Oversampling Analog to Digital Converters

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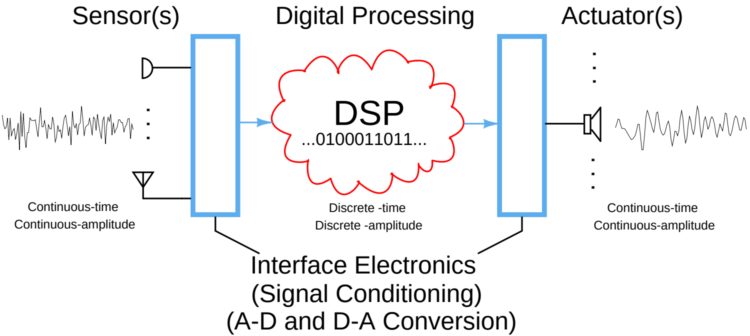
Outline

- Introduction to sampling and quantization
  - Quantization noise spectral density
  - Oversampling
  - Noise shaping-$\Delta\Sigma$ modulation
- High order multi bit $\Delta\Sigma$ modulators
- Stability of $\Delta\Sigma$ A/D converters
- Implementation of $\Delta\Sigma$ A/D converters
  - Loop filter design
  - Multi bit quantizer design
  - Excess delay compensation
  - Clock jitter effects
- Mitigation of feedback DAC mismatch
  - Dynamic element matching
  - DAC calibration
- Case study
  - 15 bit continuous-time $\Delta\Sigma$ ADC for digital audio

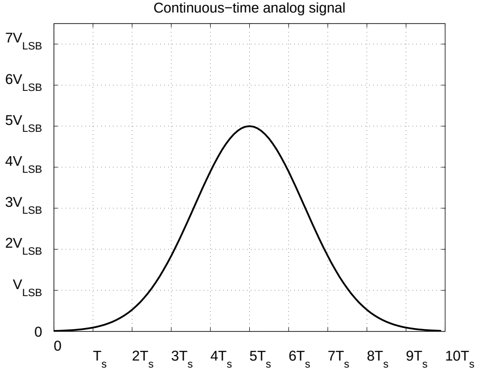
Signal processing systems



Signal processing systems

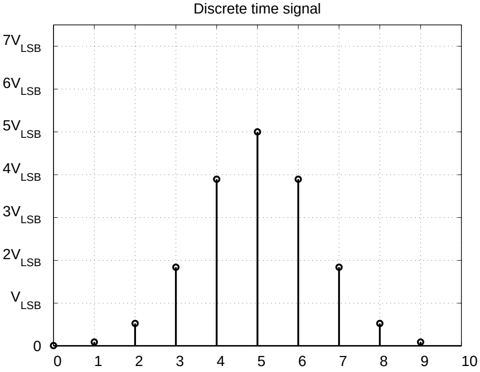
- Natural world: continuous-time analog signals
- Storage and processing: discrete-time digital signals
- Data conversion circuits interface between the two
- Wide variety of precision and speed

Continuous time signals



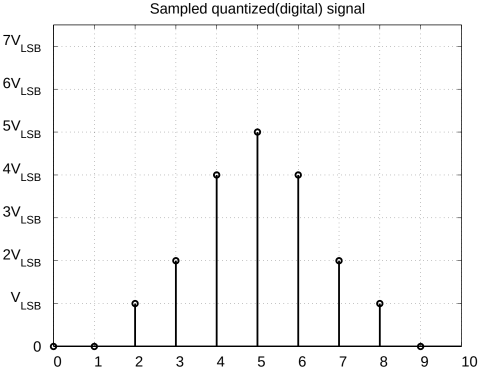
- Signals defined for all *t*
- Signals can take any value in a given range

Discrete time signals



- Signals defined for discrete instants *n*
- Signals can take any value in a given range

Digital signals



- Signals defined for discrete instants n
- Signals can take discrete values kV_{LSB}

Sampling and quantization

- A segment of a continuous-time signal has an infinite number of points of infinite precision
- Discretization of time (sampling) and amplitude (quantization) results in a finite number of points of finite precision
- Sampling and quantization = *Analog to digital conversion*
- Errors in the process?

Signals in time and frequency domains

- Continuous time signal $x_{ct}(t)$
- Frequency domain representation using its Fourier transform $X_{ct}(f)$

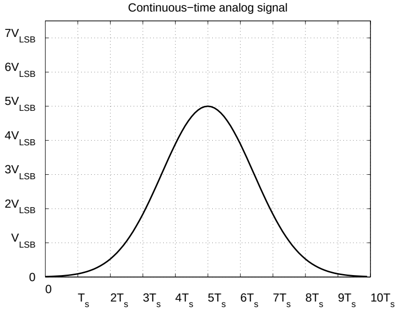
$$X_{ct}(f) = \int_{-\infty}^{\infty} x_{ct}(t)\exp(-j2\pi ft)dt$$

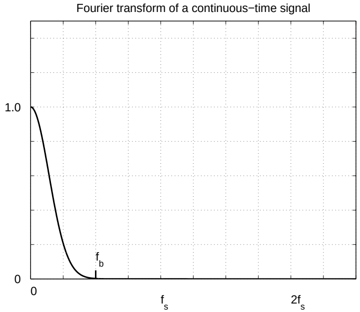
- Discrete time signal $x_d[n]$
- Frequency domain representation using its Fourier transform $X_d(\nu)$

$$X_d[\nu] = \sum_{n=-\infty}^{\infty} x_d[n]\exp(-j2\pi\nu n)$$

- $X_d[\nu]$ periodic with a period of 1

Signals in time and frequency domains

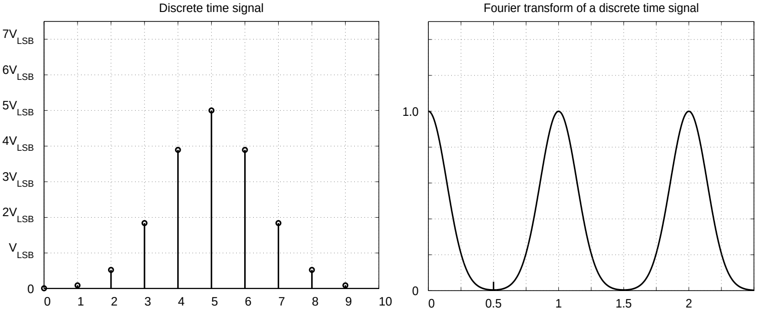




$$X_{ct}(f) = \int_{-\infty}^{\infty} x_{ct}(t) \exp(-j2\pi ft) dt$$

- Signal bandwidth f_b: $|X_{ct}(f)| = 0$ for $f > f_b$

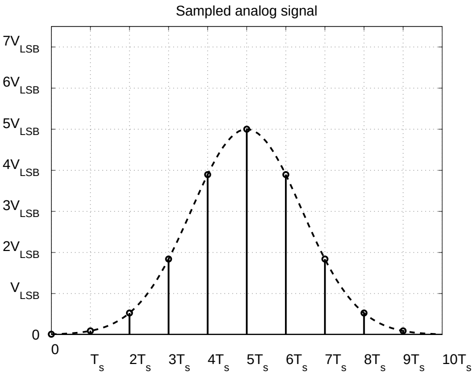
Signals in time and frequency domains



$$X_d[\nu] = \sum_{n=-\infty}^{\infty} x_d[n] \exp(-j2\pi\nu n)$$

- $X_d[\nu]$ periodic with a period of 1
- $X_d[\nu]$, $0 \leq \nu \leq 0.5$ completely defines real $x_d[n]$

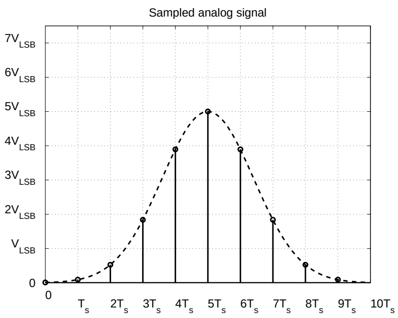
Sampling an analog signal



$$x_d[n] = x_{ct}(nT_s)$$

- Analog signal sampled to obtain a discrete-time signal

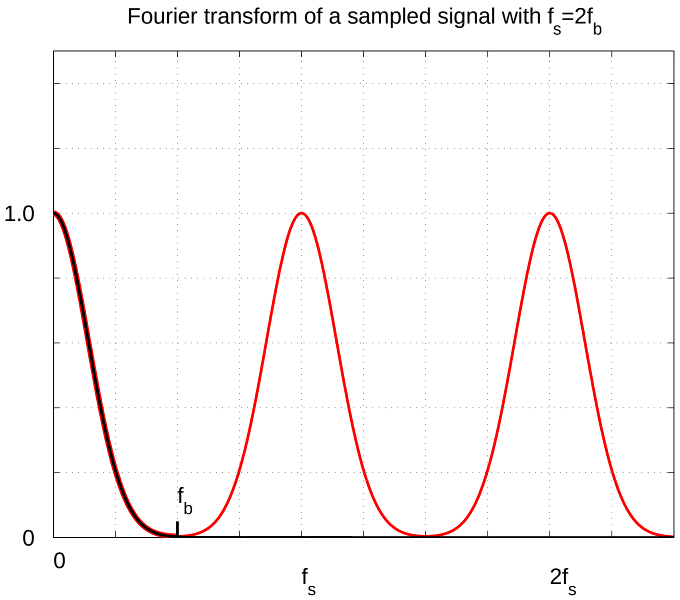
Sampling



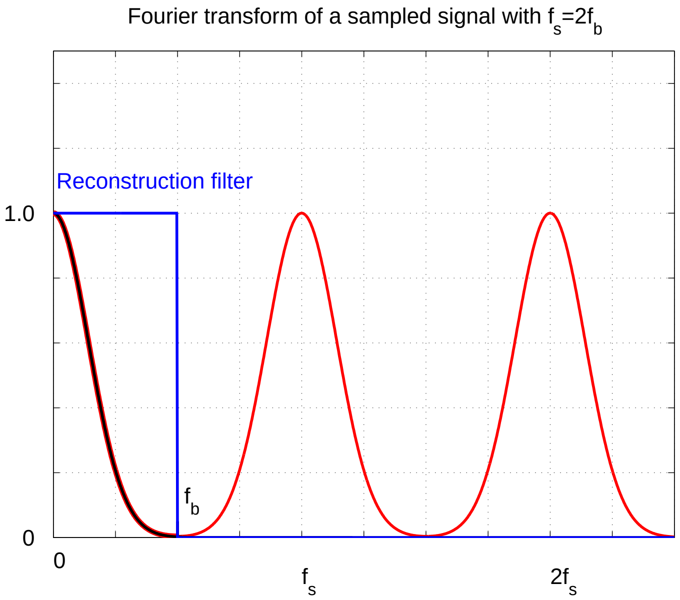
$$X_d[\nu] \quad = \quad \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X_{ct}(\nu f_s - n)$$

- Copies of signal spectrum at $nf_s = n/T_s$
- Perfect reconstruction possible for $f_s \geq 2f_b$

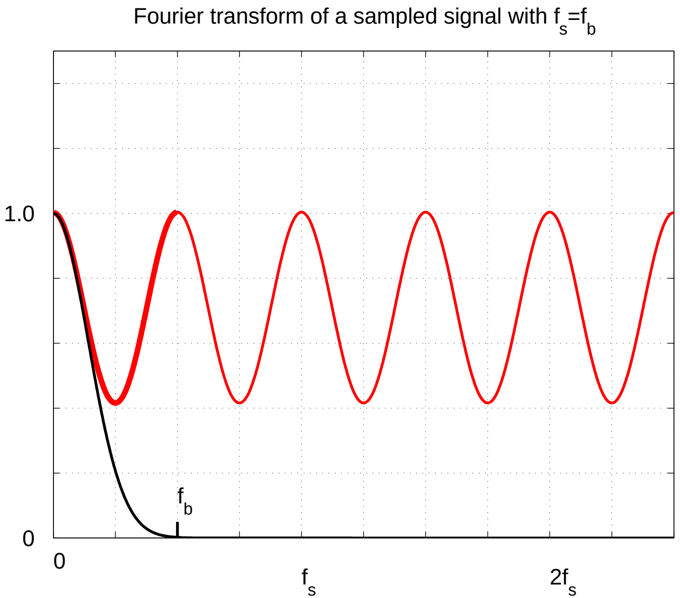
Sampling without aliasing



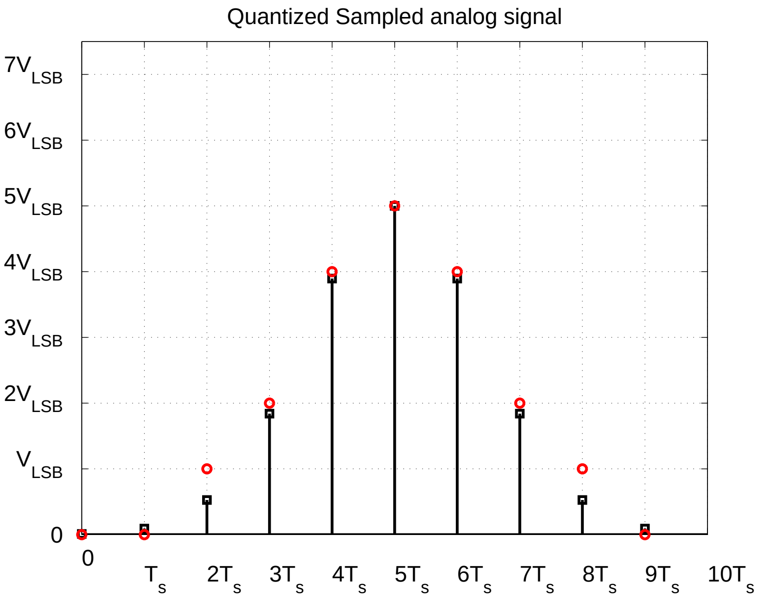
Reconstruction from sampled signal



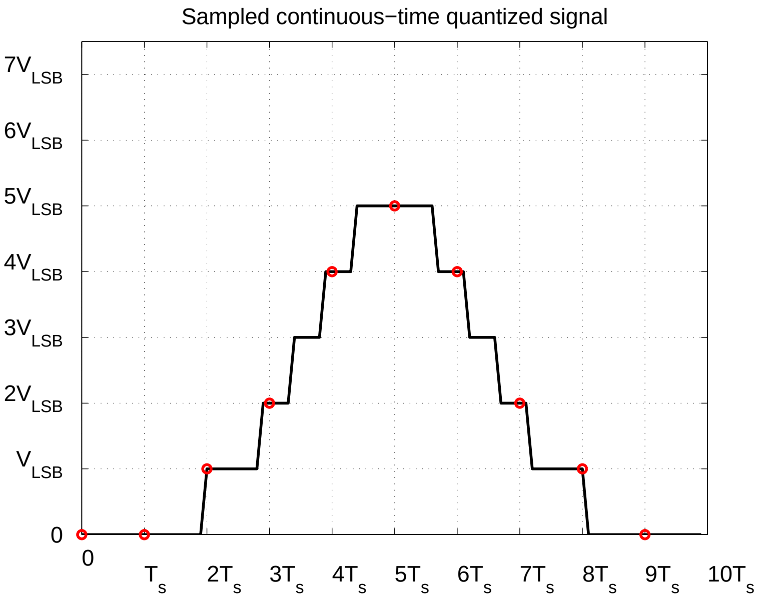
Aliasing during sampling



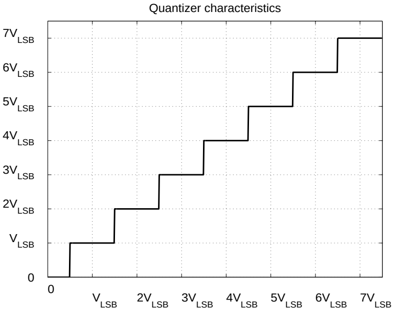
Sampling followed by quantization

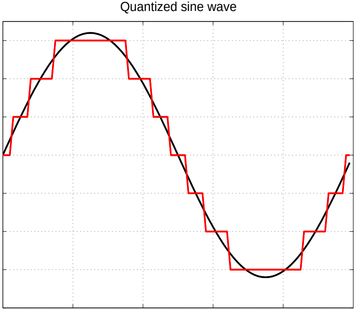


Quantization followed by sampling



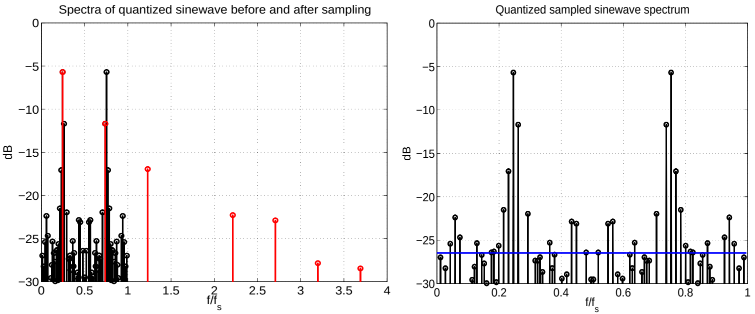
Quantization



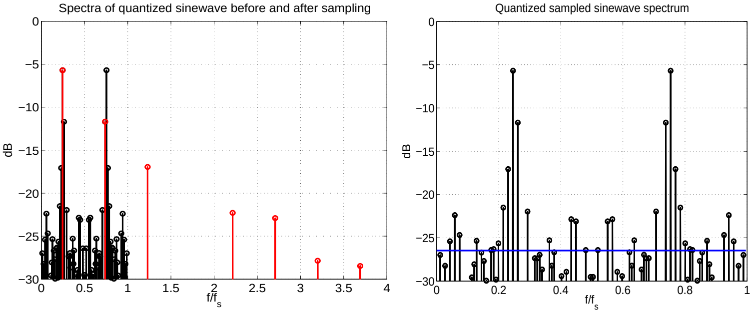


- Nonlinearity results in harmonic distortion
- Harmonics folded about the sampling frequency

Sampling and Quantization-Spectral density

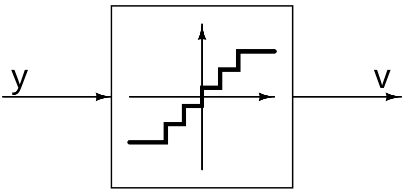


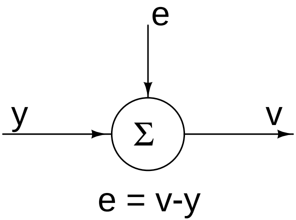
Sampling and Quantization-Spectral density



- $f_s/f_{in} = p/q$, large p, q: Closely spaced tones $\sim$ noise
- f_s/f_{in} irrational: Continuous spectrum
- Approximated by a constant spectral density

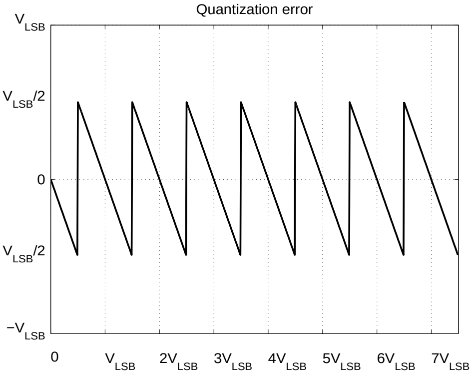
Quantization error model

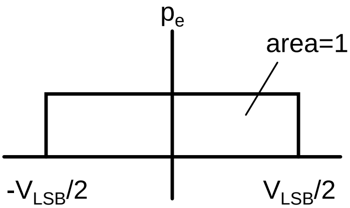




- Modelled as an additive error

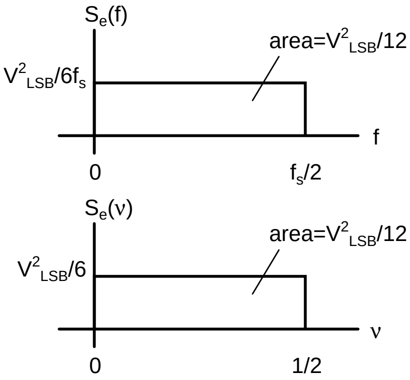
Quantization error distribution





- Quantization error in the range $[-V_{LSB}/2, V_{LSB}/2]$
- Uniform distribution
- Mean squared value of $V_{LSB}^2/12$

Sampling and Quantization-Error

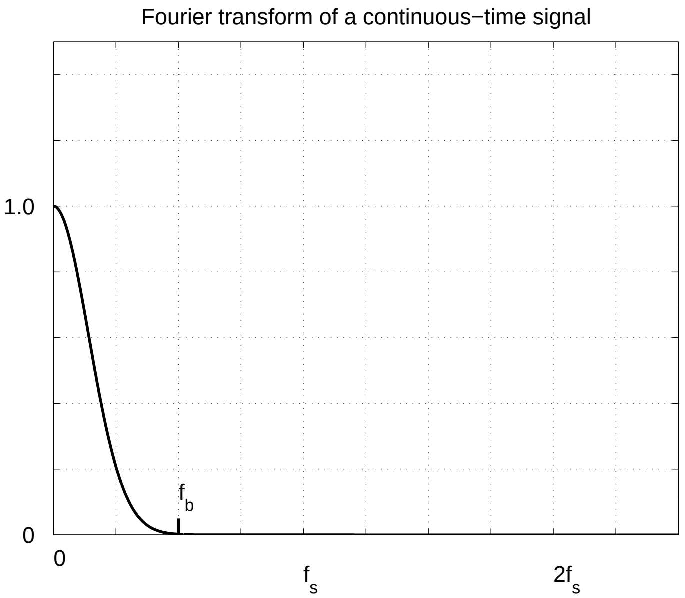


- Fully correlated to the input signal
- Statistics independent of the input signal
  - Uniform distribution; mean $= 0$; variance $= V_{LSB}^2/12$
- White spectral density
- Modelled as uncorrelated additive white noise

