Image recovery under nonlinear and non-Gaussian degradations

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A new two-dimensional recursive filter for recovering degraded images is proposed that is based on particle-filter theory. The main contribution of this work lies in evolving a framework that has the potential to recover images suffering from a general class of degradations such as system nonlinearity and non-Gaussian observation noise. Samples of the prior probability distribution of the original image are obtained by propagating the samples through an appropriate state model. Given the measurement model and the degraded image, the weights of the samples are computed. The samples and their corresponding weights are used to calculate the conditional mean that yields an estimate of the original image. The proposed method is validated by demonstrating its effectiveness in recovering images degraded by film-grain noise. Synthetic as well as real examples are considered for this purpose. Performance is also compared with that of an existing scheme.

1. INTRODUCTION

The goal of image recovery is to recover an image from a distorted version of itself. This typically involves modeling the degradations and applying an inverse procedure to obtain the original image. Several techniques exist in the literature to handle linear degradation. These include mainly the inverse filter, the classical Wiener filter, and the constrained least-squares filter. Iterative and recursive approaches also exist. Recursive estimation techniques are quite popular, as they permit spatial adaptivity to be easily incorporated into the image model. Also, the computational load in recursive filtering is less than that of iterative methods. The extension of the Kalman filter, which is a recursive filter in two dimensions, and its application to image recovery have received a great deal of attention. The two-dimensional (2-D) scalar Kalman filter and its computationally efficient versions, namely, the reduced-update Kalman filter and the reduced-order-model Kalman filter, have been found to be very useful in recovering images degraded by additive white Gaussian noise.

The degradation model has been commonly assumed to be linear and the noise additive Gaussian. However, in its most general form, image distortion is nonlinear, and the noise can be either nonadditive and/or non-Gaussian. For example, the imperfections in recording an original scene arise from the nonlinear input–output characteristics of most imaging sensors and scanners. The more complex problem of image estimation under nonlinear/non-Gaussian conditions has received much less attention than the linear case. Among the various possible choices for the estimator, the conditional mean that is a minimum-mean-square estimator is known to be very good for estimating the original image from the given observation. Under linear and Gaussian conditions, the conditional mean reduces neatly to a linear function of the observation. The case of nonlinear/non-Gaussian estimation is considerably more complex, as the conditional mean is typically a nonlinear estimator, and a closed-form solution is usually elusive.

In recent years, interesting work has been going on in the area of one-dimensional (1-D) particle filters to address nonlinear/non-Gaussian problems. Particle filters are attractive owing to their simple form and generality. They offer an arbitrarily good approximation given enough computing power. Techniques such as the extended Kalman filter, approximate grid-based methods, and Gaussian sum approximation have been developed for recursive nonlinear estimation problems in one dimension, but their performance has been shown to be clearly inferior to that of the particle filter. Particle filters are being applied increasingly to a large number of interesting problems in the areas of computer vision, communications, control, and neural networks, among others.

The Kalman filter is known to be optimal only for linear/Gaussian Bayesian situations. The use of the 2-D Kalman filter constrains both the image-formation model and the observation model to be linear and driven by additive white Gaussian noise. The extended Kalman filter involves linearization of the state, the measurement equation, or both, which may result in a distortion of the true underlying behavior and can lead to filter divergence. In this paper we propose and develop a general 2-D recursive filtering framework for image recovery that is based on the particle filter. The proposed method has the potential to handle general types of image degradation such as those due to sensor nonlinearity and non-Gaussian noise (not necessarily additive). The original image is modeled as an autoregressive (AR) process with a nonsymmetric half-plane (NSHP) support. The degradation can, in general, be nonlinear/non-Gaussian.
The problem of estimating the original image is posed as a dynamic state-estimation problem within a recursive Bayesian framework. By use of the state model and the measurement model, the posterior state density is represented by a set of samples or "particles" with corresponding weights. The conditional mean of the posterior is computed from these samples and their weights to yield an estimate of the original image. The 2-D Kalman filter5 and the proposed particle-filter-based approach are filters and not smoothers, as the estimate of the state at a particular pixel location is based only on the measurements that have been processed up to that location. In that sense, these approaches are approximate.

To demonstrate the effectiveness of the proposed method, we consider the recovery of scanned photographic images degraded by film-grain noise.19–21 This is an important practical example of sensor nonlinearity, which also manifests itself as multiplicative non-Gaussian noise. Both synthetic and real cases are considered, and the performance of the proposed method is found to be quite good. Important issues such as the choice of the state vector, the resampling strategy, and the boundary values are discussed. The performance of our algorithm is also compared with that of the modified linear Wiener (MLW) filter proposed by Tekalp and Pavlovic.21 The MLW filter is optimal among the class of linear filters for handling film-grain noise.

The material in this paper is presented as follows. In Section 2, we discuss image and degradation models. Section 3 formulates a particle-filter-based framework for image recovery. Section 4 describes in detail the image-estimation process. Experimental results are given in Section 5, and Section 6 concludes the paper.

