $n-1$ th closest BS of the typical UE to the distance of its closest BS. It can be shown that the PDF of the random variable β is

$$g_\beta(\beta)=2(n-1)\beta^{1-2n}(\beta^2-1)^{n-2},\ \ \beta>1.$$

Hence from (6), the coverage probability for $n>1$ also equals

$$\mathtt{P}_c(T,\alpha)=\mathbb{E}_\beta\left[{}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};-T\beta^{-\alpha}\right)^{-n}\right]. \tag{7}$$

The ratio of the distance to the serving BS plays to the closest interferer plays a crucial role in determining the coverage. The average value of β is given by

$$\mathbb{E}[\beta] = \frac{\sqrt{\pi}\Gamma(n)}{\Gamma(n-1/2)} \approx \sqrt{(n-1)\pi}.$$

In Figure 2, the coverage probability given by (6) is plotted

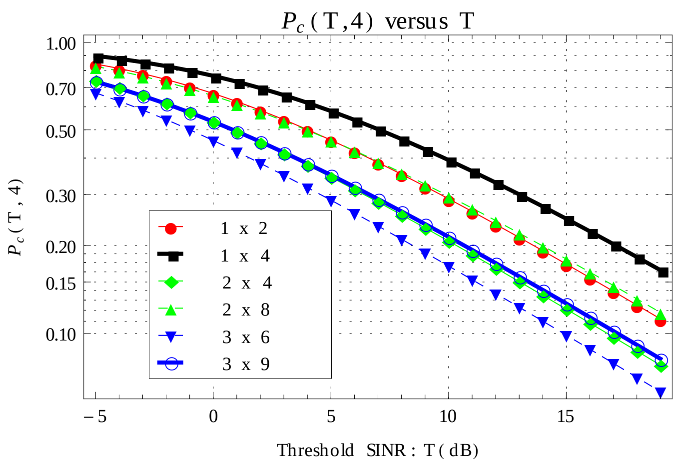

Fig. 2: Coverage probability versus T for 1×2, 1×4, 2×4, 2×8, 3×6, 3×9 antenna configurations with $\sigma^2 = 0$ and $\alpha = 4$.

for various antenna configurations. While a single interferer is cancelled in $1\times2, 2\times4, 3\times6$ configurations, we observe that 1×2 has the best performance followed by 2×4 and 3×6. This is because of the increased aggregate interference across the antenna as N_r increases. We also observe that cancelling more interferers increases the coverage. Also we can see that 2×8 has a similar performance to 1×2, even though three interferers are cancelled in 2×8 compared to a single interferer in 1×2. Similar observation can be made for 2×4 and 3×9 configurations.

IV. Average ergodic rate

In this section, we compute the ergodic data rate achievable over a cell for a given user and the rate CDF, assuming N_t users are served by the BS in a cell. For computing the rate, we consider the interference as noise. We also assume that the modulation and coding is chosen so that they achieve Shannon bound $\log_2(1+\mathtt{SINR})$, by treating residual interference as noise.

A. Average Achievable Rate per user

We begin by the theorem to find the ergodic capacity of typical mobile user and also consider some special cases of importance.

Theorem 2. *The average ergodic rate of a typical mobile user and its associated BS in the downlink, in bits/sec/Hz, is given by*

$$\mathcal{C}(\lambda,\alpha,N_t,n) \stackrel{\triangle}{=} \mathbb{E}\left[\log_2(1+\mathtt{SINR})\right] = \int_0^\infty \mathrm{P}_c(2^t-1,\alpha)\mathrm{d}t. \quad (8)$$

Proof: The proof follows from the CDF of the positive random variable $\log_2(1+\mathtt{SINR})$. ■

We now will discuss some special cases of determining average achievable rate as in coverage analysis. When $n=1$ and $\sigma^2=0$, the averge rate is

$$\mathcal{C}(\lambda,\alpha,N_t,1) = \int_0^\infty {}_2F_1\left(N_t, -\frac{2}{\alpha}; \frac{\alpha-2}{\alpha}; 1-2^t\right)^{-1}\mathrm{d}t,$$

which can be simplified when $\alpha=4$. For $n>1$, the average rate with $\sigma^2=0$ is

$$\mathcal{C}(\lambda,\alpha,N_t,n) = \int_0^\infty\int_1^\infty \frac{2(n-1)\beta^{1-2n}(\beta^2-1)^{n-2}}{{}_2F_1\left(N_t,-\frac{2}{\alpha};\frac{\alpha-2}{\alpha};(1-2^t)\beta^{-\alpha}\right)^n}\mathrm{d}\beta\mathrm{d}t.$$

B. Comparison with single stream transmission

In our system model, we considered a multi-user spatial multiplexing where N_t streams are transmitted to the N_t users in the cell. In this sub-section we want to compare this with a single-stream transmission (SST). In SST, the BS has only one antenna *i.e.*, $N_t=1$. Hence it can serve only one stream and hence one user. So all the users are served by dividing the resources either in time (TDMA) or frequency (FDMA). Hence in this case, each user has $1/N_t$ time or frequency slice.