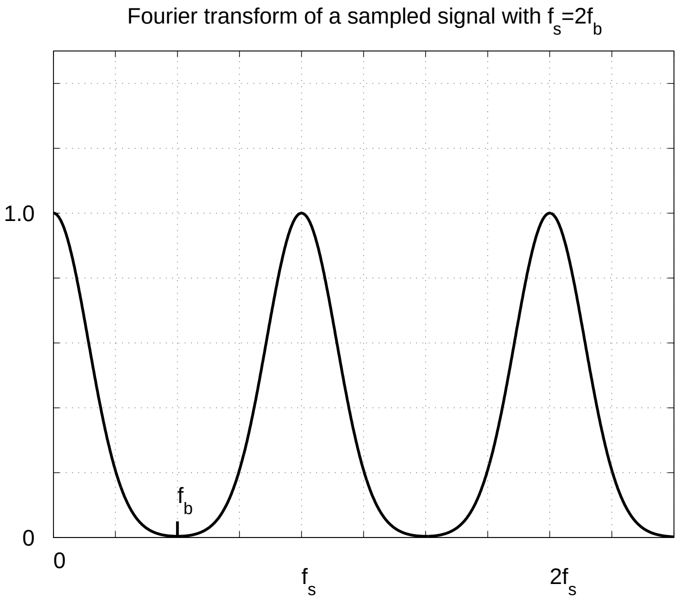
Sampling and Quantization- *SNR*

- 2^N level quantizer with V_{LSB} spacing
- Full scale sinewave input—amplitude $(2^{N-1}V_{LSB})$
- Mean squared signal: $\left(2^{N-1}V_{LSB}\right)^2/2$
- Mean squared noise: $V_{LSB}^2/12$
- $SNR = \frac{3}{2}2^{2N} = 6.02\,N + 1.78\,\mathrm{dB}$

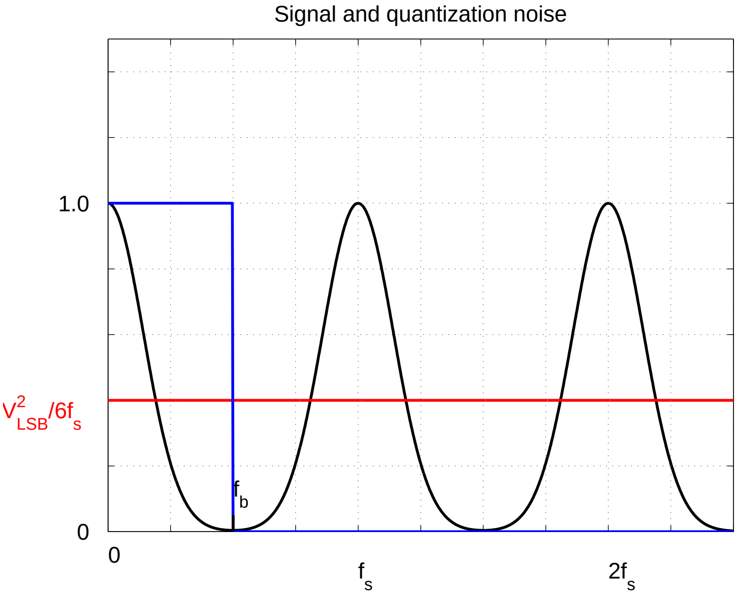
Sampling and Quantization



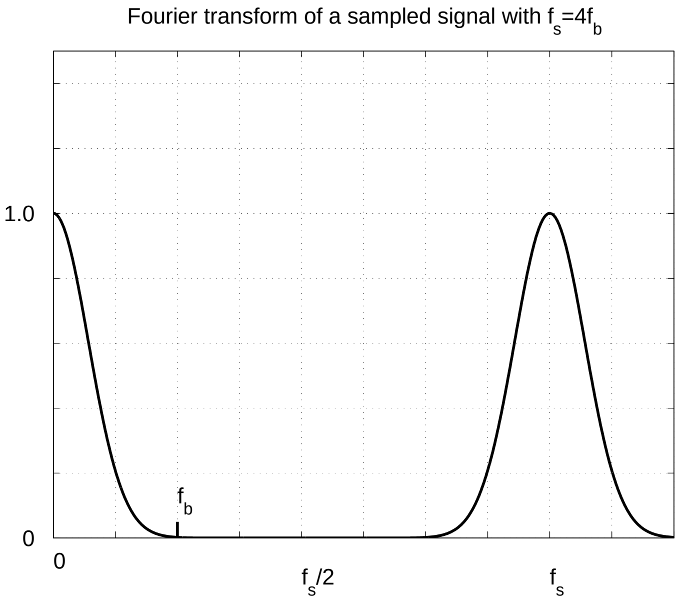
Sampling and Quantization



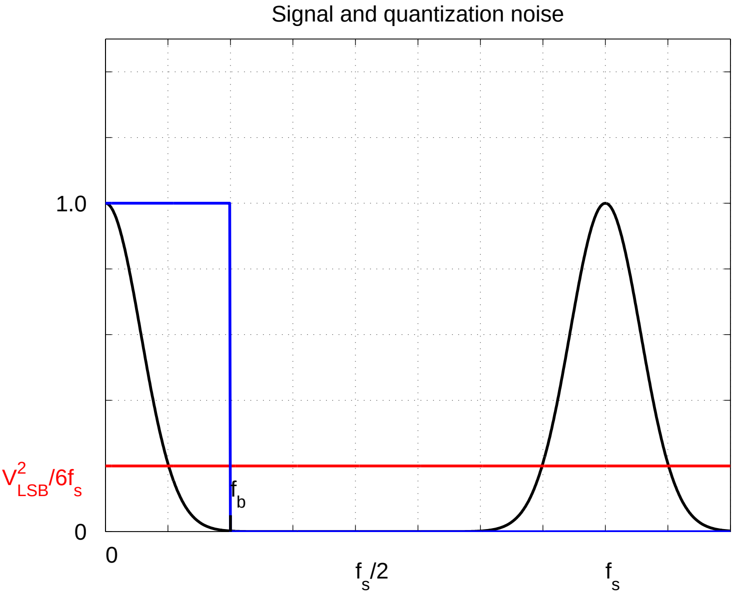
Sampling and Quantization



Oversampling and Quantization



Oversampling and Quantization



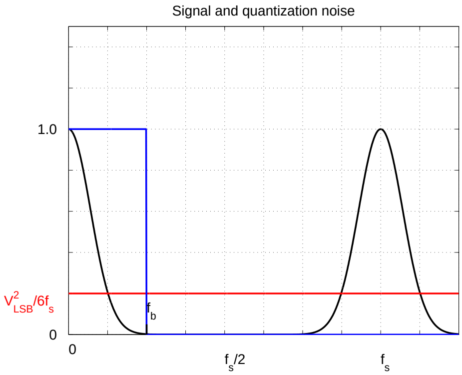
Oversampling

- Sample at $f_s \gg 2f_{in}$
- Oversampling ratio $OSR = f_s/2f_{in}$
- Filter the noise using a filter of bandwidth f_b
- Mean squared value of error $= V_{LSB}^2/12/OSR$
- Increased signal to quantization noise ratio

Oversampling and Quantization- *SNR*

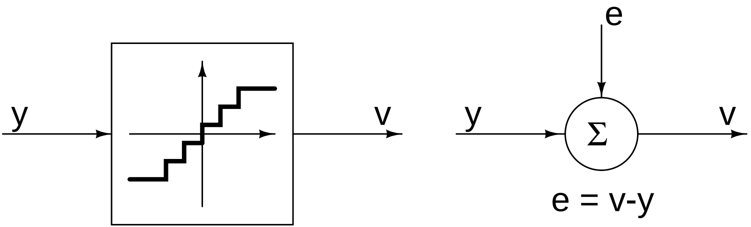
- 2^N level quantizer with V_{LSB} spacing
- Full scale sinewave input—amplitude $= 2^{N-1}V_{LSB}$
- Oversampling ratio OSR
- Mean squared signal: $\left(2^{N-1}V_{LSB}\right)^2/2$
- Mean squared noise: $V_{LSB}^2/12/OSR$
- $SNR = \frac{3}{2}2^{2N}\,OSR = 6.02\,N + 10\log OSR + 1.76\,\mathrm{dB}$

Oversampling and Quantization



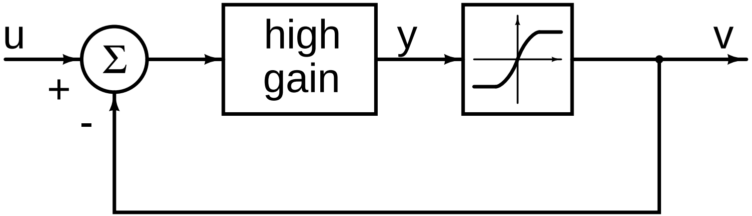
- Move quantization error to filter stopband?

Quantizer



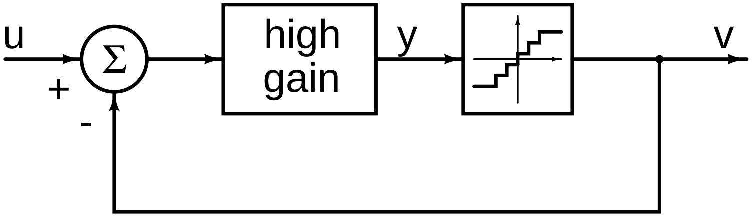
- Hard nonlinearity
- Modelled as additive error

Linearization of soft nonlinearity



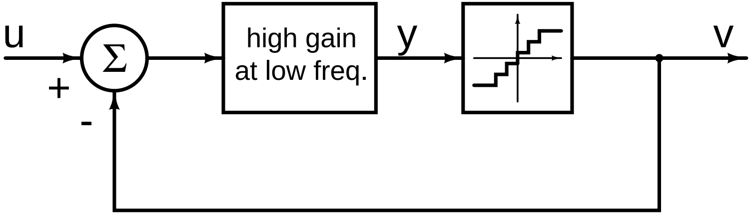
- Negative feedback loop
- Loop gain $\rightarrow \infty \Rightarrow$ Error $u - v \rightarrow 0$

Linearization of hardnonlinearity



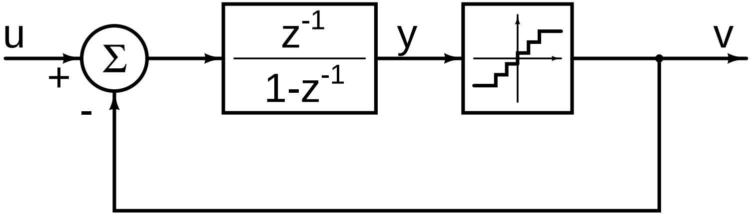
- Quantizer output cannot equal the input
- Loop gain $\rightarrow \infty \Rightarrow$ Error $|u - v| \rightarrow \infty$

Reduce error to zero only in the signal band



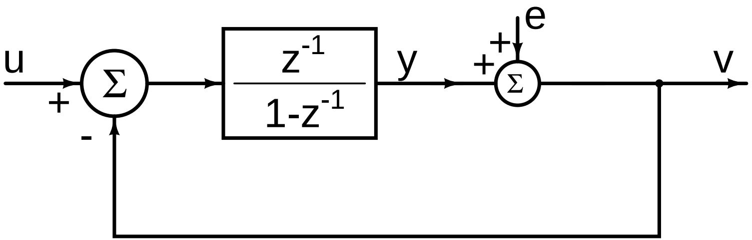
- Negative feedback loop with dc loop gain $\rightarrow \infty$
- Small loop gain at high frequencies
- Error $|u - v| \rightarrow 0$ at low frequencies

First order ΔΣ modulator



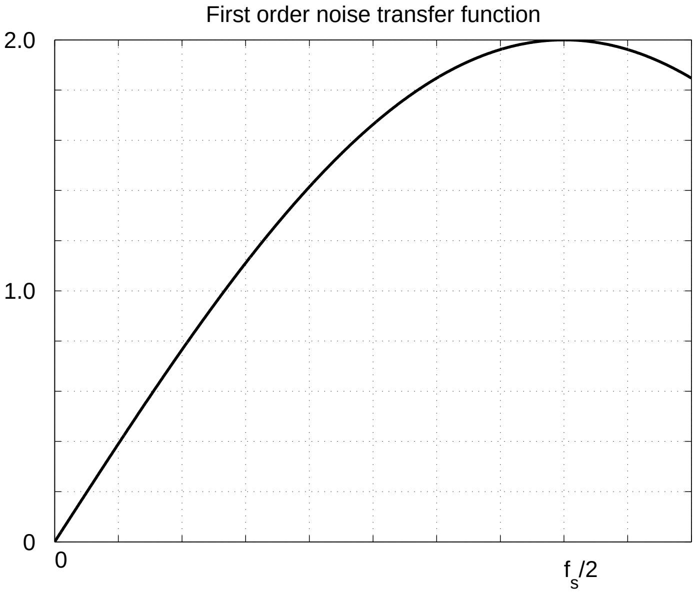
- Loop filter is an accumulator
- Error $|u - v| \to 0$ at low frequencies
- Differencing followed by accumulation–ΔΣ modulator

Noise and Signal transfer functions

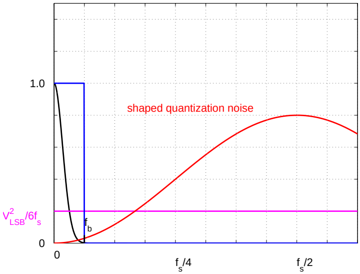


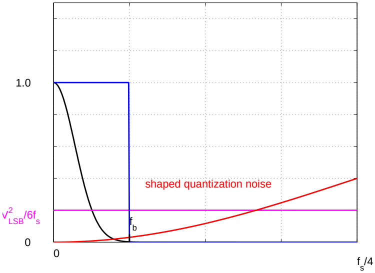
$$
\begin{aligned}
STF &= \frac{V}{U} = \frac{z^{-1}/1-z^{-1}}{1+z^{-1}/1-z^{-1}} \\
&= z^{-1} \\
NTF &= \frac{V}{E} = \frac{1}{1+z^{-1}/1-z^{-1}} \\
&= 1-z^{-1}
\end{aligned}
$$

Noise transfer function



Output noise spectral density





$$
\begin{aligned}
S_{v_e}(\nu) &= S_e(\nu)|1 - exp(-j2\pi\nu)|^2 \\
&= 4S_e(\nu)\sin^2(\pi\nu) \\
S_{v_e}(f) &= 4S_e(f)\sin^2(\pi f/f_s)
\end{aligned}
$$

Output noise in the signal band

$$
\begin{aligned}
v_e^2 &= \int_0^{f_b} S_{v_e}(f)df \\
&= 4\frac{V_{LSB}^2}{6f_s}\int_0^{f_b}\sin^2(\pi f/f_s)df \\
&\approx 4\frac{V_{LSB}^2}{6f_s}\int_0^{f_b}(\pi f/f_s)^2 df \\
&= \frac{V_{LSB}^2}{12}\frac{\pi^2}{3}\left(\frac{2f_b}{f_s}\right)^3 \\
&= \frac{V_{LSB}^2}{12}\frac{\pi^2}{3}\left(\frac{1}{OSR}\right)^3
\end{aligned}
$$

Oversampling with noise shaping

- Output noise $\propto OSR^{-3}$ with first order noise shaping
- Output noise $\propto OSR^{-1}$ with no noise shaping
- Output noise $\propto OSR^{-(2L+1)}$ with L^{th} order noise shaping

Tremendous increase in signal to noise ratio with oversampling

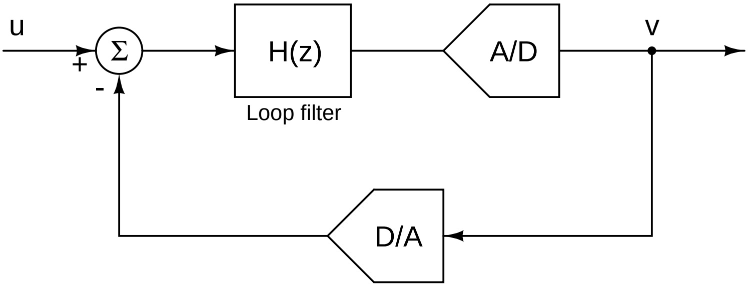
Oversampling, Noise shaping, and Quantization- *SNR*

- 2^N level quantizer with V_{LSB} spacing
- Full scale sinewave input—amplitude $= 2^{N-1}V_{LSB}$
- Oversampling ratio OSR
- First order noise shaping
- Mean squared signal: $\left(2^{N-1}V_{LSB}\right)^2/2$
- Mean squared noise: $(V_{LSB}^2/12)(\pi^2/3)1/OSR^3$
- $SNR = \frac{9}{2\pi^2}2^{2N}\,OSR^3 = 6.02\,N + 30\log OSR - 3.4\,\mathrm{dB}$

Noise transfer functions

- $1 - z^{-1}$ for a first order $\Delta\Sigma$ modulator
- Higer order differencing ($\sim \left(1 - z^{-1}\right)^{N}$) in higher order modulators
- Crucial quantity in the design of delta sigma modulators

ΔΣ analog to digital converter



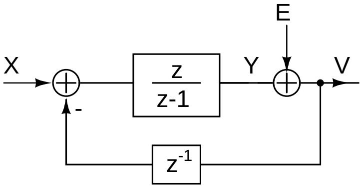
- Analog to digital converter (Flash) in the forward path
- Digital to analog converter in the feedback path
- Output noise in signal band suppressed by noise shaping

Output of the analog to digital converter is the oversampled digital output v

Summary

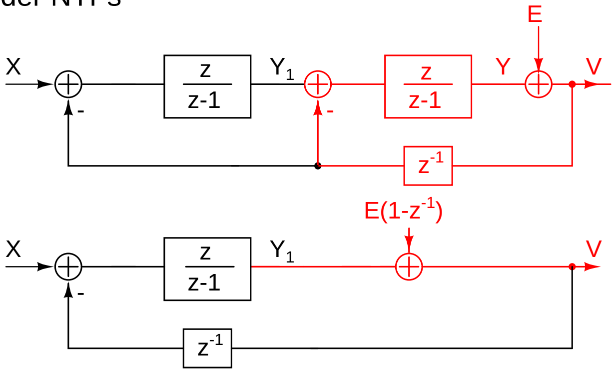
- Sampling preserves the signal if $f_s \geq 2f_b$
- Quantization adds an error $V_{LSB}^2/12$
- Quantization error modelled as additive white noise
- Oversampling and filtering reduces quantization error in the signal band
- Oversampling, noise shaping, and filtering provides a much higher reduction of quantization error in the signal band

High Order NTFs



- For the first order loop
- $V(z) = X(z) + (1 - z^{-1})E(z)$
- STF = 1, NTF = $1 - z^{-1}$
- Can we do better ?

High Order NTFs



- $V(z) = X(z) + (1 - z^{-1})^2 E(z)$
- Second Order Noise Shaping
- Can be extended to higher orders

High Order NTFs

In-band quantization noise for a first order NTF is

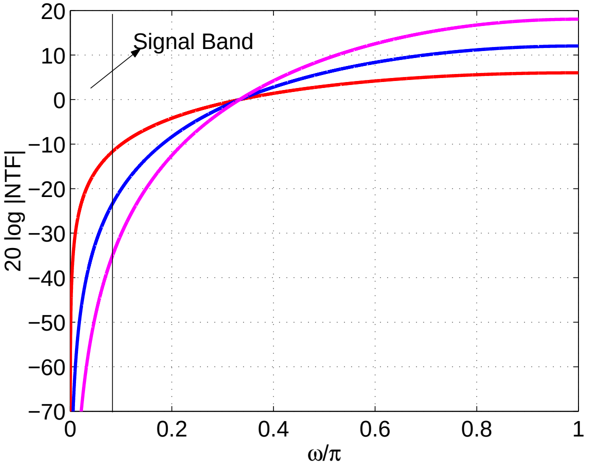
$$Q \approx \frac{\Delta^2}{12\pi}\int_0^{\frac{\pi}{OSR}} \omega^2 d\omega = \frac{\Delta^2}{36\pi}\left(\frac{\pi}{OSR}\right)^3$$

What if the NTF was of the form $(1 - z^{-1})^N$?

$$Q \approx \frac{\Delta^2}{12\pi}\int_0^{\frac{\pi}{OSR}} \omega^{2N} d\omega = \frac{\Delta^2}{12(2N+1)\pi}\left(\frac{\pi}{OSR}\right)^{2N+1}$$

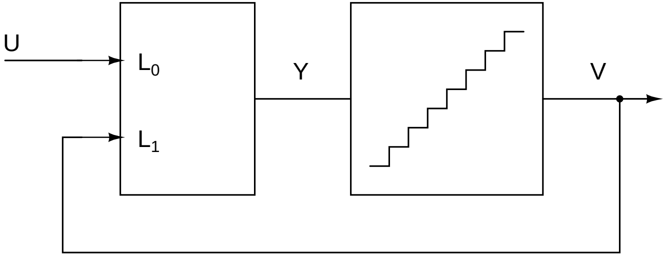
Increasing order can dramatically reduce in-band quantization noise.