2. IMAGE AND DEGRADATION MODELS

A typical 2-D $N \times N$ field $F$ can be partitioned into the boundary field $B$ and the image field $I$ as shown in Fig. 1(a). Most image-recovery methods make use of some a priori knowledge about the structure of the original image. One way of incorporating a priori knowledge is to model the original image by a linear AR process. The AR model has been used successfully to model images in many previous studies.5,6,12

For an NSHP support, a pixel at $(m, n)$ can be predicted as

$$s(m, n) = \sum_{(i,j) \in \Gamma} c_{i,j} s(m-i, n-j) + \left(1 - \sum_{(i,j) \in \Gamma} c_{i,j}\right) \bar{s} + w(m, n), \quad (1)$$

where $s(m, n)$ is the image intensity at location $(m, n)$, $\Gamma$ is an NSHP model support of order $M$ at the location $(m, n)$ and is defined as

$$\Gamma = \{(m-i, n-j)|(M \leq i \leq M, 1 \leq j \leq M) \cup (0, 1 \leq j \leq M)\},$$

the scalars $c_{i,j}$ are the AR model coefficients (assumed stationary) and computed from a prototype image by using the Yule–Walker equations.22 $\bar{s}$ is the average intensity of the prototype image, and $w(m, n)$ is uncorrelated zero-mean white Gaussian noise driving the AR process. The term $w(m, n)$ can also be regarded as the modeling error between the image and its predicted value.

Since image-recording systems are never perfect, both deterministic and stochastic degradations occur. Most studies assume a linear degradation model for the deterministic degradation. However, imaging sensors and scanners typically have a nonlinear response.1,9 Scanner noise is a form of stochastic degradation introduced during the recording mechanism. Film-grain noise is a photochemical noise that affects photographic films. In photoelectronic systems, the major sources of noise are thermal noise and amplifier electronic noise.

The observation model relating the degraded output and the original image can be written as

$$d(m, n) = g(s(m, n)) \circ v(m, n), \quad (2)$$

where $d(m, n)$ is the intensity of the degraded image at location $(m, n)$, $g(\cdot, \cdot, \cdot)$ is a function that captures the nonlinear behavior of the imaging sensor, $s(m, n)$ is the original image, and $v(m, n)$ is the observation noise. The operator $\circ$ is usually additive or multiplicative.

Blurring, which is a linear form of degradation, is not considered here.

### A. Film-Grain Noise

In this subsection we discuss image degradation due to film-grain noise. When a photographic film is used as the image-recording medium, there is a well-known nonlinear relationship between the incoming light intensity...
The nonlinear function \( g(\cdot, \cdot) \) is approximated by a logarithmic function in regions where the film response is linear with respect to the logarithm of the incident intensity. This relationship is provided by the manufacturer in the form of an optical density. The \( D \) curve is given by \( D(E) = a \log_{10}(E) + \beta \), where \( a \) represents the slope of the linear region and \( \beta \) is the density offset of the extrapolated region. Film-grain noise can be approximated as additive white Gaussian in the density domain, but the variance of the noise is not a constant and varies with the mean value of the signal. Thus the resulting noise consists of a signal-dependent as well as a signal-independent part.

Following Eq. (2), the degraded image \( d(m, n) \) recorded in the density domain can be expressed as

\[
d(m, n) = a \log_{10}[s(m, n)] + \beta + v(m, n),
\]

where \( v(m, n) \) is white Gaussian noise, statistically independent of the image. The degraded image can be alternatively represented as

\[
s(m, n) = [s(m, n), s(m - 1, n), \cdots, s(1, n), s(N, n - 1), s(N - 1, n - 1), \cdots, s(1, n - 1), s(N - M + 1), s(N - 1, n - M + 1), \cdots, s(1, n - M + 1), s(N - M), s(N - 1, n - M), \cdots, s(m - M, n - M), s(M + 1), s(M - 1, n + 1), \cdots, s(1, n + 1), s(N, n), s(N - 1, n), \cdots, s(N - M + 1, n), s(N, N), s(N - 1, N), \cdots, s(N - M + 1, N)]^T.
\]

where \( e(m, n) \) is the degraded image in the exposure domain and \( v(s(m, n)) = 10^{\frac{v(m, n)}{a}} \) is the noise that multiplies the signal in the exposure domain. Note that even though \( v(m, n) \) is Gaussian, \( v(s(m, n)) \) is not. From Eqs. (3) and (4) we observe that the sensor nonlinearity manifests itself as multiplicative non-Gaussian noise in the exposure domain. Interestingly, the observation model can be viewed as being nonlinear with additive Gaussian noise (in the density domain) or as linear with multiplicative non-Gaussian noise (in the exposure domain).

### 3. Development of the Two-Dimensional Particle Filter

In this section we propose a general recursive framework for image recovery by extending the basic idea behind the 1-D particle filter to two dimensions. Our aim is to estimate the original image field \( I \) from a degraded image field \( I_d \).