Total rate with SM: In SM each user decodes a single stream and hence achieves an ergodic rate $\mathcal{C}(\lambda,\alpha,N_t,n)$, $N_t>1$. *Hence for* N_t *users*, the rate CDF is given by

$$F_{SM}(c) = \mathbb{P}(N_t\log_2(1+\mathtt{SINR}(N_t,n)) \le c), \quad (9)$$

where $\mathtt{SINR}(N_t,n)$ denotes the $\mathtt{SINR}$ with N_t transmit and nN_t receive antenna. The above distribution can be easily computed from the $\mathtt{SINR}$ CCDF in Theorem 1. It is easy to see that the total average downlink rate is given by

$$\mathcal{C}_{SM} = N_t\mathcal{C}(\lambda,\alpha,N_t,n).$$

Total rate with SST: In SST, since the resources have to be divided among the users, each user achieves an average rate $N_t^{-1}\mathcal{C}(\lambda,\alpha,1,n)$. *Hence for* N_t *users* the average total downlink rate achieved is

$$\mathcal{C}_{SST} = \mathcal{C}(\lambda,\alpha,1,n).$$

The rate CDF is given by

$$F_{SST}(c) = \mathbb{P}(\log_2(1+\mathtt{SINR}(1,n)) \le c),$$

where $\mathtt{SINR}(N_t,n)$ denoted the $\mathtt{SINR}$ with 1 transmit and n receive antenna. The above distribution can be easily computed from the $\mathtt{SINR}$ CCDF in Theorem 1.

The average ergodic rates achievable for various configurations is shown in the Table II. We make the following observations.

1) $\mathcal{C}(\lambda,\alpha,N_t,n)$ increases as a function of N_t. So adding more transmit antennas at BSs improves the mean rate, but the rate profile obtained from the rate CDF gives us more insight.
2) The mean rate for 1×4 is 2.78 while it is 4.3 for 2×4 and 4.6 for 4×4. Hence the returns are diminishing with

Two users the cell

$N_t \times N_r$	SM/SST	Mean	5%	50%	80%
1×2	SST	2.79	0.113	1.82	4.59
2×2	SM	3.16	0.076	1.42	4.93

Four users in the cell

$N_t \times N_r$	SM/SST	Mean	5%	50%	80%
1×4	SST	3.5	0.187	2.61	5.72
2×4	SM	4.31	0.115	2.23	7.30
4×4	SM	4.6	0.076	1.55	6.57

Six users in the cell

$N_t \times N_r$	SM/SST	Mean
1×6	SST	3.9
2×6	SM	5.05
3×6	SM	5.51
6×6	SM	5.74

TABLE II: Rate profile for various configurations. The rates are in bits/sec/Hz and are computed for $\sigma^2 = 0$ and $\alpha = 4$. For a general $N_t \times nN_t$ system with k users, the average sum rate for comparison is $\frac{N_t}{k}\mathcal{C}(\lambda, \alpha, N_t, n)k = N_t\mathcal{C}(\lambda, \alpha, N_t, n)$. This follows from the fact that the users are divided into k/N_t group and resources are divided between them. Each group is served using SM of N_t streams.

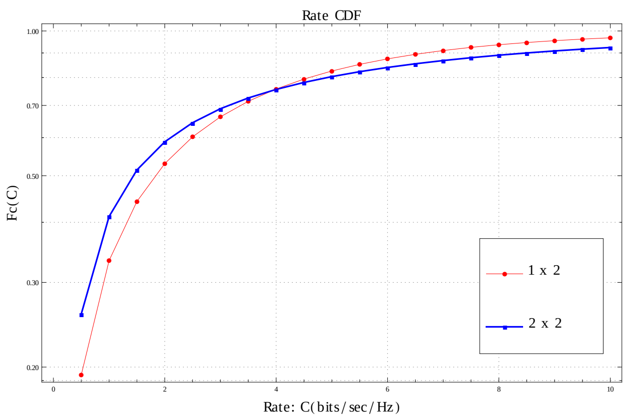

Fig. 3: Rate CDF of 1×2 and 2×2 with $\lambda = 1$, $\alpha = 4$and $\sigma^2 = 0$.

increasing N_t. Similar observation can be made for the $k \times 6$ for $k = 1, 2, 3, 6$.

3) Increasing N_t and hence increasing the number of streams in SM degrades the network performance. It can be seen that the 5 percentile rate of the network is better for 1×2 and the 1×4 cases. The same is true for the case of 50 percentile point too. In both cases as N_t is increasing, the rate is highly reduced. But for 80 percentile $N_t = 2$ and 4 are better, but the rates are comparable. This implies increasing N_t and using the multiple transmit antenna for transmitting more streams will hurt the cell edge users.

In Figures 3 and 4, the CDFs of the rate are plotted for various configurations. The rate profile tells that increasing N_t not only provide lesser increase in mean rate but also degrades the performance of the network. From the rate CDF it is interesting to see that SM with ZF receivers degrades the rate for more than 60 percentage of in the presence of other

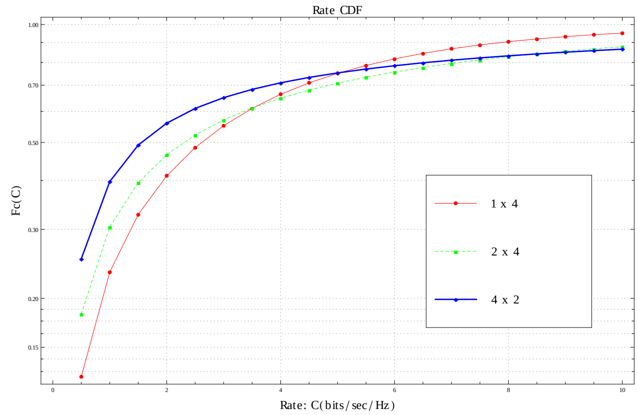

Fig. 4: Rate CDF of 1×4, 2×4 and 4×4 $\lambda = 1$, $\alpha = 4$and $\sigma^2 = 0$.

cell interference. This might be because of using ZF which is a suboptimal receiver. It will be interesting to analyse the performance with MMSE receiver.

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