High Order NTFs



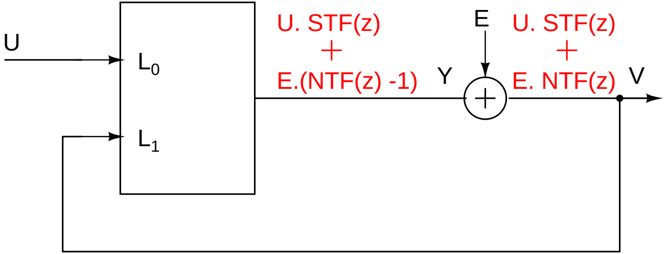
- Higher order $\Rightarrow$ Reduced in-band noise
- NTF gain increases at high frequencies (around $\omega \approx \pi$).
- Why cant one go on increasing order ?

Stability of ΔΣ Modulators



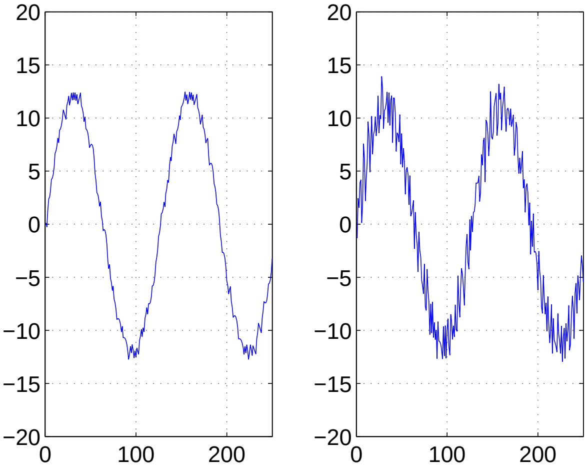
- $Y(z) = L_0(z)U(z) + L_1(z)V(z)$
- v is the quantized version of y.

Stability of ΔΣ Modulators



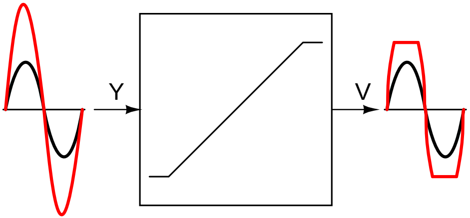
- Quantizer is modeled as an additive noise source.
- $V(z) = U(z)STF(z) + E(z)NTF(z)$
- $Y(z) = U(z)STF(z) + E(z)(NTF(z) - 1)$
- In the signal band, $STF(z) \approx 1$
- Quantizer Input $\approx$ (ADC input) + (Shaped Noise)

Stability of ΔΣ Modulators



- Quantizer input for OBG=1.5 and OBG=3.5

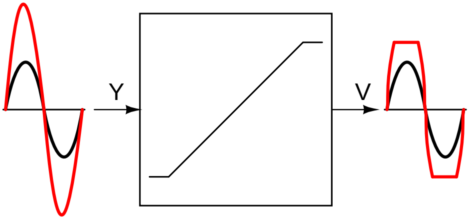
Gain of a Nonlinear Characteristic



Gain = ?

- Assume an infinite precision quantizer with saturation.
- What is its gain ?
- Gain depends on signal.
- Black sinewave : Gain = 1
- Red sinewave : Gain < 1

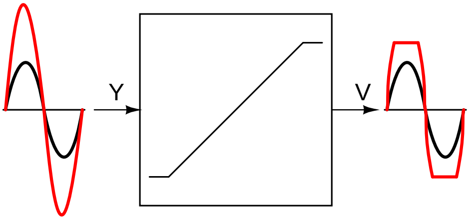
Gain of a Nonlinear Characteristic



Gain = ?

- $Gain = \frac{E(v.y)}{E(y.y)}$.
- Makes intuitive sense.
- $E(v.y)$ is the average value of $v.y$.
- $E(v.y)$ is a measure of how much the output “resembles” the input.

Gain of a Nonlinear Characteristic



Gain = ?

If input to the quantizer exceeds the quantizer range

- Quantizer gain falls.
- If quantizer gain falls, system poles can move out of the unit circle.
- Modulator will become unstable.
- Signal level dependent loop stability has to be **expected**.

Intuition about Loop Stability

- Loop becomes unstable if the quantizer saturates.
- Saturation occurs if the quantizer input exceeds the quantizer range.
- Quantizer Input = ADC Input + Shaped Noise.
- Conclusions -
  - The maximum ADC input **must be smaller** than the quantizer range. (called the Maximum Stable Amplitude (MSA)).
  - More "shaped" noise $\rightarrow$ More likelihood of instability.
- More shaped noise $\rightarrow$ Lesser in-band noise.
- **An aggressive NTF will have a reduced MSA.**

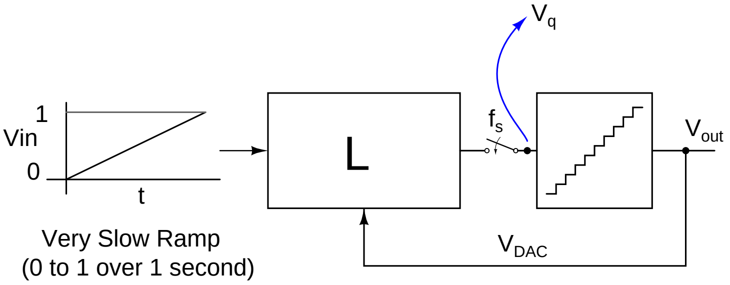
Estimating Maximum Stable Amplitude (MSA)

- Simulation is the best way.
- Keep stepping up the input sinewave amplitude.
  - For every amplitude, compute in-band SNR.
  - Beyond the MSA, the closed loop poles move out of the unit-circle.
  - Noise shaping is lost $\Rightarrow$ In-band SNR falls.
  - Quantizer input tends to infinity.
- Time consuming.

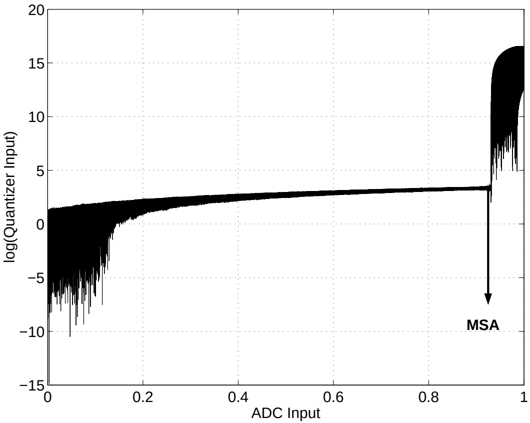
Estimating MSA Without Sinewave Inputs

- Originally proposed by Lars Risbo.
- Put a slowly increasing ramp into the ADC.
  - Beyond the MSA, the closed loop poles move out of the unit-circle.
  - Quantizer input tends to infinity very rapidly.
  - The value of the ADC input when the quantizer input *blows up* is the MSA.
- Found (empirically) to result in an MSA close to that predicted by the sinewave method.
- Much quicker than the sinewave technique.

Estimating MSA Without Sinewave Inputs

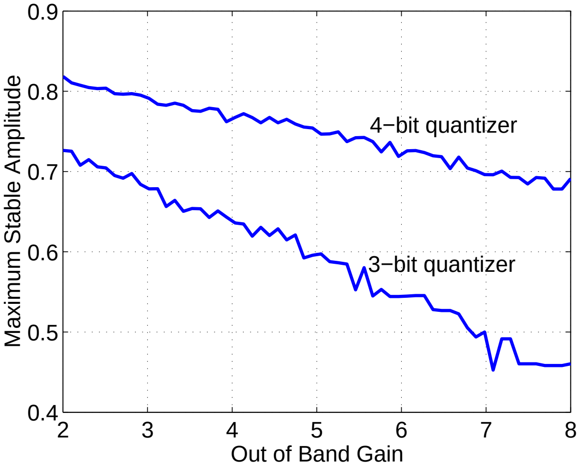


Estimating MSA Without Sinewave Inputs



log(Quantizer Input) versus ADC Input
MSA is about 90% of the quantizer range

MSA vs OBG for a Third Order NTF



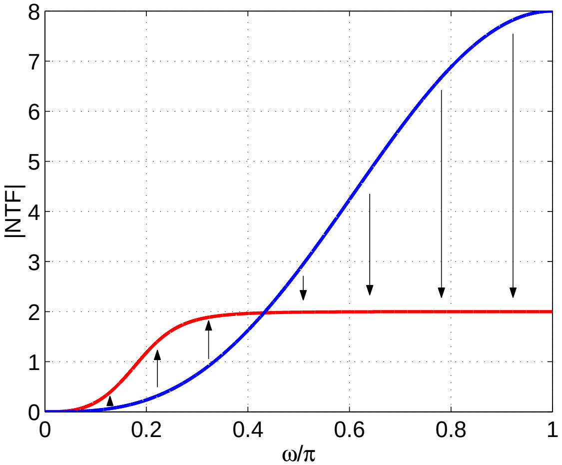
A Systematic NTF Design Procedure

- NTFs of the form $(1 - z^{-1})^N$ have stability problems.
- Why ?
- The OBG is too high (2^N).
- This saturates the quantizer even for small inputs, causing instability.
- The MSA is small.
- Worse for low quantizer resolutions.

A Systematic NTF Design Procedure Solution

- Introduce poles into the NTF.
- $NTF(z) = \frac{(1 - z^{-1})^N}{D(z^{-1})}$.
- Recall that $NTF(\infty) = 1$.
- $\Rightarrow D(z = \infty) = 1$.

Why do poles help ?



- Properly chosen poles reduce OBG of the NTF, enhancing stability.
- However, stability comes at the expense of increased in-band noise.

A Systematic NTF Design Procedure

- Commonly used pole positions : Butterworth, Chebyshev, Inv. Chebyshev etc.
- Coefficients for these approximations readily gotten from MATLAB.
- Schreier's Delta-Sigma Toolbox is an invaluable design aid.
- One should understand what the toolbox does.

A Systematic NTF Design Procedure

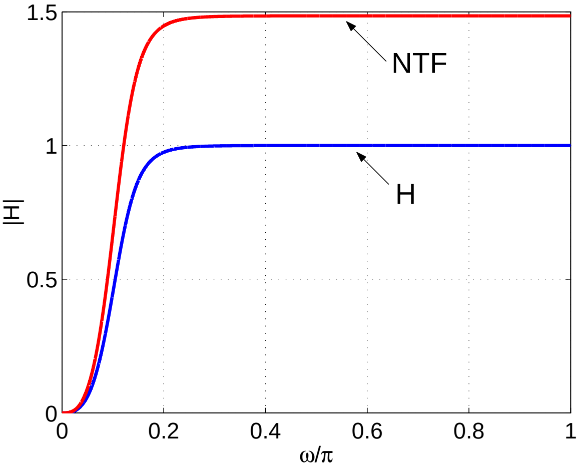
- Choose the order of the NTF.
- OSR, number of levels (n) and desired SNR are known.
    - Example : Order = 3, OSR = 64, $n = 16$, SNR = 115 dB.
- Basically, the NTF is a high-pass filter transfer function.
    - Example : Choose a Butterworth Highpass.
- Choose the 3 dB corner of the high pass filter -
    - Example : $\omega_{3dB} = \frac{\pi}{8}$.
    - For a Butterworth NTF, specifying the cutoff specifies the complete transfer function.

A Systematic NTF Design Procedure

- Get the transfer function from MATLAB
  - `[b,a]=butter(3,1/8,'high')`
  - $H(z) = \dfrac{0.6735 - 2.0204z^{-1} + 2.0204z^{-2} - 0.6735z^{-3}}{1 - 2.2192z^{-1} + 1.7151z^{-2} - 0.4535z^{-3}}$
  - MATLAB sets $|H(e^{j\pi})| = 1$.
- Recall that for $H(z)$ to be a valid NTF, $H(\infty) = 1$.

A Systematic NTF Design Procedure

- Scale $H(z)$ by $\frac{1}{0.6735}$ to obtain $NTF(z)$.
  - $NTF(z) = \dfrac{(1 - 3z^{-1} + 3z^{-2} - z^{-3})}{1 - 2.2192z^{-1} + 1.7151z^{-2} - 0.4535z^{-3}}$



A Systematic NTF Design Procedure

- Find loop filter using $\frac{1}{1+L(z)} = NTF(z)$.
- Simulate the equations describing the modulator.
- Compute the peak SNR.
    - In our example, we obtain SNR=102 dB after simulation.
    - MSA = 0.85.

A Systematic NTF Design Procedure

- If SNR is not enough, repeat the entire procedure above with a higher cutoff frequency for the Butterworth high pass filter.
  - This will increase the OBG (intuition on this later).
  - The MSA will reduce.

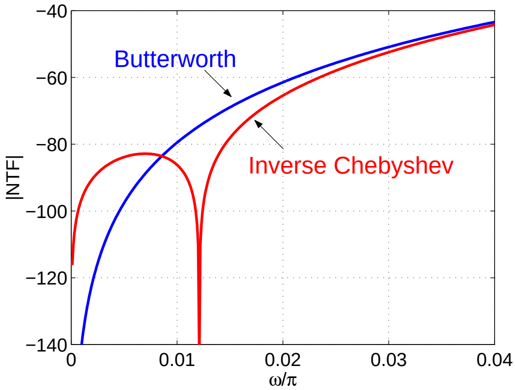
- If SNR is too high, repeat the entire procedure above with a lower cutoff frequency for the Butterworth high pass filter.
  - This will decrease the OBG (intuition on this later).
  - The MSA will increase.

A Systematic NTF Design Procedure

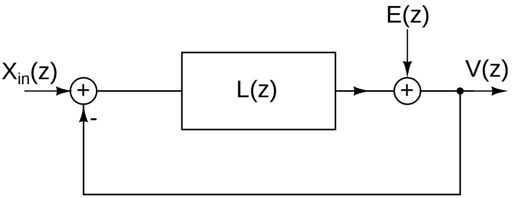
- SNR obtained with 3 dB cutoff of $\frac{\pi}{8}$ is inadequate.
- So, we increase the cutoff frequency to $\frac{\pi}{4}$.
- The peak SNR is around 116 dB.
- OBG = 2.25, MSA = 0.8.
- We are done.
- This iterative process is coded into `synthesizeNTF` in Schreier's toolbox.

A Systematic NTF Design Procedure : Remarks

- Butterworth is one of several candidate high pass filters.
  - All the zeros of transmission are at the origin.
- Another useful family is the inverse Chebyshev approximation.
  - Has complex zeros (on the unit circle).

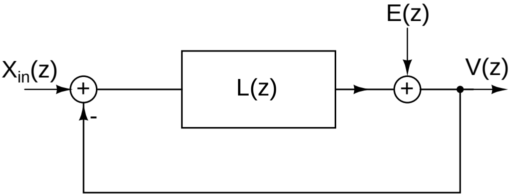


The Sensitivity of a Feedback Loop



- E is a disturbance injected into the feedback loop.
- $V(z) = X(z)\frac{L(z)}{1+L(z)} + E(z)\frac{1}{1+L(z)}$.
- If $L(z) = \infty$, $V(z) = X(z)$.
- The loop rejects $E(z)$, or the loop is *insensitive* to $E(z)$.

The Sensitivity of a Feedback Loop



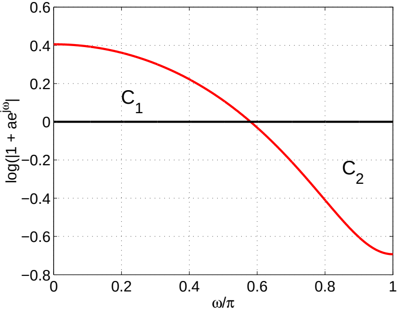
- $L(z)$ cannot be ∞ at all frequencies.
- $V(z) = X(z)\frac{L(z)}{1+L(z)} + E(z)\frac{1}{1+L(z)}$.
- The loop rejects E at frequencies where the loop gain is high.
- How effectively this is done is called the sensitivity function.
- Sensitivity is $\frac{1}{1+L(e^{j\omega})}$

The Sensitivity of a Feedback Loop

- In a $\Delta\Sigma$ loop, sensitivity is the same as the NTF.
- Recall : The first sample of the NTF impulse response is 1.
- Equivalent to $NTF(\infty) = 1$
- The NTF can be written as $\frac{(1+a_1z^{-1})(1+a_2z^{-1}+a_3z^{-2})\cdots}{(1+b_1z^{-1})(1+b_2z^{-1}+b_3z^{-3})\cdots}$
- Poles must be within the unit circle (for a stable loop).
- The zeroes are on the unit circle (or inside).

The Sensitivity of a Feedback Loop

- It can be shown that $\int_0^{\pi} \log(|1 + a_1 e^{-j\omega}|)\, d\omega = 0$, if $|a_1| \leq 1$.



The area above the 0 dB in the log magnitude plot is equal to the area below the 0 dB line.

The Sensitivity of a Feedback Loop

- $\int_0^{\pi} \log(|1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega}|)\, d\omega = 0$
  if the roots of $1 + a_2 z^{-1} + a_3 z^{-2}$ lie within (or on) the unit circle.
- Straightforward to derive, if one accepts the previous result.

The Sensitivity of a Feedback Loop

$$\int_0^{\pi} \log|NTF(e^{j\omega})|d\omega =$$

$$\int_0^{\pi} \log\left|\frac{(1+a_1e^{-j\omega})(1+a_2e^{-j\omega}+a_3e^{-2j\omega})\cdots}{(1+b_1e^{-j\omega})(1+b_2e^{-j\omega}+b_3e^{-3j\omega})\cdots}\right| =$$

$$\int_0^{\pi} \log(|1+a_1e^{-j\omega}|)\,d\omega + \int_0^{\pi} \log(|1+a_2e^{-j\omega}+a_3e^{-j2\omega}|)\,d\omega -$$
$$\int_0^{\pi} \log(|1+b_1e^{-j\omega}|)\,d\omega - \int_0^{\pi} \log(|1+b_2e^{-j\omega}+b_3e^{-j2\omega}|)\,d\omega + \cdots$$

The Sensitivity of a Feedback Loop

$$\int_0^{\pi} \log|NTF(e^{j\omega})|d\omega =$$

$$\int_0^{\pi} \log\left|\frac{(1+a_1e^{-j\omega})(1+a_2e^{-j\omega}+a_3e^{-2j\omega})\cdots}{(1+b_1e^{-j\omega})(1+b_2e^{-j\omega}+b_3e^{-3j\omega})\cdots}\right| =$$

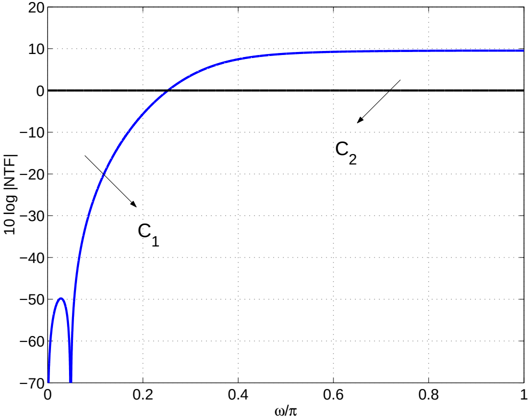
$$\int_0^{\pi} \log(|1+a_1e^{-j\omega}|)\,d\omega + \int_0^{\pi} \log(|1+a_2e^{-j\omega}+a_3e^{-j2\omega}|)\,d\omega -$$
$$\int_0^{\pi} \log(|1+b_1e^{-j\omega}|)\,d\omega - \int_0^{\pi} \log(|1+b_2e^{-j\omega}+b_3e^{-j2\omega}|)\,d\omega + \cdots$$

= Zero

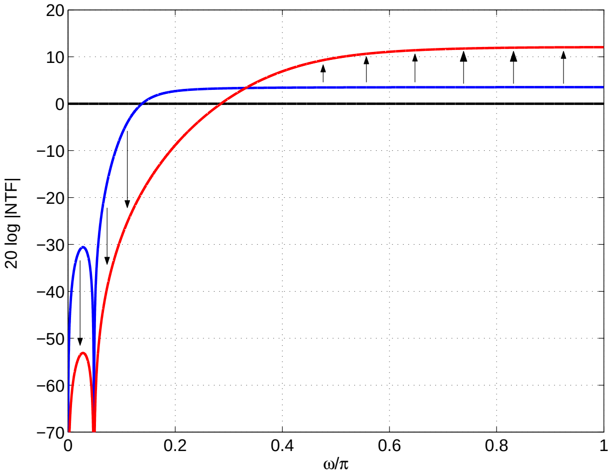
The Bode Sensitivity Integral

$$\int_0^{\pi} \log |NTF(e^{j\omega})| d\omega = 0$$

The Integral of the Log Magnitude of an NTF is 0

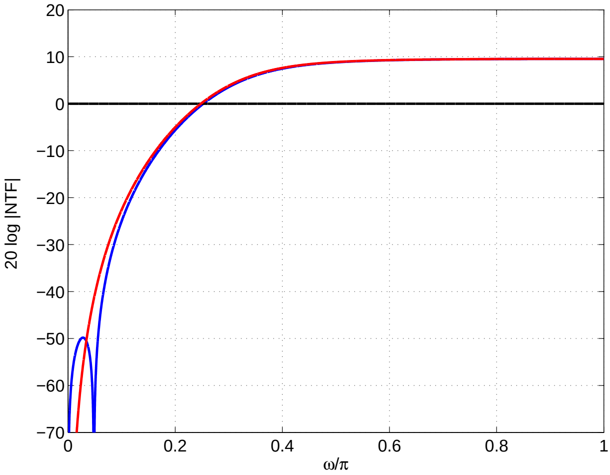


The Bode Sensitivity Integral



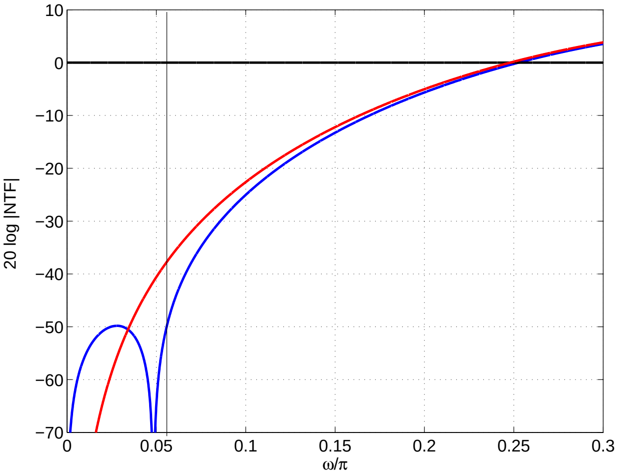
Good inband performance at the expense of poor out-of-band performance.