#### A. State-Space Representation

We raster scan the image field \( I \), i.e., scan from top to bottom, advance one column, then repeat. The recursive framework requires that an appropriate state vector and a measurement vector be conceptualized. The state model, which is Markov, describes the evolution of the state and is given by

\[
s(m, n) = f_{m-1,n}(s(m - 1, n), w(m - 1, n)),
\]

where \( s(m, n) \) and \( s(m - 1, n) \) are the state vectors of the system at pixel location \((m, n)\) and \((m - 1, n)\), respectively; \( f_{m,n} \) is a known system transition function, and \( w(m, n) \) is zero-mean white noise independent of the past and the current states. The state should take into account the support of the model and the order of scanning of the image field. The Markov-chain assumption must be valid even as the filter makes a transition from the end of a column to the beginning of the next column in the image field \( I \). We choose the state vector \( s(m, n) \) at location \((m, n)\) as

\[
y(m, n) = h_{m,n}(s(m, n), v(m, n)),
\]

The pixels that the state vector contains are also shown pictorially in Fig. 1(b). Note that for this conception of the state, the property of Markovianity holds throughout the chain, even at the end of a scanned column. The sense of continuity as the filter passes from end of a scan line to the beginning of the next scan line in the image field \( I \) is taken care of by the boundary pixels. All the boundary field pixels that will be needed in the future are included in the state vector. This conception of the state vector for recursive filtering incorporates the scalar image-formation model [Eq. (1)] in the form of a Markov chain. By the use of this state and the raster scanning procedure, the 2-D problem has been converted into an equivalent 1-D problem.

The measurement model that relates the noisy observations to the state is
where \( y(m, n) \) is a known measurement at \((m, n)\), \( h_{m,n} \) is a known measurement function, and \( v(m, n) \) is zero-mean white measurement noise independent of the past and the current states as well as of the state noise. The measurement \( y(m, n) \) is a scalar since the degradation considered in this paper is pointwise. The state model predicts the pixel intensities in the current state, whereas the measurement model updates it with the help of a set of observations.

B. Propagation of the State Density

Let, \( \chi(m, n) = \{ s(1, 1), s(2, 1), \ldots, s(m, n) \} \) be the set of state vectors up to the current pixel \((m, n)\). The process dynamics [Eq. (5)] forms a Markov chain so that

\[
p(s(m, n) | \chi(m - 1, n)) = p(s(m, n) | s(m - 1, n)).
\]

(7)

The observations \( y(m, n) \) given by the measurement model of Eq. (6) are independent both mutually and with respect to \( \chi(m - 1, n) \). The available information at \((m, n)\) is the history of measurements \( Y_{m,n} = \{ y(1, 1), y(2, 1), \ldots, y(m, n) \} \). These measurements are the set of present and past intensities observed during the conventional raster scan in the degraded image field \( I_d \). The objective is to construct the posterior conditional probability density function (pdf) \( p(s(m, n) | Y_{m,n}) \) of the current state \( s(m, n) \), given \( Y_{m,n} \), \( f_{m,n} \), and \( h_{m,n} \), along with the initial pdf. The required posterior state density \( p(s(m, n) | Y_{m,n}) \) can be obtained recursively in two stages: prediction and update. Suppose that the posterior state density of the previous state, \( p(s(m - 1, n) | Y_{m-1,n}) \) is available, the prior pdf \( p(s(m, n) | Y_{m-1,n}) \) of the current state can be obtained by using the state model. Then, with the measurement made on the system at location \((m, n)\), the predicted prior is updated to obtain the required posterior state density of the current state \( p(s(m, n) | Y_{m,n}) \).

1. Prediction

We first express \( p(s(m, n) | Y_{m-1,n}) \) in terms of the available pdf \( p(s(m - 1, n) | Y_{m-1,n}) \). From basic probability theory we have

\[
p(s(m, n) | Y_{m-1,n}) = \int p(s(m, n) | Y_{m-1,n}, s(m - 1, n))
\times p(s(m - 1, n) | Y_{m-1,n}) \, ds(m - 1, n).
\]

Since

\[
p(s(m, n) | Y_{m-1,n}, s(m - 1, n)) = p(s(m, n) | s(m - 1, n)),
\]

we get

\[
p(s(m, n) | Y_{m-1,n}) = \int p(s(m, n) | s(m - 1, n))
\times p(s(m - 1, n) | Y_{m-1,n})
\times ds(m - 1, n).
\]

(8)

By assumption, \( p(s(m - 1, n) | Y_{m-1,n}) \) is known. Also,

\[
p(s(m, n) | s(m - 1, n))
= \int p(s(m, n) | s(m - 1, n), w(m - 1, n))
\times p(w(m - 1, n) | s(m - 1, n)) \, dw(m - 1, n).
\]

(9)

Since the state noise \( w(m - 1, n) \) is independent of the state,

\[
p(w(m - 1, n) | s(m - 1, n)) = p(w(m - 1, n)).
\]

Hence Eq. (9) becomes

\[
p(s(m, n) | s(m - 1, n))
= \int p(s(m, n) | s(m - 1, n), w(m - 1, n))
\times p(w(m - 1, n)) \, dw(m - 1, n),
\]

where \( p(s(m, n) | s(m - 1, n), w(m - 1, n)) \) can be evaluated directly from Eq. (5).