The Bode Sensitivity Integral



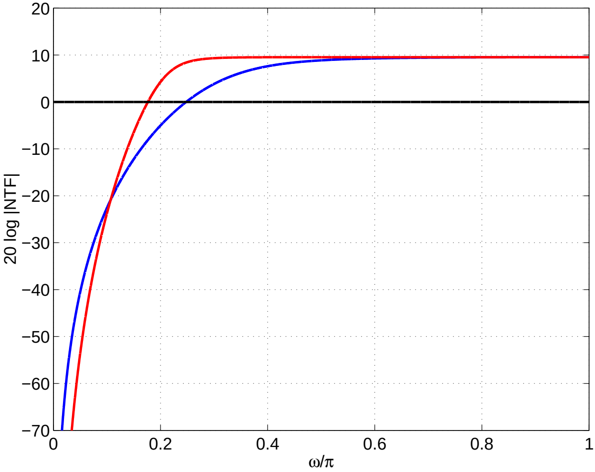
Complex zeros better than choosing all NTF zeros at the origin.

The Bode Sensitivity Integral



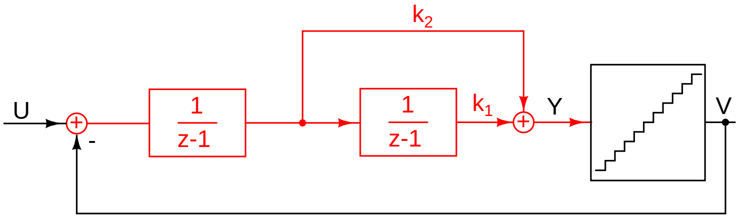
Complex zeros better than choosing all NTF zeros at the origin.

The Bode Sensitivity Integral



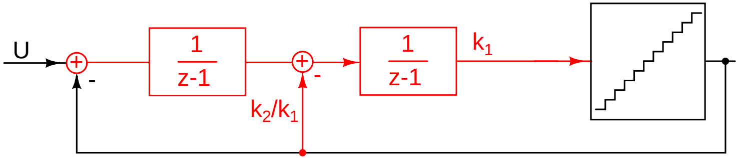
Higher order $\Rightarrow$ less in-band noise.

Loop Filter Architectures



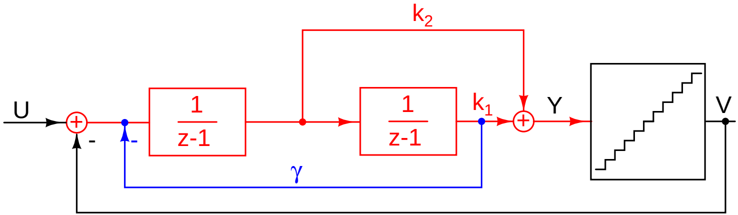
- Remember : A quantizer = ADC + DAC.
- Needs ONE DAC.
- Loop filter gain goes to infinity at DC, with order 2.
- Both NTF zeros at DC ($z = 1$).
- Called CIFF (Cascade of Integrators Feed Forward)

Loop Filter Architectures



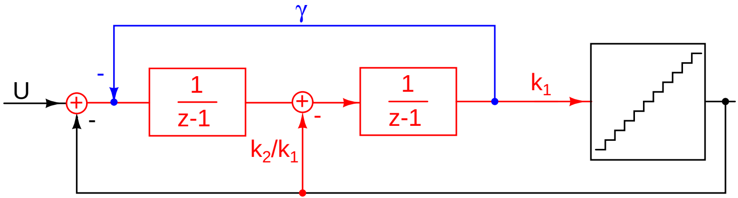
- Remember : A quantizer = ADC + DAC.
- Needs TWO DACs.
- Loop filter gain goes to infinity at DC, with order 2.
- Both NTF zeros at DC ($z = 1$).
- Called CIFB (Cascade of Integrators Feed Back).

Loop Filter Architectures



- CIFF loop with complex zeros.
- NTF zeros are at $1 \pm j\sqrt{\gamma}$.

Loop Filter Architectures



- CIFB loop with complex zeros.
- NTF zeros are at $1 \pm j\sqrt{\gamma}$.

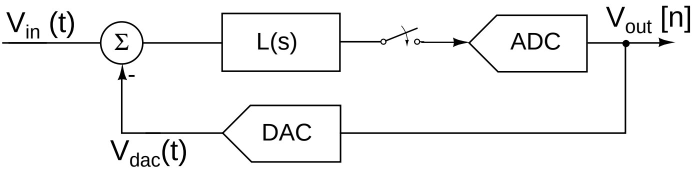
Loop Filter Implementation

- Traditionally done in discrete-time.
- Implemented using switched-capacitor techniques.
- Switched capacitor circuits have several advantages.
  - Exact nature of settling is irrelevant, only the settled value matters.
  - Pole-zero locations of the loop filter are set by capacitor ratios, which are exteremely accurate.
  - Insensitive to clock jitter, as long as complete settling occurs.
  - Easier to simulate.

Loop Filter Implementation Switched capacitor loop filters have disadvantages too -

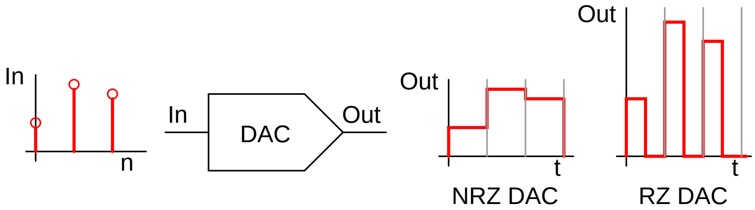
- Difficult to drive from external sources due to the large spike currents drawn.
- Upfront sampling : requires an anti-alias filter.
- Integrator opamps consume more power than continuous-time counterparts.
- Require large capacitors to lower kT/C noise.

Continuous-time Loop Filters



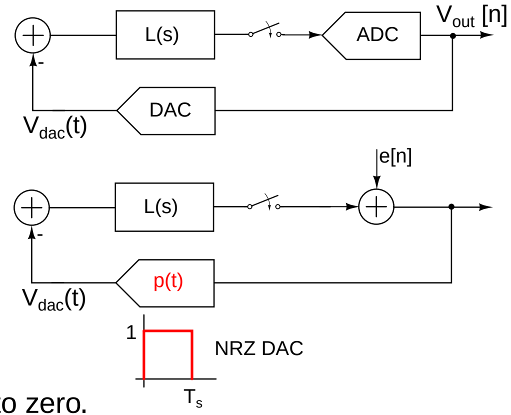
- What is the NTF ?
- How does one design such a loop ?
- How does this compare with a discrete-time loop filter ?

DAC Modeling



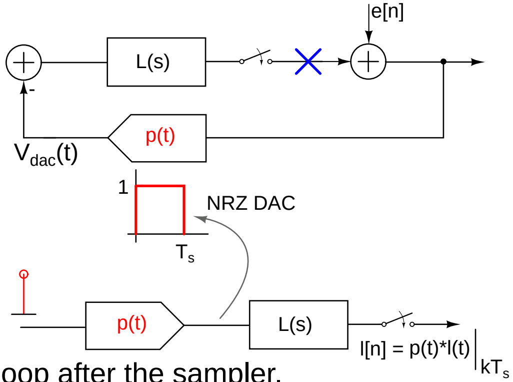
- The input to the DAC is a digital code a_k that changes every T_s.
- The DAC output is an analog waveform.
- Output = $\sum_k a_k p(t - kT_s)$
- $p(t)$ is called the pulse-shape.
- Commonly used shapes are the Non-Return to Zero (NRZ) and Return-to-Zero (RZ) pulses.

Loop Modeling



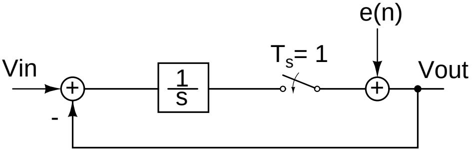
- Set input to zero.
- Replace ADC-DAC with quantization noise $e(n)$.
- DAC is modeled as a filter with impulse response $p(t)$.

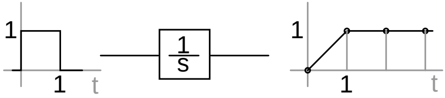
Loop Modeling



- Break the loop after the sampler.
- Apply a discrete time impulse.
- What comes back is $l[n] = p(t) * l(t)|_{kT_s}$.
- The z-transform of $l[n]$ is the equivalent discrete time loop filter.

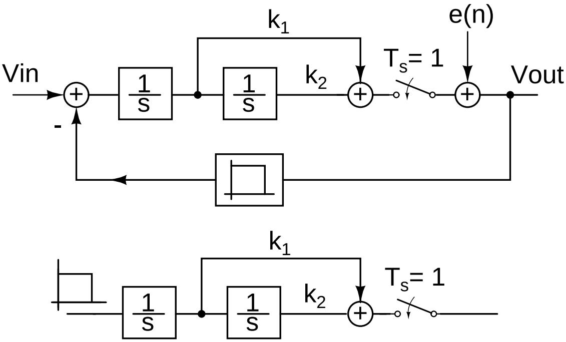
A First Order Example





- Discrete-time equivalent impulse response of the loop filter 0, 1, 1, 1, 1 $\cdots$
- $L(z) = \frac{z^{-1}}{1-z^{-1}}$
- $NTF(z) = \frac{1}{1+L(z)} = 1 - z^{-1}$

A Second Order Example



- Say we need $NTF(z) = (1 - z^{-1})^2$.
- Discrete-time impulse response through k_1
  $k_1(r_1(t) - r_1(t-1)) = \{0, k_1, k_1, k_1, k_1 \cdots\}$
- Discrete-time impulse response through k_2
  $k_2(r_2(t) - r_2(t-1)) = \frac{1}{2}\{0, k_2, 3k_2, 5k_2 \cdots\}$

A Second Order Example

- Discrete-time impulse response through k_1

  $$k_1(r_1(t) - r_1(t-1)) = \{0, k_1, k_1, k_1, k_1 \cdots\} \Rightarrow \frac{k_1 z^{-1}}{1 - z^{-1}}.$$

- Discrete-time impulse response through k_2

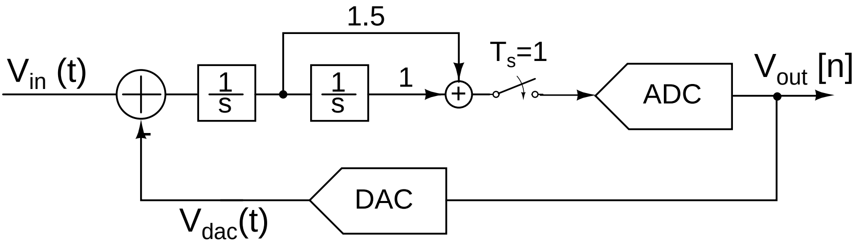
  $$k_2(r_2(t) - r_2(t-1))$$

  $$= \tfrac{1}{2}\{0, k_2, 3k_2, 5k_2, 7k_2 \cdots\} \Rightarrow \frac{k_2 z^{-1}}{(1 - z^{-1})^2} - \frac{0.5k_2 z^{-1}}{1 - z^{-1}}.$$

- $$L(z) = \frac{(k_1 + 0.5k_2)z^{-1} + (-k_1 + 0.5k_2)z^{-2}}{(1 - z^{-1})^2}.$$

A Second Order Example

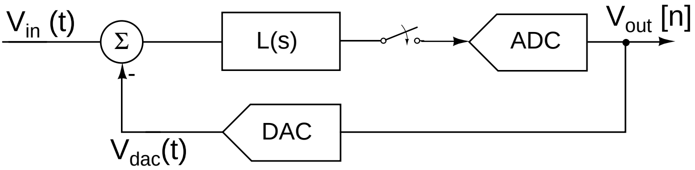
- $L(z) = \dfrac{(k_1 + 0.5k_2)z^{-1} + (-k_1 + 0.5k_2)z^{-2}}{(1 - z^{-1})^2}$.
- To achieve $NTF(z) = (1 - z^{-1})^2$, we need $L(z) = \dfrac{2z^{-1} - z^{-2}}{(1 - z^{-1})^2}$.
- $\Rightarrow k_1 = 1.5, k_2 = 1$.

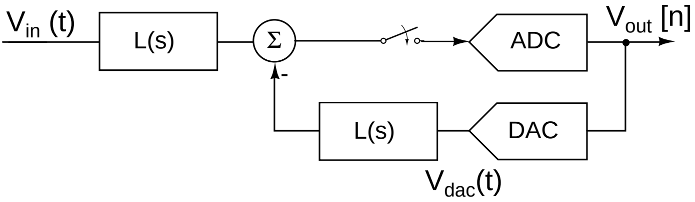


Continuous-time Sigma-Delta Summary

- It is possible to “emulate” a D-T loop filter with a C-T one.
- The equivalence depends on the DAC pulse shape.
- The technique can be extended to high order NTFs -
  - From the desired $NTF(z)$, find $L(z)$
  - Convert $L(z)$ into $L(s)$ using the DAC pulse shape
  - The MATLAB command `d2c` will do it for you, for an NRZ DAC.
  - Implement $L(s)$ using any one of the loop filter topologies.
- A CT loop filter has several other advantages ... listen on.

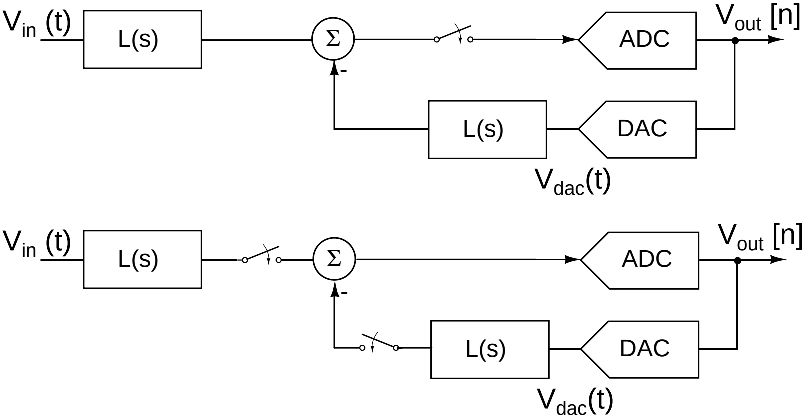
The Anti-Aliasing Feature of CT ΔΣ Modulators





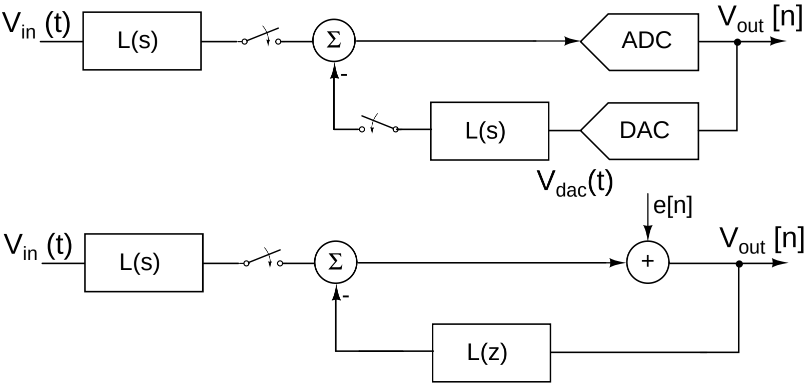
- Move $L(s)$ outside the loop

The Anti-Aliasing Feature of CT ΔΣ Modulators



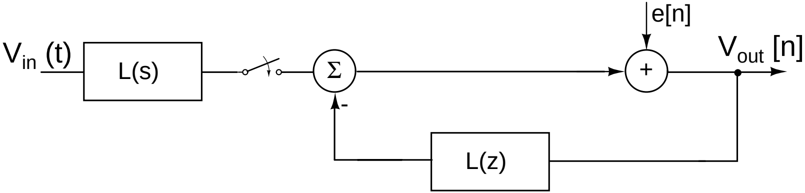
- Move the sampler outside the loop

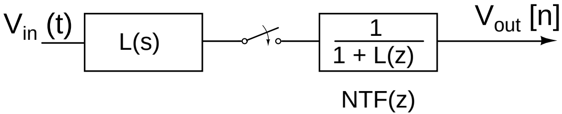
The Anti-Aliasing Feature of CT ΔΣ Modulators



- Replace the cascade of the DAC and $L(s)$ by the equivalent discrete-time filter $L(z)$.

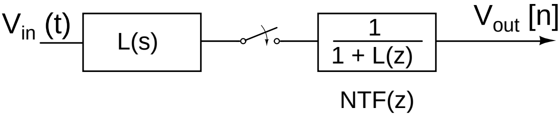
The Anti-Aliasing Feature of CT ΔΣ Modulators





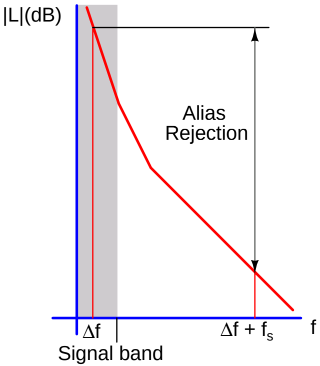
- $NTF(z) = 1/(1 + L(z))$

The Anti-Aliasing Feature of CT $\Delta\Sigma$ Modulators



- Consider a tone at frequency Δf in the signal band.
- Response to frequency Δf is $L(\Delta f)NTF(\Delta f)$.
- In a general ADC, a tone $(\Delta f + f_s)$ can alias as Δf.
- What about a CTDSM ?
- Response to frequency $(\Delta f + f_s)$ is $L(\Delta f + f_s)NTF(\Delta f)$

The Anti-Aliasing Feature of CT $\Delta\Sigma$ Modulators



- Alias rejection is $\left|\frac{L(\Delta f)}{L(\Delta f + f_s)}\right|$
- Implicit anti-aliasing without an explicit filter !
- Valuable feature of CT Delta-Sigma modulators.

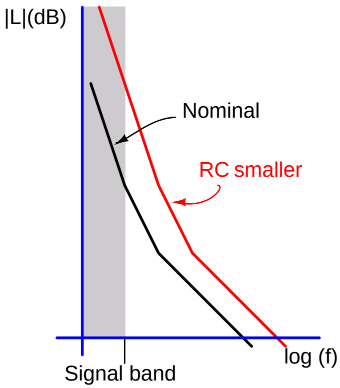
Effect of Time-Constant Variations in the Loop Filter

- On-chip RC's vary with process and temperature.
- On an integrated circuit, ratios of like elements are tightly controlled.
- We need to only worry only about quantities with "dimensions".
- What happens due to absolute variation of RC time constants ?

Effect of RC Variations : Intuitive explanation

If all RC time-constants decrease

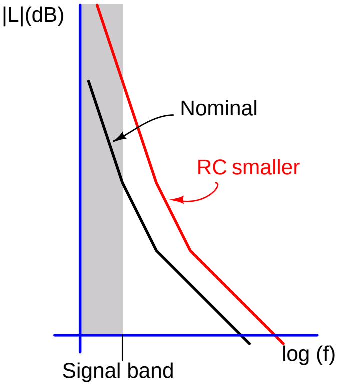
- Loop filter bandwidth increases.
- In-band loop gain increases.
- Lower in-band quantization noise - better in-band NTF.
- NTF must be worse out-of-band - higher OBG.



Effect of RC Variations : Intuitive explanation

If all RC time-constants decrease

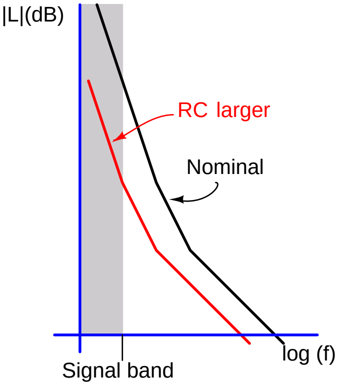
- Higher OBG for the NTF.
- Reduced maximum stable amplitude.
- Closer to instability.



Effect of RC Variations : Intuitive explanation

If all RC time-constants increase

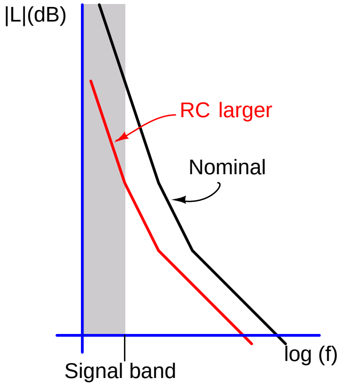
- Loop filter bandwidth decreases.
- In-band loop gain decreases.
- Higher in-band quantization noise - poorer in-band NTF.
- NTF must be better out-of-band - lower OBG.



Effect of RC Variations : Intuitive explanation

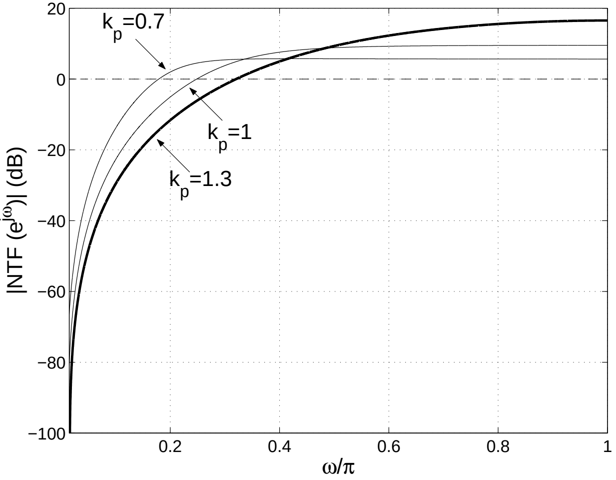
If all RC time-constants increase

- Lower OBG for the NTF.
- Increased maximum stable amplitude.
- “More” stable.



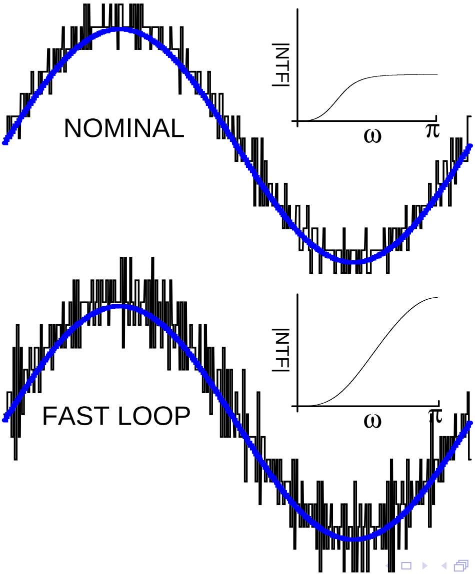
Effect of RC Variations on the NTF

Nominal NTF : Maximally flat with an OBG=3



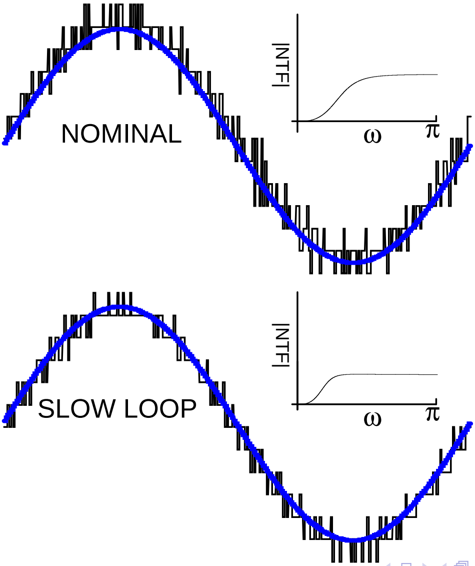
Effect of RC Variations: Time Domain Intuition

Nominal NTF : Maximally flat with an OBG=3



Effect of RC Variations: Time Domain Intuition

Nominal NTF : Maximally flat with an OBG=3

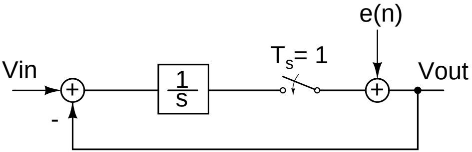


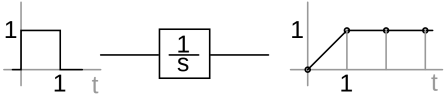
Excess Delay in CT $\Delta\Sigma$ Modulators

Why is there excess loop delay ?

- Quantizer needs time to make a decision.
- Finite operational amplifier gain-bandwidth product.
- DEM logic delay in multibit converters.

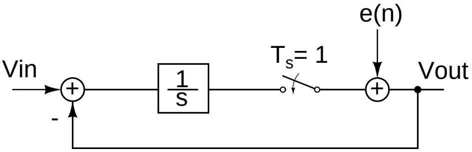
A First Order Example

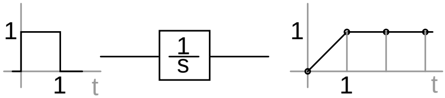




- Loop filter is an integrator.
- An NRZ DAC is used.
- Sampling Rate = 1 Hz

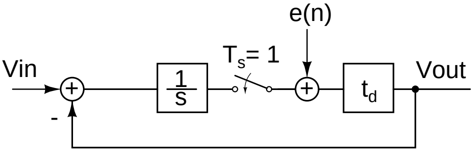
Excess Delay in CT ΔΣ Modulators

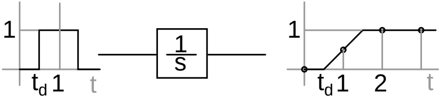




- Discrete-time equivalent impulse response of the loop filter 0, 1, 1, 1, 1 $\cdots$
- $L(z) = \frac{z^{-1}}{1-z^{-1}}$
- $NTF(z) = \frac{L(z)}{1+L(z)} = 1 - z^{-1}$

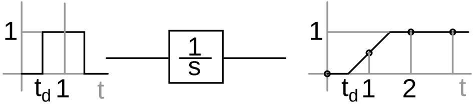
Excess Delay in CT ΔΣ Modulators





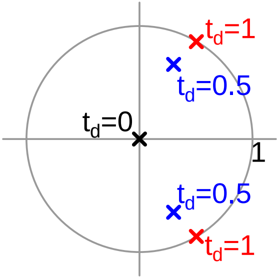
- In practice, the quantizer needs time to make a decision.
- Equivalent to a delay t_d in the loop.
- What happens to the NTF of the loop ?

Excess Delay in CT ΔΣ Modulators



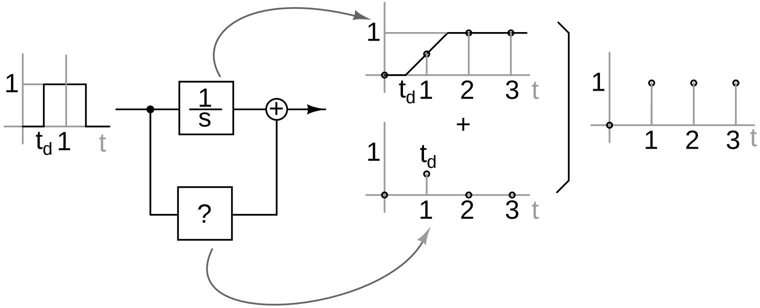
- Discrete-time equivalent impulse response of the loop filter $\{0, 1 - t_d, 1, 1, 1 \cdots\} = \{0, 1, 1, 1, 1 \cdots\} + \{0, -t_d, 0, 0, 0 \cdots\}$
- $L(z) = \frac{z^{-1}}{1-z^{-1}} - t_d z^{-1}$
- $NTF(z) = \frac{L(z)}{1+L(z)} = \frac{1-z^{-1}}{1-t_d z^{-1}+t_d z^{-2}}$

Excess Delay in CT ΔΣ Modulators



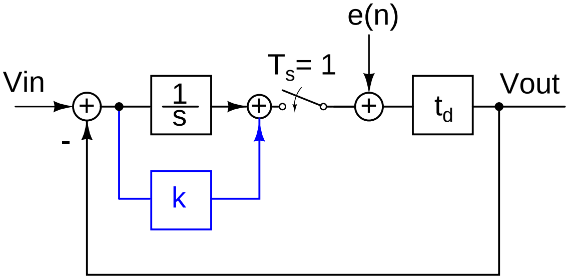
- The order of the system is increased.
- Becomes unstable for $t_d = 1$
- Not surprising - a delay in a feedback loop is always problematic.
- Aggressive NTF designs are more sensitive to excess delay.

Fix for Excess Delay : Basic Idea



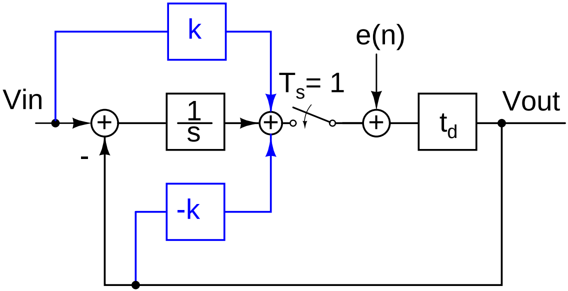
- Impulse response of the loop filter with delay $\{0, t_d, 1, 1, 1 \cdots\} = \{0, 1, 1, 1, 1 \cdots\} + \{0, -t_d, 0, 0, 0 \cdots\}$
- Add a path with discrete-time response $\{0, t_d, 0, 0, 0 \cdots\}$ to the loop filter.

Fix for Excess Delay : Basic Idea



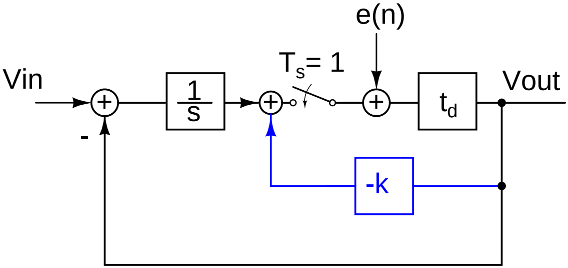
- Implementation of feedforward path in the loop.

Fix for Excess Delay : Basic Idea



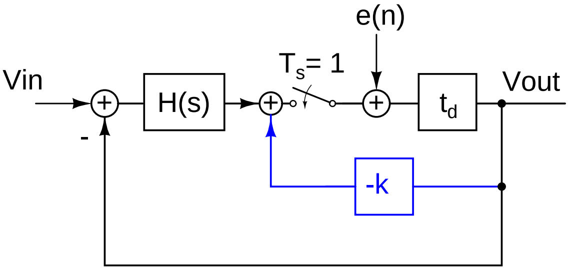
- Equivalent implementation of loop filter feedforward.

Fix for Excess Delay : Basic Idea



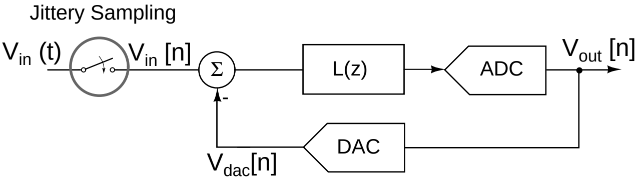
- Eliminate path from the input (small compared to the integrator output).
- **Excess delay can be compensated by adding a direct path around the quantizer.**

Excess Delay Compensation : Summary



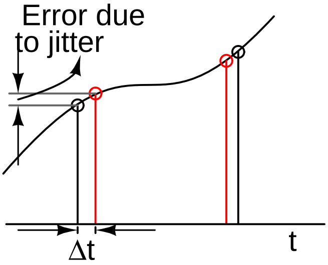
- Direct path around the quantizer.
- Modification of H(s) (coefficient tuning).
- General approach valid even for high order modulators.
- Determining coefficients and *k* best done numerically.

Clock Jitter in Discrete-time ΔΣ ADCs



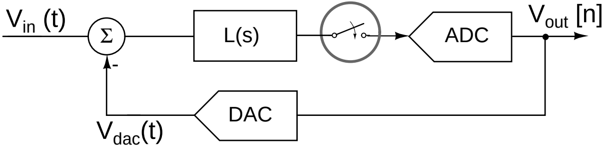
- The input is sampled outside the modulator

Clock Jitter in Discrete-time ΔΣ ADCs



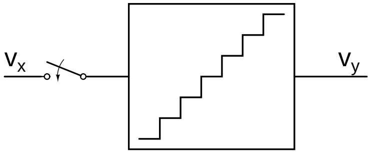
- Treat the input as a sinusoid with maximum amplitude A.
- Error due to jitter at the sampling instant is $\Delta t \frac{dA\sin(2\pi f_{in} t)}{dt}$
- Assume white clock jitter with RMS value σ_j.
- RMS value of noise due to jitter in the signal bandwidth is $\sigma_j \sqrt{2} A\pi f_{in}/OSR$

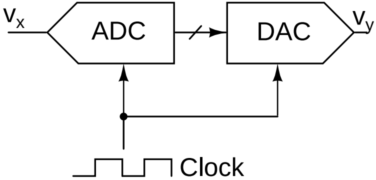
Clock Jitter in Continuous-time ΔΣ ADCs



- The input is sampled inside the modulator.

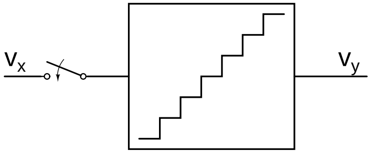
The Ideal Sampler/Quantizer

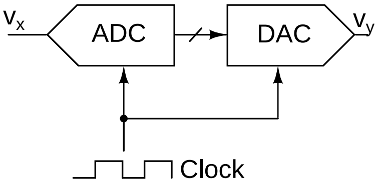




- Input is sampled in the ADC.
- ADC output code is sampled by the DAC.

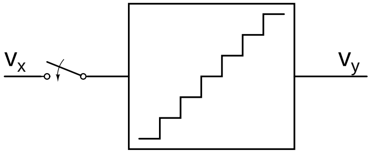
The Ideal Sampler/Quantizer

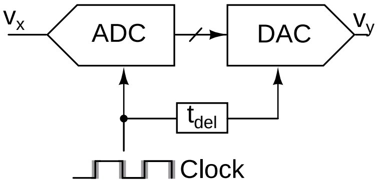




- DAC output analog waveform - fedback into the loopfilter.
- No delay in the quantizer, no clock jitter.
- ADC output code is the modulator output.

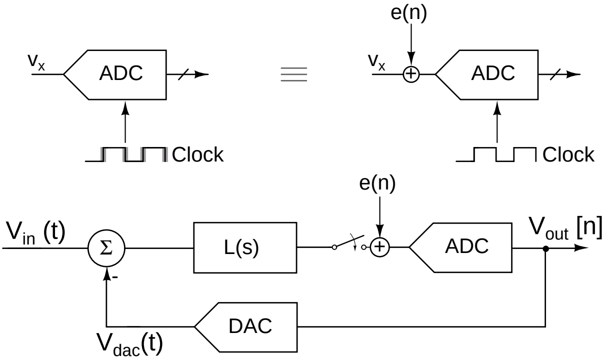
The Real Sampler/Quantizer





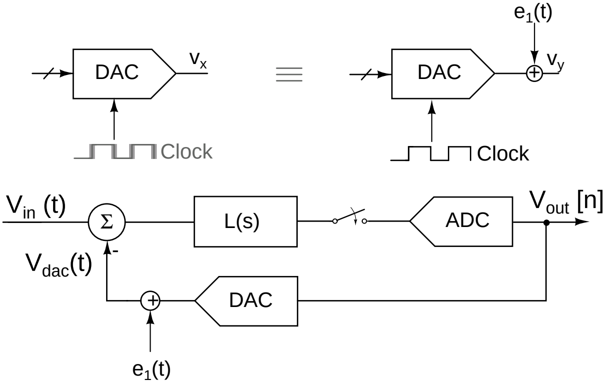
- ADC needs a finite time for conversion.
- DAC is clocked t_{del} later.
- The clock is jittery.

Effect of ADC Sampling Jitter



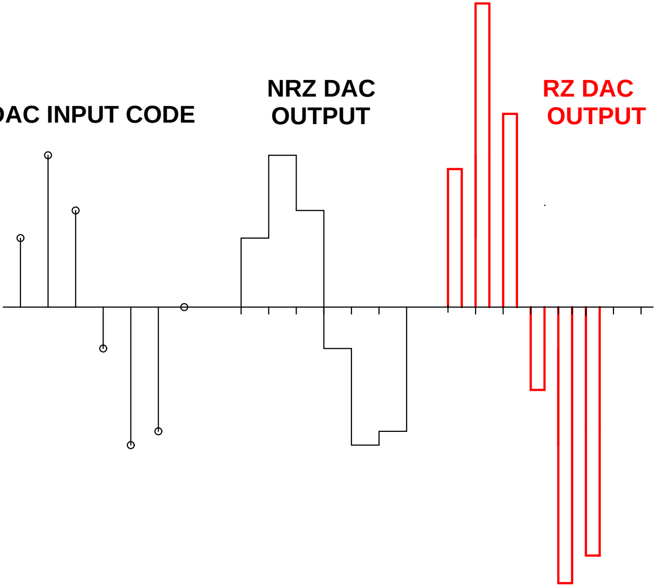
- Modelled as an error preceding the ADC.
- Noise shaped by the loop.

Effect of DAC Reconstruction Jitter

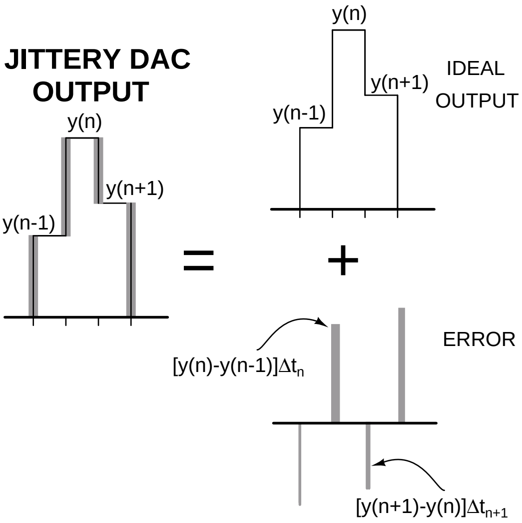


- Modelled as an error following the DAC.
- Equivalent to an error at the modulator input.
- Degrades performance.

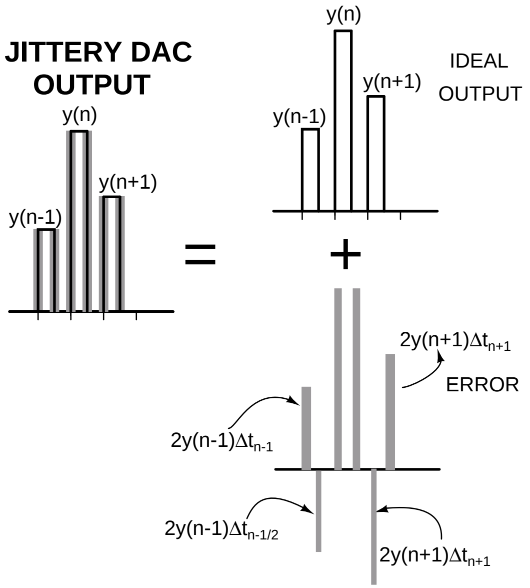
Types of DACs : NRZ versus RZ



Modeling Clock Jitter in NRZ DACs



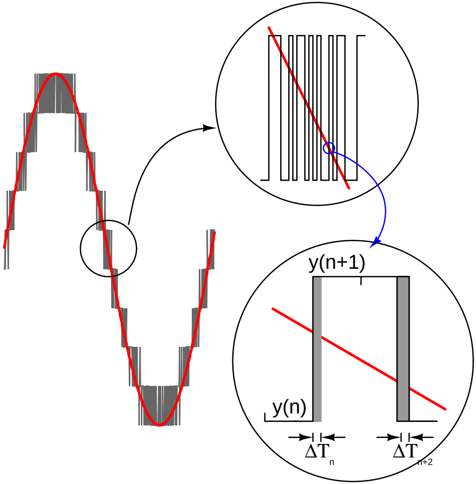
Modeling Clock Jitter in RZ DACs



Clock Jitter in NRZ versus RZ DACs

- Error depends on the height & number of transisitions in the DAC output waveform.
- NRZ DACs have a transition height $y(n) - y(n-1)$, one transistion every T_s.
- RZ DACs have a transition height $2y(n)$, two transistions every T_s.
- RZ DACs are MUCH more sensitive to clock jitter !