2. Update

Next we express the required posterior pdf \( p(s(m, n) | Y_{m,n}) \) in terms of the predicted prior \( p(s(m, n) | Y_{m-1,n}) \). First, we have

\[
p(s(m, n) | Y_{m,n}) = p(s(m, n) | Y_{m-1,n}, y(m, n))
= \frac{p(y(m, n) | s(m, n), Y_{m-1,n})p(s(m, n), Y_{m-1,n})}{p(y(m, n), Y_{m-1,n})}.
\]

Now,

\[
p(y(m, n) | Y_{m-1,n}, s(m, n)) = p(y(m, n) | s(m, n)).
\]

Therefore

\[
p(s(m, n) | Y_{m,n})
= \frac{p(y(m, n) | s(m, n)) p(s(m, n), Y_{m-1,n})}{p(y(m, n), Y_{m-1,n})}.
\]

(10)

There are three pdfs on the right-hand side of Eq. (10):

- \( p(y(m, n) | s(m, n)) \) can be evaluated directly from the measurement model,
- \( p(s(m, n) | Y_{m-1,n}) \) is the predicted prior, and
- \( p(y(m, n) | Y_{m-1,n}) \) is a normalizing constant and is given by

\[
p(y(m, n) | Y_{m-1,n}) = \int p(y(m, n) | Y_{m-1,n}, s(m, n))
\times p(s(m, n) | Y_{m-1,n}) \, ds(m, n).
\]

Equations (8) and (10) constitute a formal solution to our state-estimation problem.
4. IMAGE RECOVERY

Four distinct probability distributions are represented in the proposed recursive framework. Three of them form part of the problem specification, and the fourth constitutes the solution. The three specified distributions are

- the initial density for each pixel that belongs to the boundary field $B$,
- the image model density $p(s(m, n)|s(m - 1, n))$, and
- the degradation model density $p(y(m, n)|s(m, n))$.

The solution or the optimal estimate at each location in the image field $I$ is chosen to be the mean value of the state density $p(y(m, n)|Y_{m,n})$. Analytical solutions of this problem are tractable only for a relatively restrictive choice of system and measurement models, the most important being the Kalman filter,\textsuperscript{13,14,15} which assumes the state and the measurement functions to be linear and to be additive white Gaussian. For the simple linear-Gaussian estimation problem, the required density remains Gaussian at every iteration,\textsuperscript{14} and the Kalman filter relations propagate and update the mean and covariance of the distribution. Only when all of the above three specified distributions are Gaussian is the corresponding state density also Gaussian, in which case the 2-D Kalman filter can be used. However, as already pointed out in Section 2, the image is distorted in general by a deterministic nonlinearity and/or is corrupted by a non-Gaussian observation noise that will cause the degradation density $p(y(m, n)|s(m, n))$ to be non-Gaussian. Hence the evolving state density $p(s(m, n)|Y_{m,n})$ is also non-Gaussian.\textsuperscript{14}

Our objective is to construct the conditional posterior pdf $p(s(m, n)|Y_{m,n})$ of the state vector at location $(m, n)$, given the available information $Y_{m,n}$. Since the prior and posterior densities are non-Gaussian at all steps of prediction and update, propagating the mean and covariance of the required pdf (as in the Kalman filter) will not result in a correct estimate of the conditional mean. At every recursion, an exact representation of the pdf is required. This can be achieved within the particle-filter framework by propagating the samples of the pdf. The key ideas in particle-filter theory is to represent the required posterior state density by a set of random samples with associated weights and to compute estimates on the basis of these samples and weights. As the number of samples becomes large, this characterization tends to an exact, equivalent representation of the required density.\textsuperscript{13,14}

Consider that samples of the state vector $s(m, n)$ are available at the current pixel location $(m, n)$. Let the samples of the predicted prior $p(s(m, n)|Y_{m-1,n})$ and the posterior $p(s(m, n)|Y_{m,n})$ be denoted by $s_{pr}^{k}(m, n)$ and $s^{k}(m, n)$, respectively. The subscript $pr$ denotes the prior, and the index $k$ indicates the $k$th sample. Suppose that we have $N$ samples of the state vector at location $(m - 1, n)$, i.e., $s^{k}(m - 1, n)$, $k = 1, 2, ..., N$. The goal is to predict and update from these samples to obtain a new sample set $s^{k}(m, n)$, $k = 1, 2, ..., N$. The distribution of these samples should represent the posterior pdf $p(s(m, n)|Y_{m,n})$. We propose to construct the $k$th of the $N$ new samples as follows:

- Predict by sampling from

$$p(s(m, n)|s(m - 1, n) = s^{k}(m - 1, n))$$

to obtain $s_{pr}^{k}(m, n)$. The state vector of the current state contains all the elements of the previous state except the pixel at $(m, n)$. In Section 2 we defined the image model by a linear AR process with NSHP support. The $k$th sample of the current pixel $(m, n)$ is predicted as

$$s_{pr}^{k}(m, n) = \sum_{(i,j) \in \Gamma} c_{i,j}s^{k}(m - 1, n - j) + \left(1 - \sum_{(i,j) \in \Gamma} c_{i,j}\right)\overline{s} + w^{k}(m - 1, n), \quad (11)$$

where $w^{k}(m - 1, n)$ is a sample drawn from the known pdf of the state noise $w(m - 1, n)$. The predicted sample set $s_{pr}^{k}(m, n)$, $k = 1, 2, ..., N$ is distributed as $p(s(m, n)|Y_{m-1,n})$, the effective prior density at location $(m, n)$.

- The desired posterior density $p(s(m, n)|Y_{m,n})$ is given by Eq. (10) and is reproduced here for reference:

$$p(s(m, n)|Y_{m,n}) = \frac{p(y(m, n)|s(m, n))p(s(m, n)|Y_{m-1,n})}{p(y(m, n)|Y_{m-1,n})},$$

where the denominator quantity is a normalizing factor. What we need are samples of the pdf proportional to the product $p(y(m, n)|s(m, n))p(s(m, n)|Y_{m-1,n})$. To obtain this pdf, we invoke an important theorem by Smith and Gelfand.\textsuperscript{23} They address the implementation of Bayes’s theorem as a weighted bootstrap. The following steps describe the use of this theorem in our algorithm:

1. Consider the prior samples $s_{pr}^{k}(m, n)$, $k = 1, 2, ..., N$, drawn from the continuous density function $p(s(m, n)|Y_{m-1,n})$.

2. Consider the known function $p(y(m, n)|s(m, n))$. Evaluate the normalized weight for the $k$th sample as

$$\Omega_{m,n}^{(k)} = \frac{p(y(m, n)|s_{pr}^{k}(m, n))}{\sum_{i=1}^{N} p(y(m, n)|s_{pr}^{i}(m, n))}, \quad k = 1, 2, ..., N, \quad (12)$$

Note that the above expression for the normalized weight uses the prior as the importance function. The prior is obtained by modeling the original image as an AR process with an appropriate NSHP support. The samples of the prior are obtained by using Eq. (11). This particular choice of the prior allows us to suitably place the particles in the state space. If the observation model is given by Eq. (3), then

$$\Omega_{m,n}^{(k)} = \frac{p_{o}(d(m, n) - \alpha \log_{10}[s^{k}(m, n)] - \beta)}{\sum_{i=1}^{N} p_{o}(d(m, n) - \alpha \log_{10}[s^{i}(m, n)] - \beta)},$$

(13)
3. Resample $N$ times to generate the posterior samples $s^k(m, n), k = 1, 2, \ldots, N$ such that for any $j$, the probability of

$$\{s^k(m, n) = s^k(m, n)\} = p_{m,n}^k.$$

The idea behind resampling is to eliminate particles that have small weights and to concentrate on particles with large weights. Moreover, the interdependencies among the distributions of the various pixels in the state are taken care of by the resampling stage of the particle filter. As $N$ tends to infinity, the pdf constructed from these samples approaches that of the product term $p(y(m, n)\mid s(m, n))p(s(m, n)\mid Y(m-1, n))$ except for a scale factor. This product is proportional to the required posterior density $p(s(m, n)\mid Y(m, n))$.

Resampling is implemented by

(a) Generating a uniform random number $r \in [0, 1]$.
(b) Finding the smallest $j$ for which $c(j) \geq r$.
(c) Setting $s^k(m, n) = s^k(m, n)$.

Here, $c(0) = 0$ and $c(k) = c(k - 1) + \Omega_{m,n}^k, k = 1, 2, \ldots, N$.

- Estimate the moments of the pixel intensity at the current location $(m, n)$ as

$$E[f(s(m, n))] = \sum_{k=1}^{N} \Omega_{m,n}^k f(s^k(m, n)).$$

Since we are interested only in the conditional mean, an optimal estimate of the pixel intensity of the original image at $(m, n)$ can be obtained by substituting $f(s(m, n)) = s(m, n)$. Note that this is a scalar particle filter since only one pixel is estimated at a time.

To initiate the algorithm, $N$ samples each are drawn from the boundary field $B$, and, following raster scanning, the interior pixels are estimated recursively. Each pixel in $B$ is initially modeled as a Gaussian. The variance of the Gaussian reflects the confidence in the chosen mean. The mean itself can be estimated in any of the following ways:

- The mean may be fixed at the (global) average of some prototype image.
- The mean may be the same as that measured from the image.
- The mean may be the same as the intensity in some prototype image.