Clock Jitter in Modulators with NRZ DACs



Effect of Jitter on SNR

$$e_j(n) = [y(n) - y(n-1)]\,\frac{\Delta t(n)}{T}$$

$$\sigma_{ej}^2 = \sigma_{dy}^2\,\frac{\sigma_{\Delta t}^2}{T^2}$$

$$y(n) = v_{in}(n) + e_q(n) * h(n)$$

- v_{in} is the input.
- e_q is the quantization noise sequence.
- $h(n)$ is the impulse response corresponding to the NTF.

$$y(n) - y(n-1) = v_{in}(n) - v_{in}(n-1) + (e_q(n) - e_q(n-1)) * h(n)$$

Due to oversampling, $v_{in}(n) \approx v_{in}(n-1)$

$$y(n) - y(n-1) \approx (e_q(n) - e_q(n-1)) * h(n)$$

$e_q(n)$ is a white sequence with mean square value σ_{lsb}^2.

$$\sigma_{dy}^2 \approx \frac{\sigma_{lsb}^2}{\pi} \int_0^{\pi} |(1 - e^{-j\omega}) NTF(e^{j\omega})|^2 d\omega$$

The in-band noise due to jitter (J) is

$$J \approx \frac{\sigma_{\Delta T_s}^2}{T^2} \frac{\sigma_{lsb}^2}{\pi OSR} \int_0^{\pi} |(1 - e^{-j\omega}) NTF(e^{j\omega})|^2 d\omega$$

Effect of Jitter on SNR

$$J = \frac{\sigma^2_{\Delta T_s}}{T^2} \frac{\sigma^2_{lsb}}{\pi OSR} \int_0^{\pi} \left|(1 - e^{-j\omega})\, NTF(e^{j\omega})\right|^2 d\omega \qquad (1)$$

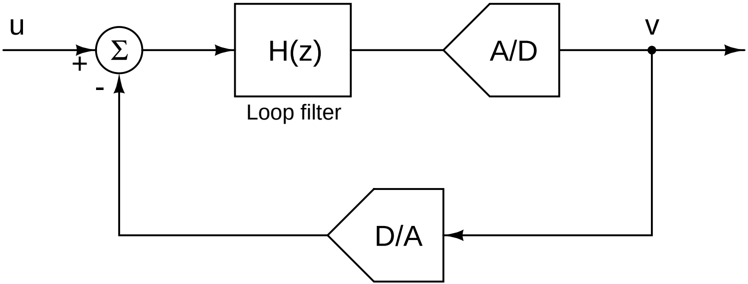
- Observation : The NTF at high frequencies (close to $\omega = \pi$) contributes the most to J.
- $\Rightarrow$ NTFs with high OBG result in more jitter noise.
- Smaller LSB, less jitter noise $\rightarrow$ multibit modulator less sensitive to jitter.

Example Calculation

- Audio modulator, 24 kHz bandwidth.
- OSR = 64 ($f_s = 3.072\,MHz$), 4-bit quantizer.
- Quantizer input range is 2 V.
- LSB size is 2/16 $\rightarrow \sigma_{lsb}^2 = \frac{(2/16)^2}{12}$
- Assume 100 ps RMS jitter.
- J = $(1.28\,\mu\mathrm{V})^2$.
- Maximum Signal Amplitude is 0.83 V peak.
- Signal to Jitter Noise Ratio is $20\log(\frac{0.83/\sqrt{2}}{1.28\,\mu\mathrm{V}}) = 113\,\mathrm{dB}$
- Conclusion : 100 ps RMS Jitter is not an issue for 15 bit resolution.

Feedback DAC nonlinearity

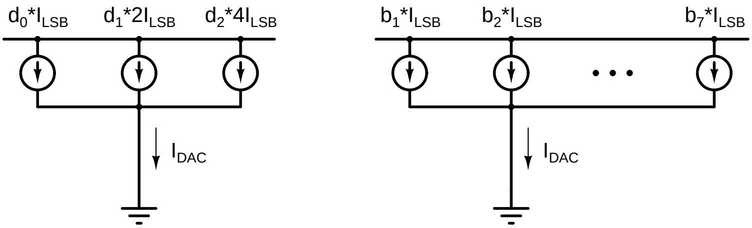
ΔΣ analog to digital converter



- Typically 4 bits (16 levels) or less in the quantizer

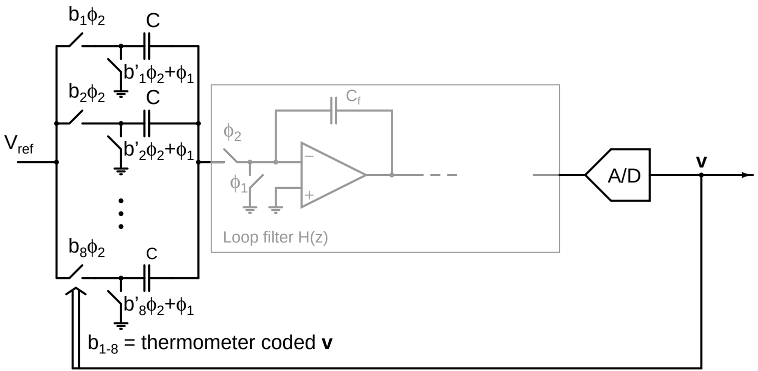
Feedback DAC architecture

quantizer output v = $d_{2\text{-}0}$ [binary] = $b_{1\text{-}7}$ [thermometer]



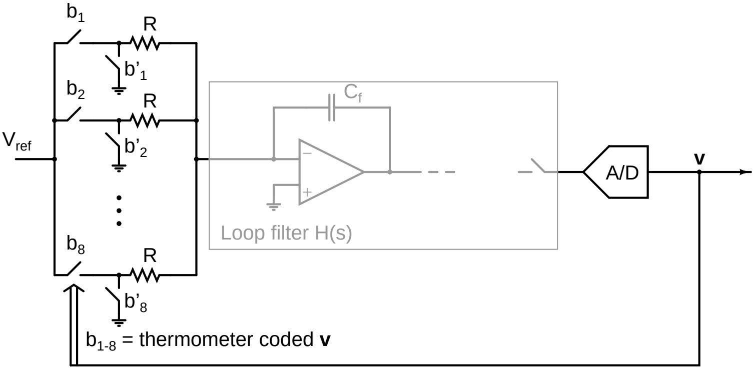
$I_{DAC} = kI_{LSB}$, k={0,1,...,7}

- Flash quantizer gives a thermometer coded output
- Thermometer coded DAC: high accuracy and small loop delay



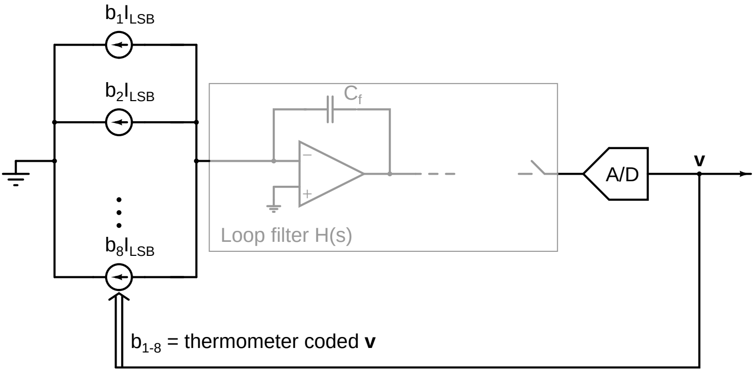
- Array of M capacitors for $M + 1$ levels
- Flash quantizer output v
- v capacitors charged to V_{ref} and $M - v$ to zero volts

Continuous-time ΔΣ modulator



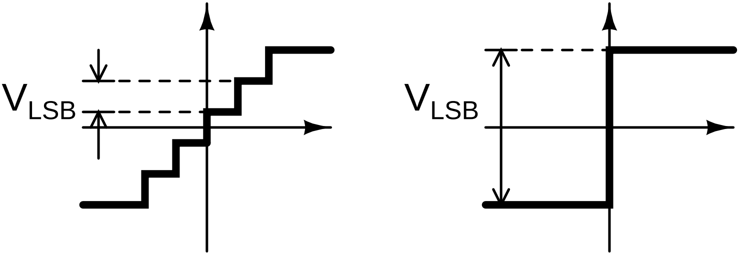
- Array of M resistors for $M+1$ levels
- Flash quantizer output v
- v resistors connected to V_{ref} and $M-v$ to ground

Continuous-time ΔΣ modulator



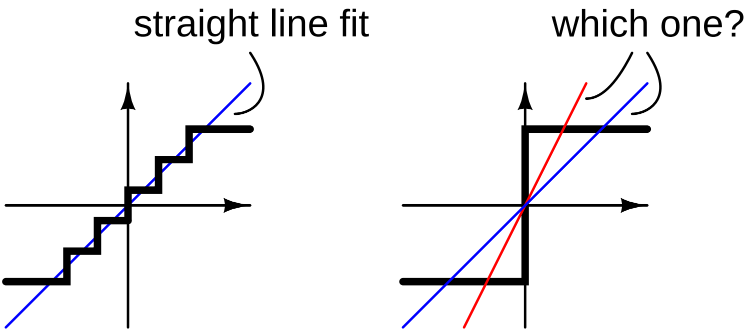
- Array of M current sources for $M + 1$ levels
- Flash quantizer output v
- v current sources turned on and $M - v$ turned off

Multi bit versus single bit quantizer



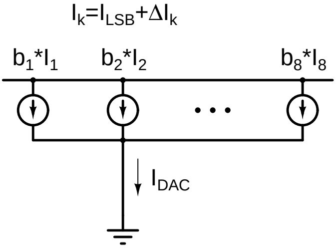
- Multi bit: smaller LSB $\Rightarrow$ lower quantization noise
- Single bit: larger LSB $\Rightarrow$ higher quantization noise

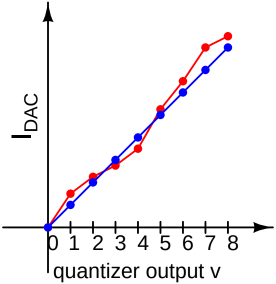
Multi bit versus single bit quantizer

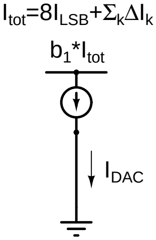


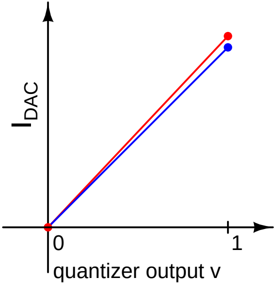
- Multi bit quantizer
  - Clearly defined gain
  - Conforms to prediction using linear models
- Single bit quantizer
  - Signal dependent quantizer gain
  - Deviates from prediction using linear models

Multi bit versus single bit quantizer





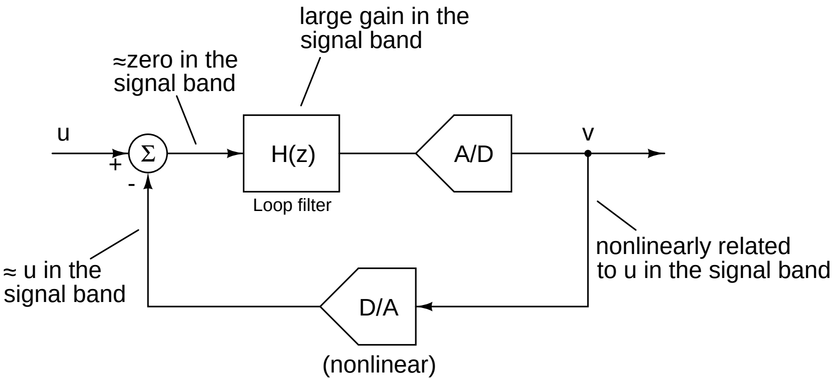




Multi bit versus single bit quantizer

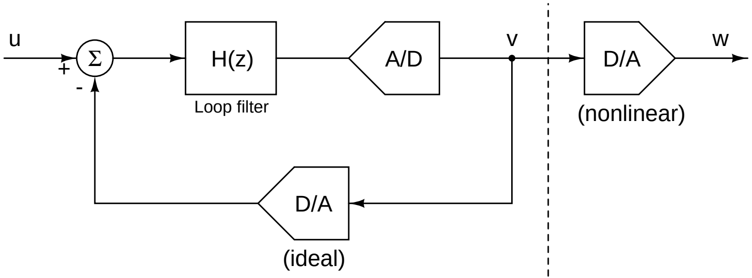
- Multi bit quantizer
  - Characteristics not linear due to mismatch
- Single bit quantizer
  - Characteristics always linear

Effect of DAC nonlinearity



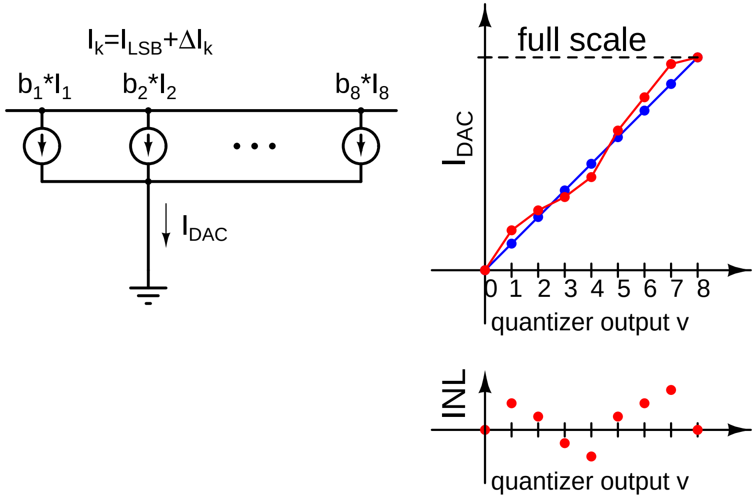
- DAC output equals the input u
- v related to the input u by inverse nonlinearity of the DAC

Modeling the effect of DAC nonlinearity



- Nonlinear DAC driven by an ideal $\Delta\Sigma$ modulator and its output w analyzed

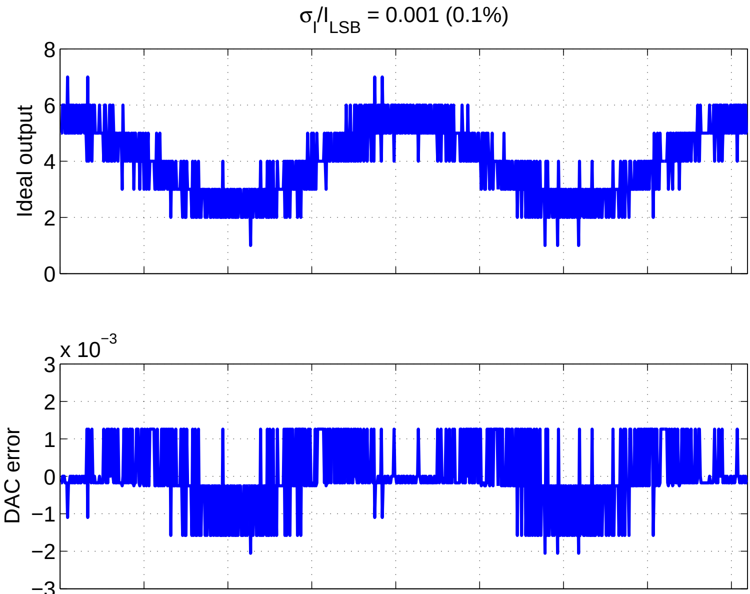
Multi bit feedback DAC nonlinearity



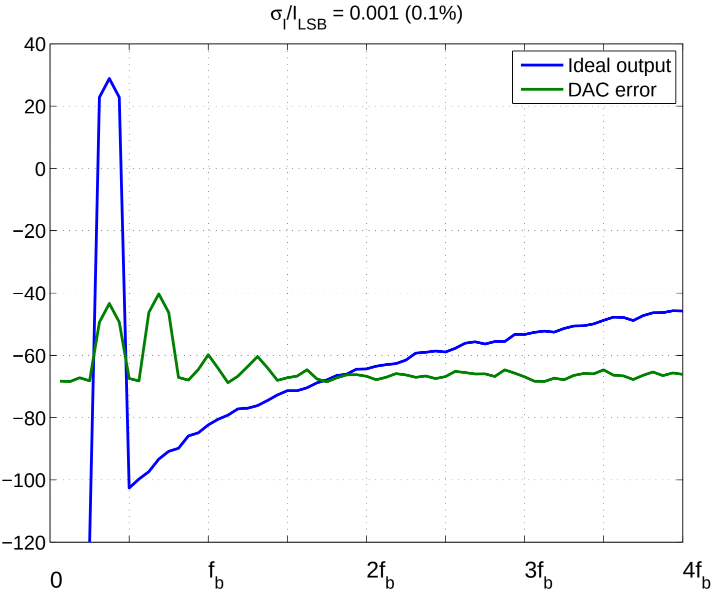
Multi bit feedback DAC nonlinearity

- $I_{out}[0] = 0$
- $I_{out}[8] = \sum_{n=1}^{8} I_n$
- $I_{LSB} = 1/8 \sum_{n=1}^{8} I_n$
- DNL $\Delta I_k = I_k - I_{LSB}$
- INL $I_{ek} = \sum_{n=1}^{k} I_n - nI_{LSB} = \sum_{n=1}^{k} \Delta I_k$

Effects of DAC nonlinearity



Effects of DAC nonlinearity



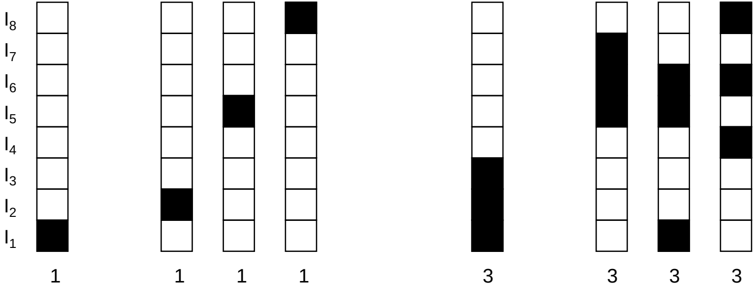
Effects of DAC nonlinearity

- Distortion
- Increased in band quantization noise

Reducing DAC nonlinearity

- Reduce relative mismatch of DAC elements
- $\sigma_I/I_{LSB}, \sigma_C/C, \sigma_R/R \propto 1/\sqrt{WL}$
- $100\times$ area increase to reduce relative mismatch by $10\times$
- Sizing alone cannot help

Representing v using a thermometer DAC



- v current sources must be on—multiple possibilities
- $M!/M!(M-v)!$ combinations can represent v
- Only one possibility for $v = 0$ (all off) and $v = 8$ (all on)

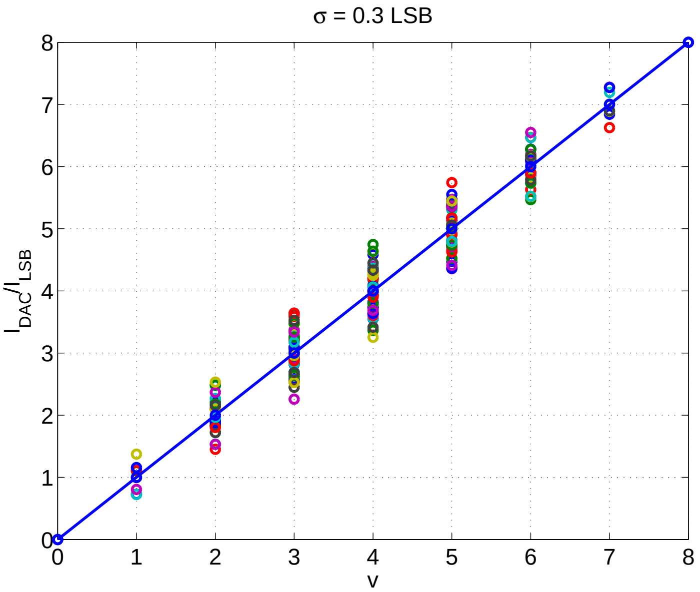
Different combinations of unit cells for a given input

- $v = 1$ can be represented by turning on any one of I_{1-8}
- Average of all possibilities

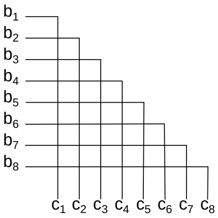
$$\frac{1}{8}\sum_{n=1}^{8} I_n = I_{LSB}$$

  is the ideal output!
- For all v, averaging all possible combinations produces the ideal output
- Use different combinations to represent a given code

Different combinations of unit cells for a given input

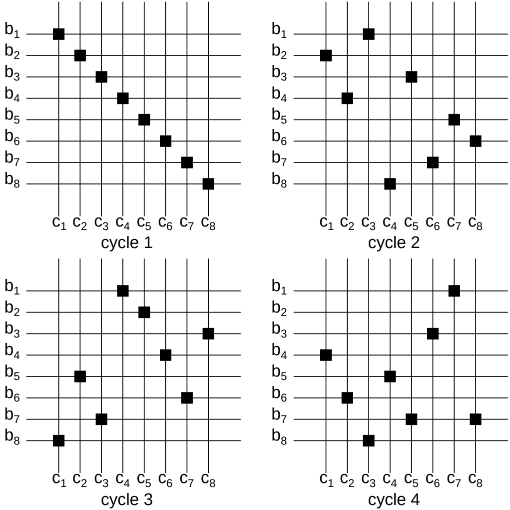


Randomization



$b_{1\text{-}8}$: Thermometer coded **v**
$c_{1\text{-}8}$: Control signals to DAC unit elements

Fixed connections

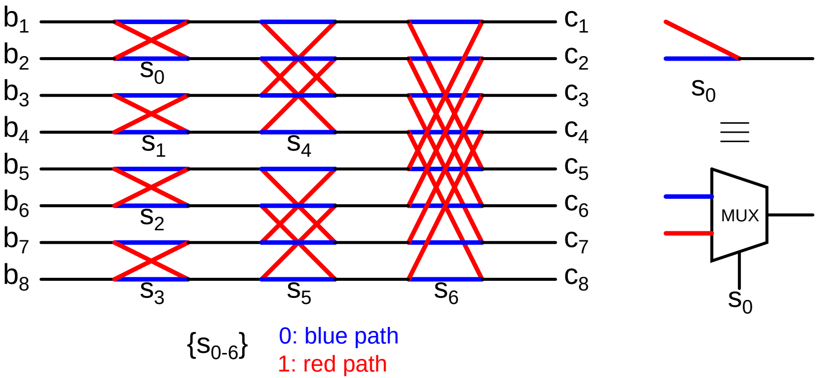


Randomized connections

Randomization

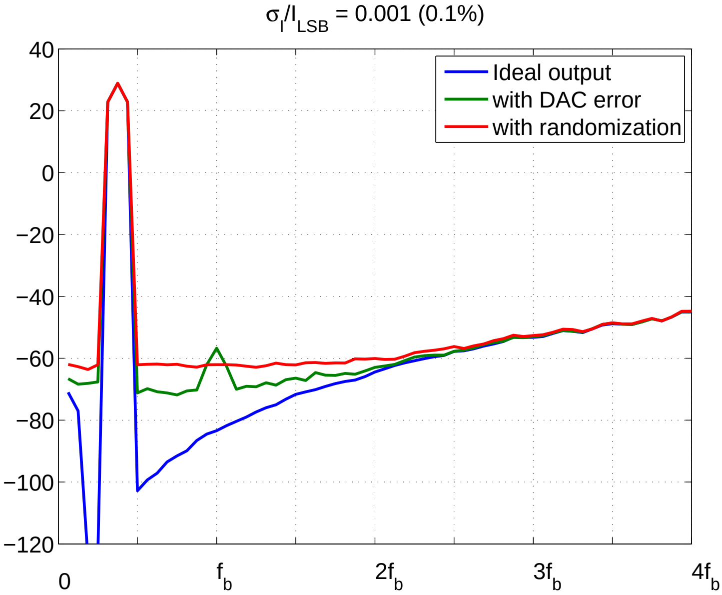
- $M \times M$ switching matrix
- In each cycle, randomly choose a set of connections
- Converts distortion to white noise
- $M!$ possible connections in the switch matrix $(9! = 362880)$—use a smaller subset
- Switch matrix introduces delay in the loop

Randomization-Butterfly scrambler

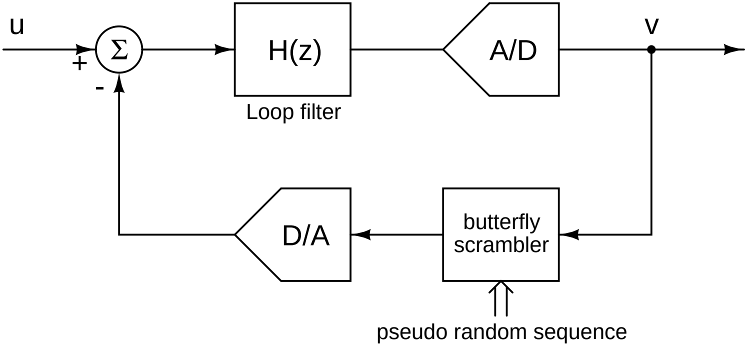


- Each stage flips across 1, 2, or 4 positions
- 7 switches instead of 64
- Only 128 combinations used—but good enough in practice

Randomization-results



ΔΣ modulator with randomization

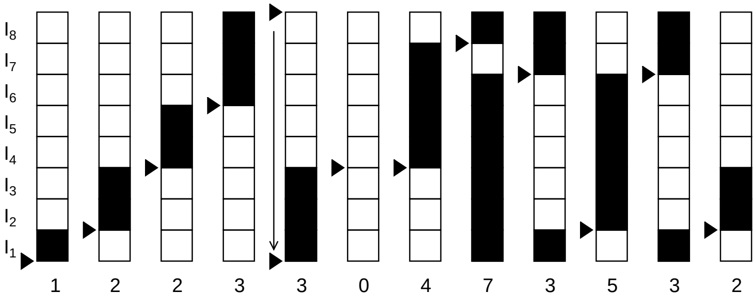


- Extra delay in the loop

Randomization-summary

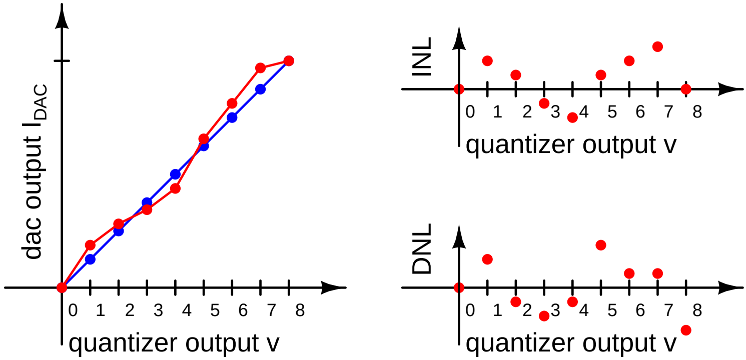
- Distortion components converted to noise
- Increased noise floor
- Additional loop delay

Data weighted averaging

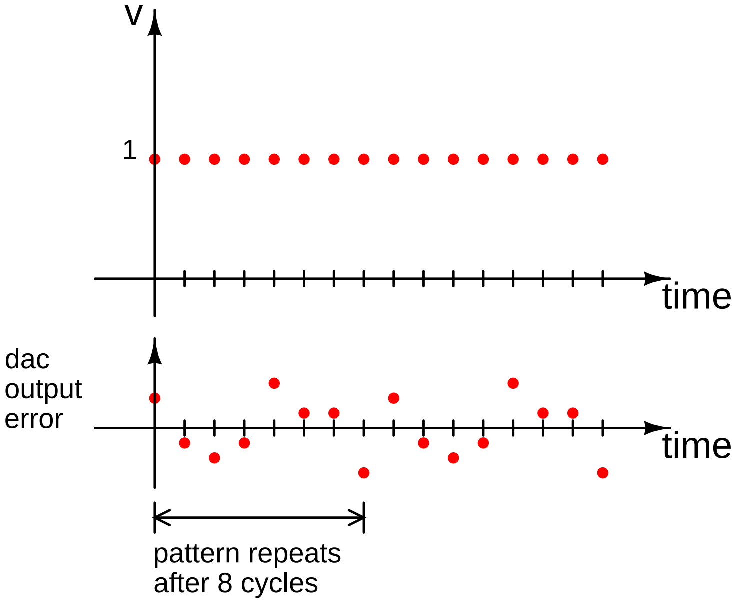


- Cycle through all the current sources as rapidly as possible

DAC nonlinearity



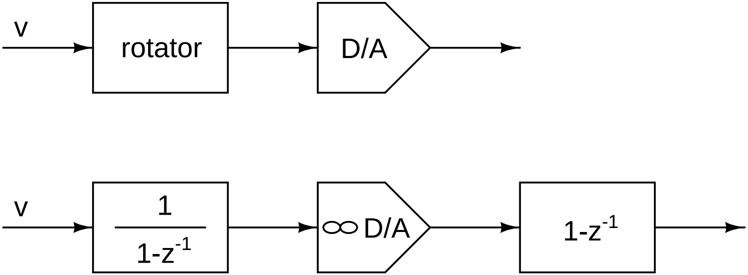
Data weighted averaging—dc input



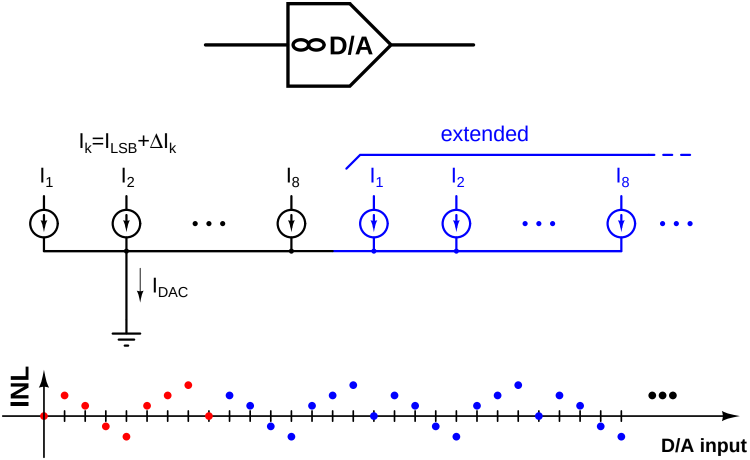
Data weighted averaging—dc input

- Accumulated error is zero after a small number of cycles
- Pattern repeats every M cycles for an $M+1$ level DAC
- Tones at f_s/M and its harmonics for $v=1$

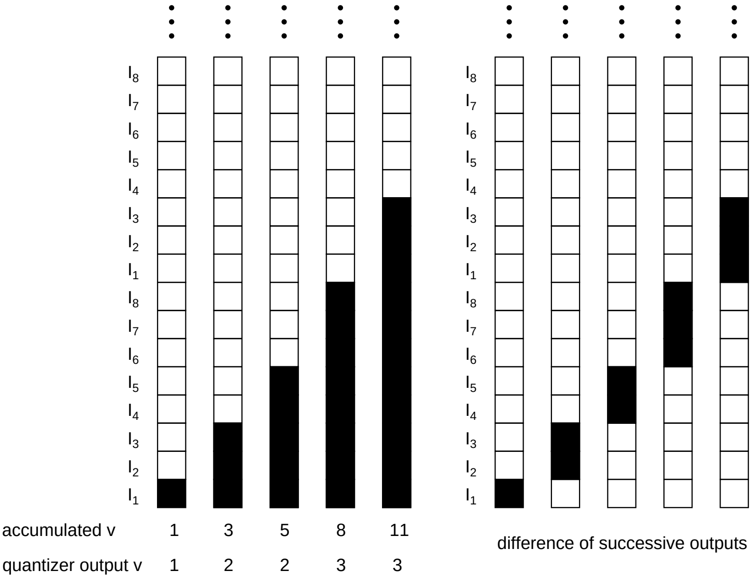
Data weighted averaging—arbitrary inputs



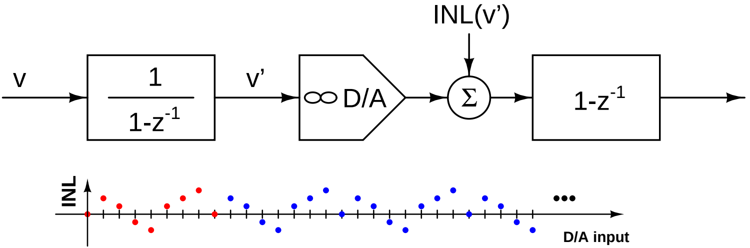
Data weighted averaging—arbitrary inputs



Data weighted averaging—arbitrary inputs

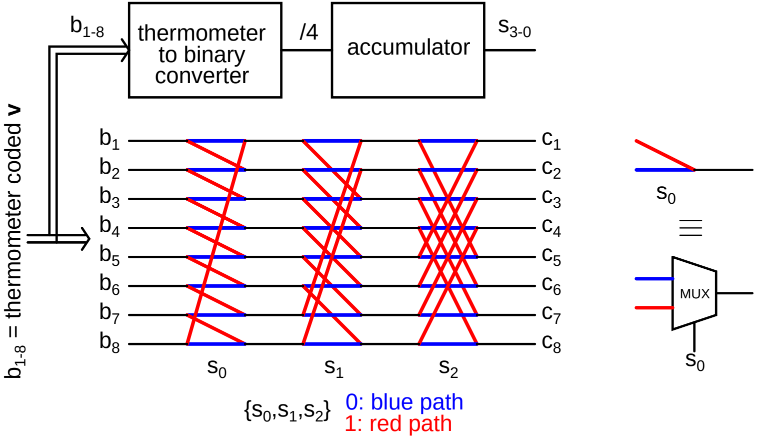


Data weighted averaging—mismatch shaping



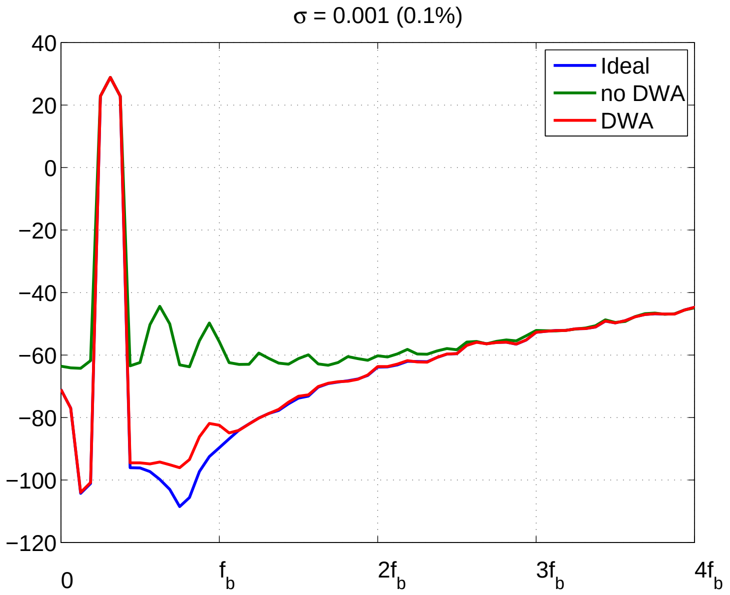
- ∞ D/A output error bounded by INL_{max}
- Finite power at all frequencies
- $1 - z^{-1}$ at the output provides first order shaping

Data weighted averaging—implementation

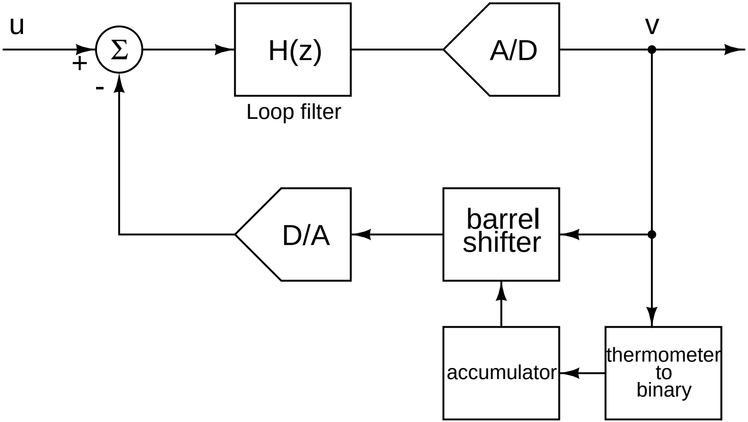


- M input barrel shifter driven by accumulated ADC output
- Loop delays from thermometer-binary converter, accumulator, barrel shifter

Data weighted averaging—results



ΔΣ modulator with data weighted averaging

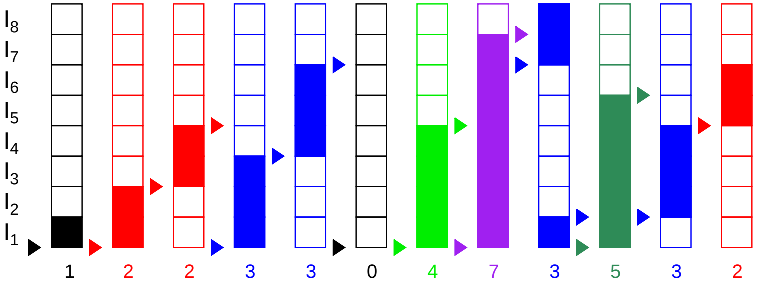


- Extra delay in the loop

Data weighted averaging-summary

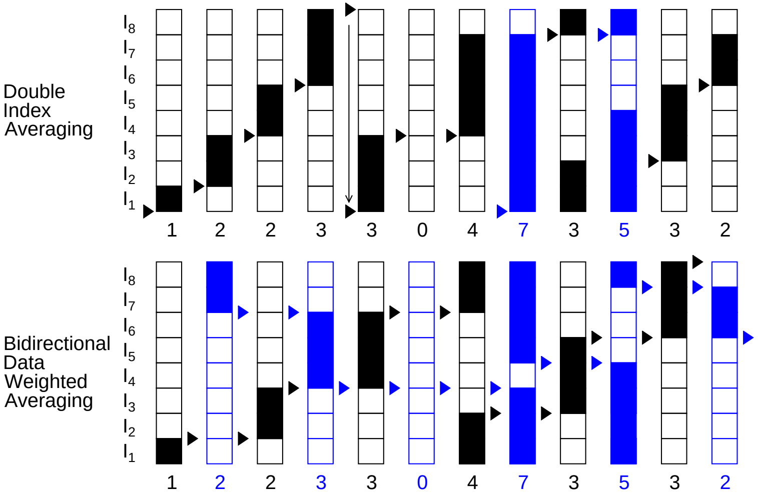
- Provides first order mismatch shaping
- Potential for tones at $\approx f_s/M$ with an $M+1$ level quantizer
- For low OSR, tones can be close to the signal band
- Additional loop delay

Individual level averaging



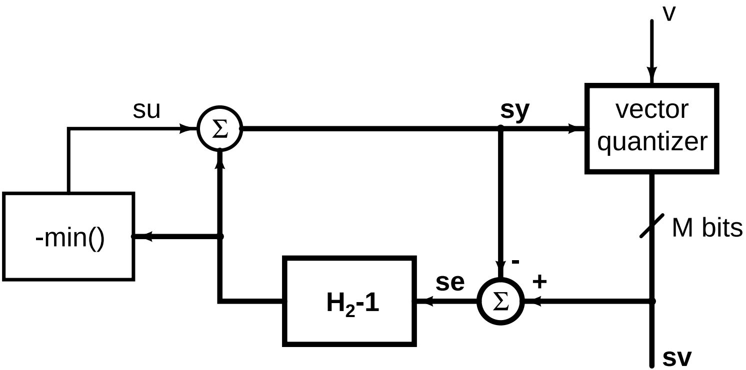
- Cycle through all current sources for each input code
- Separate pointer for each input code
- Lesser potential for tones than DWA
- More noise than DWA

Data weighted averaging—variants



- Bidirectional DWA: Opposite directions in each cycle
- Double index averaging: Separate pointers for $v > M/2$ and $v \leq M/2$
- DWA with randomization: Randomize the shifts once in every few cycles to break up tones

Higher order mismatch shaping



- Mismatch shaped by the transfer function $H_{mismatch}$
- Deviation from exact shaping due to the constraint $|sv| = |v|$
- Complex hardware

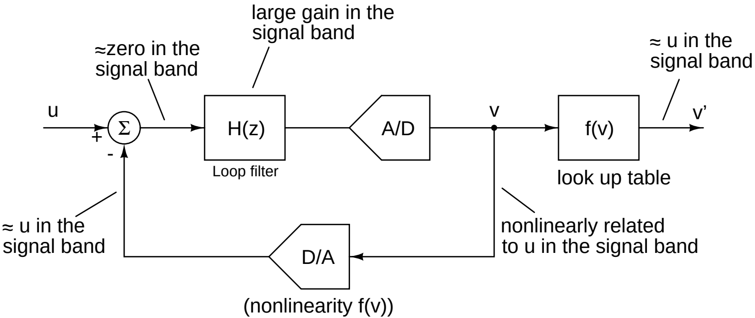
Dynamic element matching: tradeoffs

- Mismatch error reduction
  - High order noise shaping (highest)
  - DWA
  - ILA
  - Randomization (lowest)
- Potential for tones
  - Randomization (lowest)
  - High order noise shaping
  - ILA
  - DWA (highest)
- Complexity
  - High order noise shaping (highest)
  - ILA, Randomization
  - DWA (lowest)
- Excess loop delay
  - High order noise shaping (highest)
  - ILA
  - DWA
  - Randomization (lowest)