We continue these steps of prediction and update for all the pixels in the image. Note that each pixel in the state vector is represented by $N$ samples, and we deal only with these sets of samples. One of the striking features of the 2-D particle filter is that it is considerably simpler to implement than the Kalman filter. One does not have to deal with complex Riccati equations as in the Kalman filter.\textsuperscript{18}

It is a well-known fact that for practical images, the updating of the points distant from the current location in the state vector is wasteful considering the marginal improvement that occurs in the visual quality of the recovered image for a full update of the state vector.\textsuperscript{7,8} For computational efficiency, we resort to reduced update in which only those pixels that are near the current location of the state vector are updated. For updating purposes, we limit ourselves to only those pixels that are in the NSHP support of a current location $(m, n)$.

5. EXPERIMENTAL RESULTS

In this section we demonstrate the performance of the proposed method in handling nonlinear/non-Gaussian situations using several examples, both synthetic and real. We consider a specific practical application for this purpose, which is that of recovering scanned photographic images degraded by film-grain noise. This example is interesting because, on the one hand, the degradation (in the density domain) can be looked upon as being nonlinear in the presence of additive Gaussian noise. In the exposure domain, on the other hand, the model becomes linear, but the noise manifests itself as multiplicative non-Gaussian noise. The aim is to show that the proposed method can handle sensor nonlinearity as well as multiplicative non-Gaussian noise.

The problem is to recover the original image $s(m, n)$ given the degraded image in the exposure domain $e(m, n)$. The nonlinear behavior of the photographic film in terms of the $D = \log E$ characteristic curve was discussed in Section 2. The gray-level values in the linear portion of the curve ranges from 10 to 200.\textsuperscript{9} The density change over this intensity span for a typical film is 6, resulting in $\alpha = 4.61$. In our experiments, we use $\alpha = 5$. A first-order AR process with NSHP support is used to model the original focused image. The AR coefficients are obtained by using a prototype image. As mentioned above, we update only those pixels in the state vector that are in the NSHP support of the current pixel $(m, n)$. We also compare the performance of our method with an existing scheme called the modified linear Wiener (MLW) filter, proposed by Tekalp and Pavlovic\textsuperscript{21} for handling film-grain noise.

The signal-to-noise ratio (SNR) of the degraded image is given by

$$\text{SNR} = 10 \log_{10} \left( \frac{\text{variance of the degraded image}}{\text{variance of the observation noise}} \right) \text{ dB}.$$  

(16)

The recovered image is evaluated objectively by using the standard improvement in SNR (ISNR) measure, which is given by

$$\text{ISNR} = 10 \log_{10} \left( \frac{\sum_{i,j} [d(i,j) - s(i,j)]^2}{\sum_{i,j} [r(i,j) - s(i,j)]^2} \right) \text{ dB},$$

(17)

where $d(\cdot, \cdot)$, $s(\cdot, \cdot)$, and $r(\cdot, \cdot)$ represent the degraded observation, the original image, and the recovered image, respectively.

Yet another quantitative measure is a comparison between the variance of the noise in the given degraded image and the noise variance after recovery. Consider a relatively smooth region in the exposure domain in which
the noise multiplies a constant gray level (say, \( \rho \)). The mean and the second moment of this region can be expressed, respectively, as:

\[
E[pv(m, n)] = \rho \exp\left(\frac{\sigma^2 \ln^2 10}{2a^2}\right),
\]

\[
E[p^2v^2(m, n)] = \rho \exp\left(\frac{2\sigma^2 \ln^2 10}{a^2}\right).
\]

From these equations, the noise variance \( \sigma_v^2 \) in the density domain can be found as:

\[
\sigma_v^2 = \left(\frac{\ln 10}{\ln^2 10}\right) \frac{E[p^2v^2(m, n)]}{E[pv(m, n)]^2}.
\]

Using Eq. (18), one can compute (in the density domain) the noise variance \( \sigma_{v, deg}^2 \) of the degraded image and the noise variance \( \sigma_{v, rec}^2 \) of the recovered image.

In the first experiment, we consider a textured “Bark” image of size 200 × 200 pixels taken from the Brodatz album [Fig. 2(a)]. The sensor nonlinearity is activated on the original image according to Eq. (3). Observation noise corresponding to an SNR of 7 dB is then added to this image. The noisy observed image in the exposure domain is shown in Fig. 2(b). The image is first recovered by using the traditional Wiener filter; the output is

![Fig. 2.](image)
shown in Fig. 2(c) along with the ISNR value. The Wiener filter is optimal only for additive Gaussian noise, and hence its performance is not good, as expected. Even though the recovered image is sharp, it is quite noisy. Next, the nonlinear $3 \times 3$ median filter was used to recover the original image; the output is shown in Fig. 2(d). The median filter performs quite poorly. Even though it averages out the noise, it also smooths out the signal, which results in loss of finer details. This is also reflected in the ISNR value. Next, the MLW filter was used; the recovered output result is shown in Fig. 2(e). The MLW filter is a modification of the traditional Wiener filter tailored specifically to handle multiplicative noise. Its performance is better than that of both the median filter and the Wiener filter. The ISNR value for this filter is 2.35 dB.

Finally, we use the proposed algorithm to recover the original image. For each location $(m, n)$, $N$ samples of the pixel intensity $s(m, n)$ are predicted by means of the prediction model [Eq. (11)] to construct the sample set.

Fig. 3. (a) Original “Pentagon” image. Degraded image in (b) the density domain and (c) the exposure domain ($\sigma_{\text{deg}}^2 = 0.088$). (d) Image recovered with the MLW filter (ISNR = 2.82 dB, and $\sigma_{\text{rec}}^2 = 0.046$). Image recovered with the particle filter with (e) $N = 5$ (ISNR = 1.16 dB), and (f) $N = 500$ (ISNR = 3.86 dB, and $\sigma_{\text{rec}}^2 = 0.026$).
Using this predicted sample set, the observation model of Eq. (3), and the pdf of the measurement noise, we calculate the normalized weight corresponding to each sample by means of Eq. (13). We then use the samples and their normalized weights to estimate the conditional mean at pixel location $(m, n)$, using Eq. (15). This is followed by the resampling step [Eq. (14)] before we predict the next pixel. The result of image recovery with 500 samples ($N = 500$) is shown in Fig. 2(f). The corresponding ISNR value is 3.62 dB. For the particle filter to work, there should not be any degeneracy. Degeneracy is a phenomenon in which after a few iterations all but one particle will have negligible weight. We arbitrarily selected two spatial locations with a long lag, namely, $(181, 150)$ and $(181, 151)$. When the processing reaches pixel $(181, 151)$, the particles’ states contain the state relating to pixels $(181, 150)$ and $(181, 151)$. To plot the diversity of the particles’ states, we plot one point for each particle with the $x$ axis representing one site in the image and the $y$ axis representing another site for the same particle, as shown in in Fig. 2(g). It is clear from the plots that these particles have good diversity and

Fig. 4. (a) Original “Peppers” image. Degraded image in (b) the density domain and (c) the exposure domain ($\sigma^2_{\text{deg}} = 0.153$). (d) Image recovered with the MLW filter (ISNR = 3.24 dB, and $\sigma^2_{\text{rec}} = 0.078$). Image recovered with the particle filter with (e) $N = 5$ (ISNR = 2.06 dB), and (f) $N = 500$ (ISNR = 4.65 dB, and $\sigma^2_{\text{rec}} = 0.047$).
there is no degeneracy. We have also verified the same at several other locations in the image. The fact that the method works even with just 500 samples is a direct consequence of the choice of the AR model for the prior. It is important to place the particles cleverly such that there is a significant overlap between the prior and the likelihood. Otherwise, many of the samples will receive a small weight and will not be selected during resampling. Through the AR model, the particles are encouraged to be in the right place, which reduces the degeneracy effect.

Among the four methods, the proposed method performs the best. The recovered image has a higher ISNR value and is visually more pleasing than the MLW-filter output. The reason for this is that the latter is optimal only among the class of linear filters. Since the MLW filter is constrained to be linear, it can yield only a suboptimal solution. The proposed method based on the particle filter has no such constraint and directly yields an estimate of the conditional mean provided that the number of samples is sufficiently large. Henceforth we compare the output of the proposed method with that of the MLW filter only.

To show that the proposed method can also be used for recovering nontextured images, we consider the “Pentagon” image of Fig. 3(a). The degraded images in the density and in the exposure domain for an SNR of 2.5 dB are shown in Figs. 3(b) and 3(c), respectively. [The image of Fig. 3(b) has been appropriately scaled for display purposes.] Note that the observed image [Fig. 3(c)] is quite noisy. The image recovered with the MLW filter is shown in Fig. 3(d). Although the MLW filter produces some improvement, the output image is visibly noisy. The ISNR value is 2.82 dB. Next, the original image is recovered with the proposed method. When we use very few samples (N = 5), the quality of the recovered image [Fig. 3(e)] is poor, as expected (and the ISNR value is only 1.16 dB). When N is increased to 500, the quality of the output image improves significantly. The recovered image is less noisy than the MLW output, while the details are still well preserved. The ISNR value for the proposed method is 3.86 dB. The noise variances of the degraded and the recovered images were also compared over the same smooth region in the two images for both the MLW filter and the proposed method. These values are given in Figs. 3(d) and 3(f). We note that the noise variance of the recovered images is significantly less with the proposed method than with the MLW filter. For the Pentagon image, the reduction in noise variance with use of the MLW filter is 47.7%, whereas for the particle filter the reduction is 67%.

In yet another experiment, the “Peppers” image was selected as the original image [Fig. 4(a)]. The corresponding nonlinearly degraded images in the density and the exposure domains are shown in Figs. 4(b) and 4(c), respectively, for SNR = 13 dB. The image recovered with the MLW filter in the exposure domain is shown in Fig. 4(d). When the samples are few, the quality of the image recovered [Fig. 4(e)] with the particle filter is poor. However, when the number of samples is increased, the recovered image [Fig. 4(f)] looks quite good (ISNR = 4.65 dB for N = 500) and is better than the output of the MLW filter (ISNR = 3.24 dB). Note that the MLW filter output is quite spotty compared with the output of the proposed method. This is also reflected in the values of the noise variance given in Fig. 4(f). The reduction in noise variance for the Peppers image is roughly 20% more for the particle filter than for the MLW filter. A careful comparison of Figs. 4(d) and 4(f) (note the top regions of the Capsicum) reveals that the edges are crisper in the output image of the proposed method than in the MLW filter.

Fig. 5. (a) ISNR is plotted as a function of the number of samples of the particle filter for different values of SNR. (b) ISNR is plotted as a function of the input SNR. (c) Sensitivity of the particle filter output to uncertainty in α (SNR = 13 dB). The true value of α is 5.
output. Thus the overall performance of the proposed method is superior to that of the MLW filter. However, our method is computationally more intensive since it works by propagating samples.

A. Effect of Number of Samples
To demonstrate the improvement in ISNR as a function of the number of samples, the proposed particle filter-based method is used to recover the Peppers image when it is degraded according to the measurement model of Eq. (3). The image is recovered for observation noise variances corresponding to SNR values of 15 dB, 16 dB, and 17 dB. For each SNR value, the ISNR is plotted as a function of the number of samples in Fig. 5(a). From the plot, we note that the improvement in ISNR is only marginal beyond 200 samples. We have found this to be true in most examples.

B. Effect of Signal-to-Noise Ratio
To study the performance of the proposed method as a function of the SNR of the degraded image, we select SNR values between 13 and 17 dB. For these values of SNR, the degradation is quite reasonable for the Peppers image. Both the MLW filter and the particle filter are used to recover the original image. For the particle filter, the number of samples used was 200. The plot of the ISNR value as a function of the input SNR is shown in Fig. 5(b). When the SNR increases, the pixelwise difference between the original and the degraded images in the exposure domain decreases. This makes the numerator quantity in Eq. (17) small, which decreases the ISNR. From Fig. 5(b) we also note that the proposed method gives approximately a 1.5-dB improvement over that of the MLW filter for the same SNR.

C. Parameter Sensitivity
To examine the sensitivity of the proposed method to knowledge of \( \alpha \), we compute the ISNR values for different values of \( \alpha \) around its true value for the Peppers image. The percentage decrease in the ISNR value is computed with respect to its maximum value. Using the particle filter with \( N = 200 \) samples, we recover the original image for values of \( \alpha \) between 1 and 15. The true value of \( \alpha \) is 5. The percentage decrease in ISNR as a function of \( \alpha \)
has been plotted in Fig. 5(c). We note that the percentage decrease is zero when the correct value of $\alpha$ is used, and there is a decrease in the ISNR value as $\alpha$ drifts from its true value. However, the decrease in ISNR is gradual for $\alpha$ in the range 3–9 and is less than 10% for this range of $\alpha$. Hence the proposed method can be used even if the value of $\alpha$ is only approximately known, as may be the case in practical situations.

D. Real Examples
Finally, we show that the proposed method can be used effectively in real situations also. For this purpose, we consider scanned photographic images taken by using a 35-mm camera with known film characteristics. Since we need the measurement-noise pdf to find the normalized weights of the samples, we estimate the variance of the measurement noise $\sigma^2_{\text{deg}}$ [with Eq. (18)] in the density domain from the degraded observation itself by choosing a homogeneous region.

As a first example, consider the “Boat” image of size $100 \times 546$ pixels, shown in Fig. 6(a). Note the film-grain pattern in the image. The proposed method was implemented in the exposure domain for $N = 500$; the recovered image is shown in Fig. 6(b). We observe that the effect of film-grain noise is considerably reduced in the recovered image. The folds in the cloth are much more clearly visible and the edges are more clearly discernible in the recovered image. The noise variances $\sigma^2_{\text{deg}}$ and $\sigma^2_{\text{rec}}$ are also shown in Fig. 6. The noise variance of the recovered image is considerably lower than that of the given noisy image. The proposed algorithm is validated on yet another real image, namely, the “Sky” image shown in Fig. 6(c). The image recovered by the proposed method with $N = 500$ is shown in Fig. 6(d) along with the noise variances. We observe that the visual quality of the recovered image again improves significantly with the proposed method.

6. CONCLUSIONS
We have proposed a novel two-dimensional recursive filtering framework based on the particle filter for recovering images degraded under nonlinear/non-Gaussian situations. The key idea is to represent the probability density function (pdf) by a set of random samples with associated weights rather than as a function over state space. The method works by propagating samples of the posterior pdf and by using the weights of the samples to arrive at an optimal estimate of the original image. The effectiveness of the method has been demonstrated by considering the practical example of recovering images degraded by film-grain noise. The method was tested on several synthetic and real images, and its performance was found to be quite good.

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