Dynamic element matching: summary

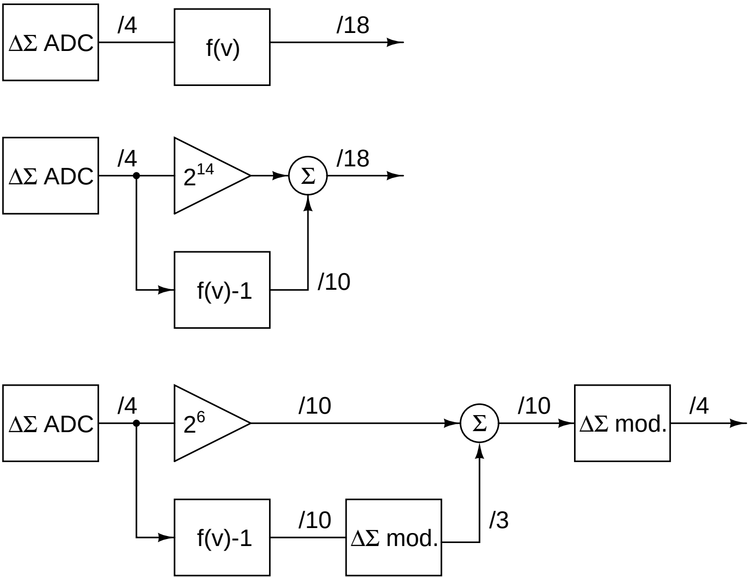
- Data weighted averaging
  - Best compromise between complexity and performance
  - Works very well with high OSR
  - Potential for tones at low OSR
- ILA, other DWA variants
  - More complex, less potential for tones
- Randomization
  - Can also be used for DACs without noise shaping

Calibration



- Measure DAC characteristics
- Duplicate its characteristics in the digital path
- $v' = v + \epsilon$; $\epsilon \ll v$; Lot more bits in v' than v

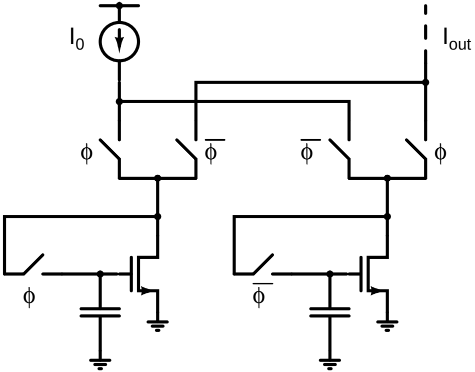
Calibration



Calibration

- Store only the error to reduce register width
- Noise shaped quantization (digital $\Delta\Sigma$ modulator) to reduce decimator input width

Analog calibration



- Calibrate all current sources against a master source
- Use $M + 1$ current sources and calibrate one at a time

Calibration: summary

- No additional components in the loop $\Rightarrow$ no excess delay
- Measuring DAC characteristics inline is challenging
- Additional digital or analog complexity

References

- **Randomization:** L. R. Carley, "A noise-shaping coder topology for 15+ bit converters," *IEEE Journal of Solid-State Circuits*, vol. 24, pp. 267 - 273, April 1989.
- **Data weighted averaging:** R. T. Baird and T. S. Fiez, "Linearity enhancement of multibit $\delta\Sigma$ A/D and D/A converters using data weighted averaging," *IEEE Transactions on circuits and systems-II*, vol. 42, pp. 753 - 762, December 1995.
- **Individual level averaging:** B. H. Leung and S. Sutarja, "Multibit Σ-Δ A/D converter incorporating a novel class of dynamic element matching techniques," *IEEE Transactions on circuits and systems-II*, vol. 39, pp. 35-51, January 1992.
- **Theoretical analysis:** O. J. A. P. Nys and R. K. Henderson, "An analysis of dynamic element matching techniques in sigma-delta modulation," *Proceedings of the 1996 IEEE International symposium on circuits and systems*, vol. 1, pp. 231-234, May 1996.
- **Comparison through simulation:** Zhimin Li, T. S. Fiez, "Dynamic element matching in low oversampling delta sigma ADCs," *Proceedings of the 2002 IEEE International symposium on circuits and systems*, vol. 4, pp. 683-686, May 2002.
- **Digitally calibrated $\Delta\Sigma$ modulator:** M. Sarhang-Nejad and G. C. Temes, "A high-resolution multibit $\Sigma\Delta$ ADC with digital correction and relaxed amplifier requirements," *IEEE Journal of Solid-State Circuits*, vol. 28, pp. 648 - 660, June 1993.
- **Analog calibrated DAC:** D. Wouter J. Groeneveld et al., "A self-calibration technique for monolithic high-resolution D/A converters," *IEEE Journal of Solid-State Circuits*, vol. 24, pp. 1517 - 1522, December 1989.
- **Higher order mismatch shaping:** R. Schreier and B. Zhang, "Noise-shaped multibit D/A convertor employing unit elements" *Electronics letters*, vol. 31, No. 20, pp. 1712-1713, 28th September 1995.
- **Additional filtering of DEM errors:** M. H. Adams and C. Toumazou, "A Novel Architecture for Reducing the Sensitivity of Multibit Sigma-Delta ADCs to DAC Nonlinearity," *Proceedings of 1995 IEEE International symposium on circuits and systems*, vol. 1, pp. 17-20, May 1995.
- **Additional filtering of DEM errors:** J. Chen and Y. P. Xu, "A Novel Noise Shaping DAC for Multi-bit Sigma-Delta Modulator,"*IEEE Transactions on Circuits and Systems II-Express Briefs*, vol. 53, no. 5, pp. 344-348, May 2006.

CASE STUDY

A 15-bit Continuous-time $\Delta\Sigma$ ADC for Digital Audio Design Targets

- Audio ADC (24 kHz Bandwidth)
- 15 bit resolution
- OSR = 64 (f_s = 3.072 MHz)
- 0.18μm CMOS process, 1.8 V supply

Continuous-time versus Discrete-time A continuous-time implementation was chosen

- Implicit anti-aliasing
- Resistive input impedance
- Low power dissipation

Architectural Choices

- Single-bit versus multibit quantization ?
- Single loop versus MASH ?
- NTF ?
- Loop Filter Architecture ?

Architecture : Single-bit vs Multibit

Single bit quantizer

- Simple hardware
- Gentle NTF
- High jitter sensitivity
- Metastability
- Opamp slew rate

Multibit quantizer

- Complex hardware
- Aggressive NTF
- Low jitter sensitivity
- Metastability : no issue
- Reduced slew rate

A 4-bit quantizer is used.

Architecture : Single Loop vs MASH

Matching of transfer functions are needed in a MASH design

- More complicated
- Might require calibration

A single loop design is chosen.

Architecture : Choice of the NTF

A maximally flat NTF is chosen

Small OBG

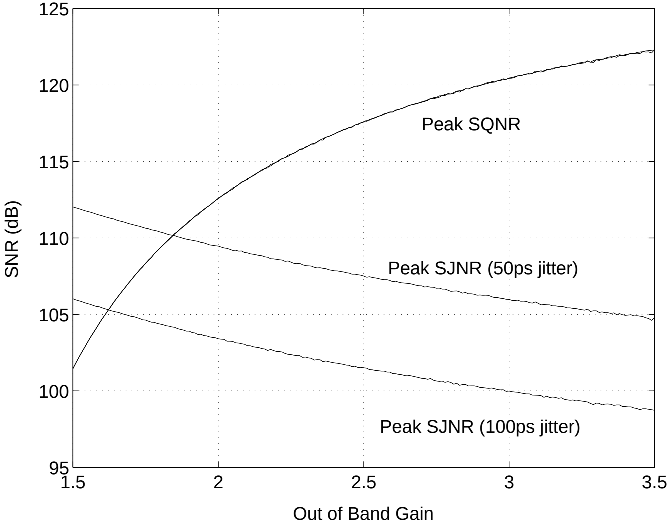
- High in-band quantization noise
- Low jitter noise
- Increased Maximum Stable Amplitude (MSA)

Large OBG

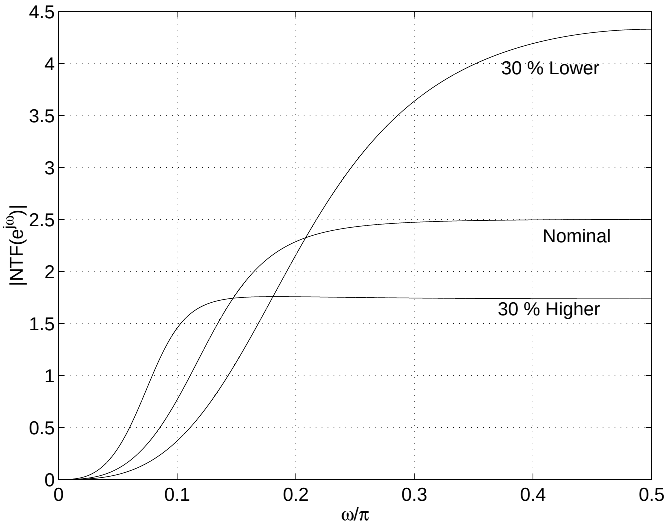
- Low in-band quantization noise
- High jitter noise
- Reduced Maximum Stable Amplitude (MSA)

An OBG of 2.5 is chosen as a compromise

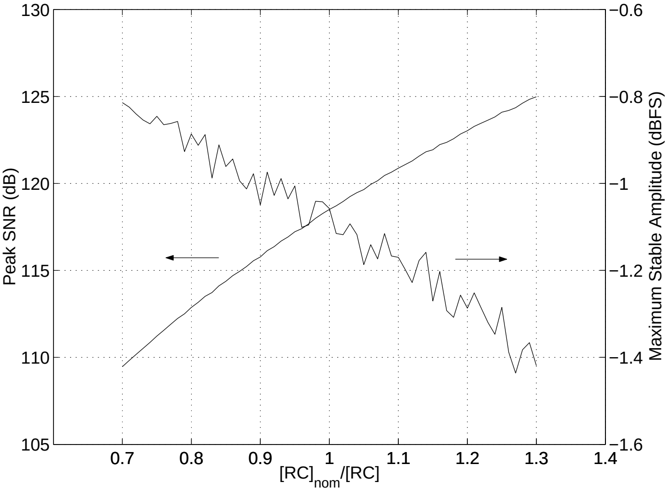
Effect Of OBG On Jitter And Quantization Noise



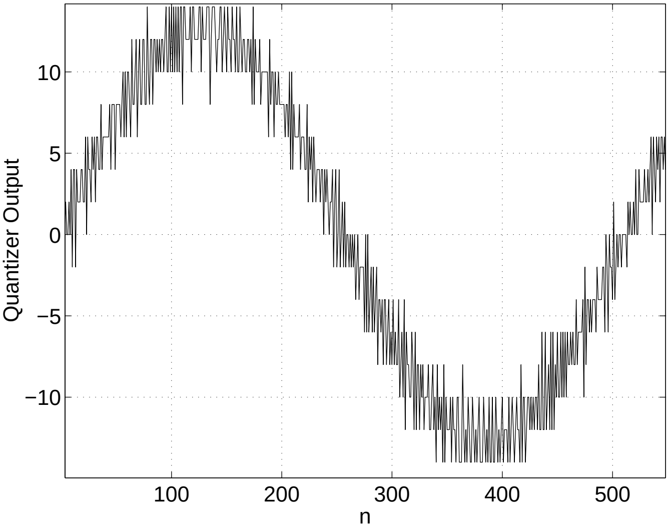
Effect Of Systematic RC Time Constant Variations On The NTF



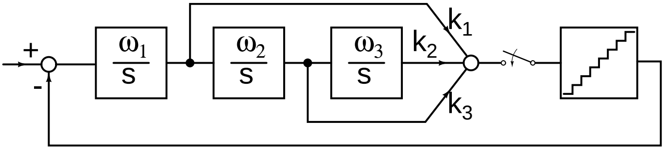
MSA And SQNR With Systematic RC Time Constant Variations



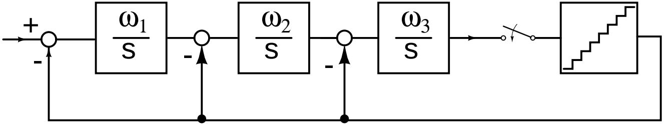
Simulated Output Bit Stream



Feedfoward versus Distributed Feedback Loopfilters



(a) $\omega_1 = 2.67,\ \omega_2 = 2.08,\ \omega_3 = 0.059$



(b) $\omega_1 = 0.34,\ \omega_2 = 0.71,\ \omega_3 = 1.225$

Feedfoward versus Distributed Feedback Loopfilters

Feedforward

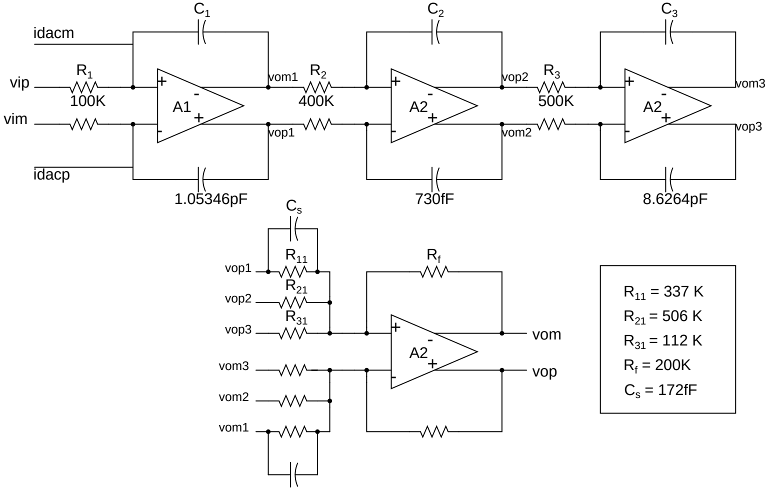
- First integrator is fasest.
- Third integrator is slowest.
- First opamp is power hungry (for noise reasons).
- Third opamp is low power (slowest integrator).
- Small capacitor area.

Distributed Feedback

- Third integrator is fastest.
- First integrator is slowest.
- First opamp is power hungry (for noise).
- Third opamp is power hungry (fastest integrator).
- Large capacitor area.

A feedforward loop filter is used.

Loop Filter



R_{11} = 337 K

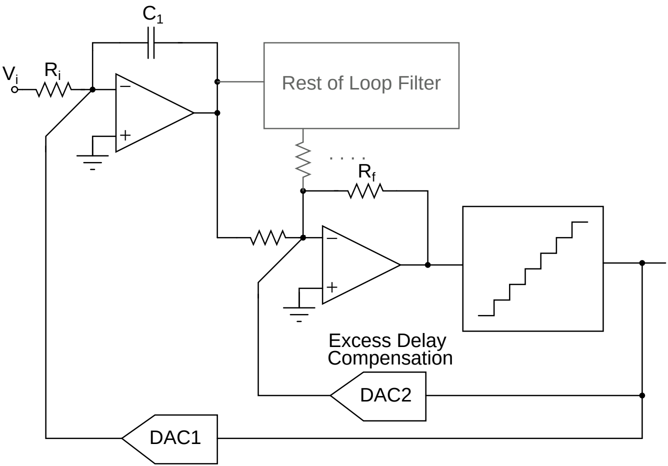
R_{21} = 506 K

R_{31} = 112 K

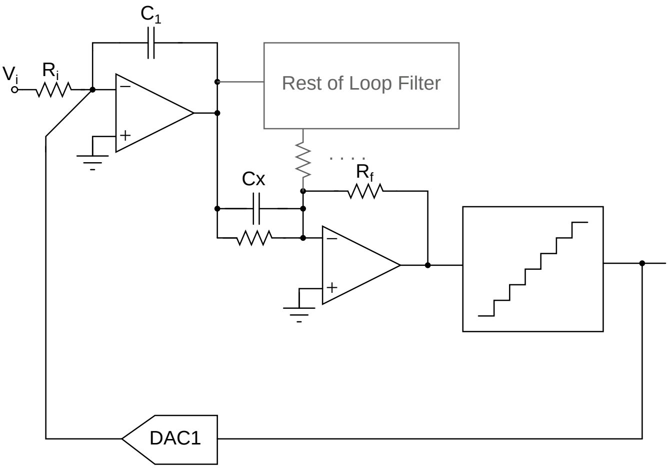
R_f = 200K

C_s = 172fF

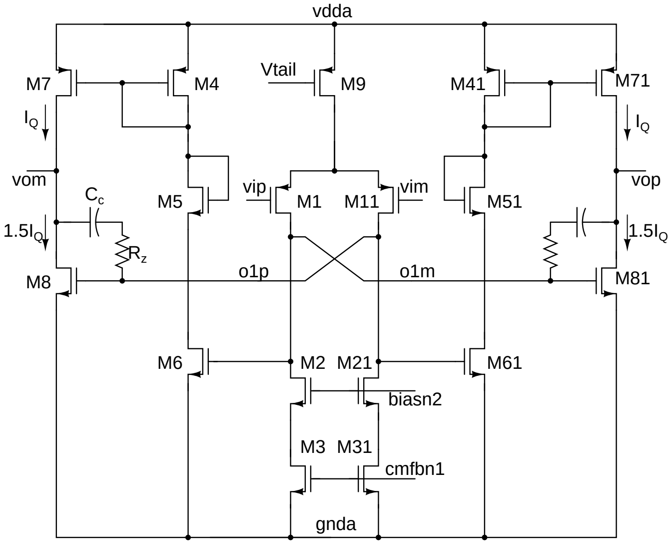
Excess Delay Compensation : Conventional



Excess Delay Compensation : Proposed

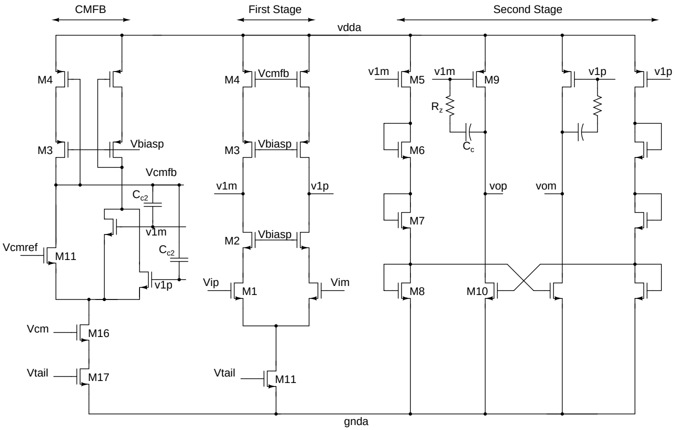


First Opamp

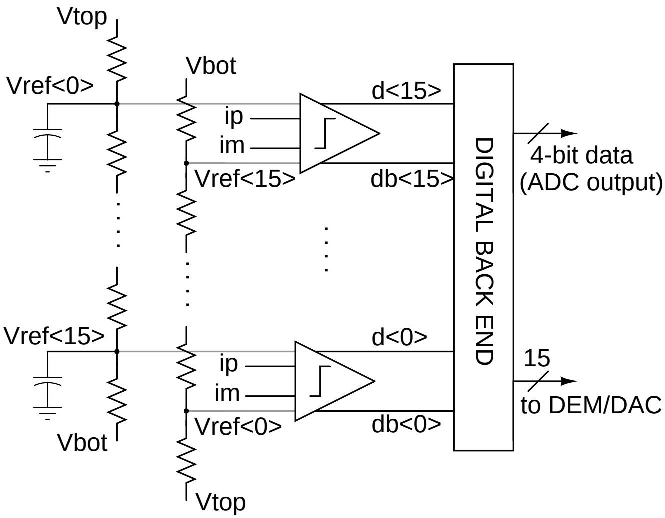


(a)

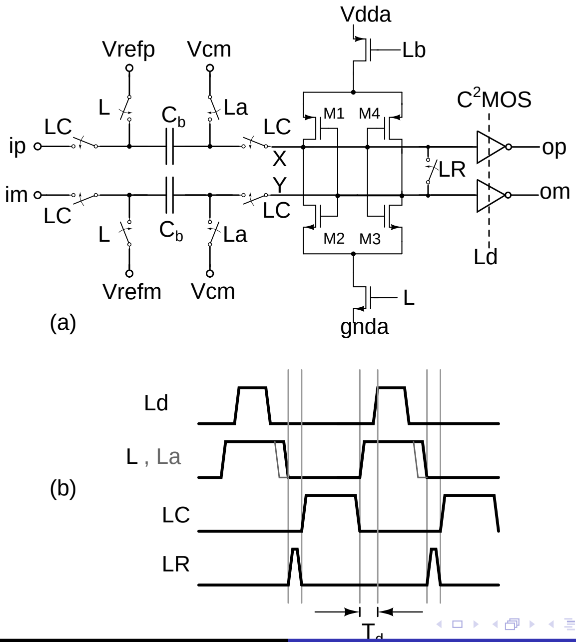
Second Opamp



Flash ADC Block Diagram

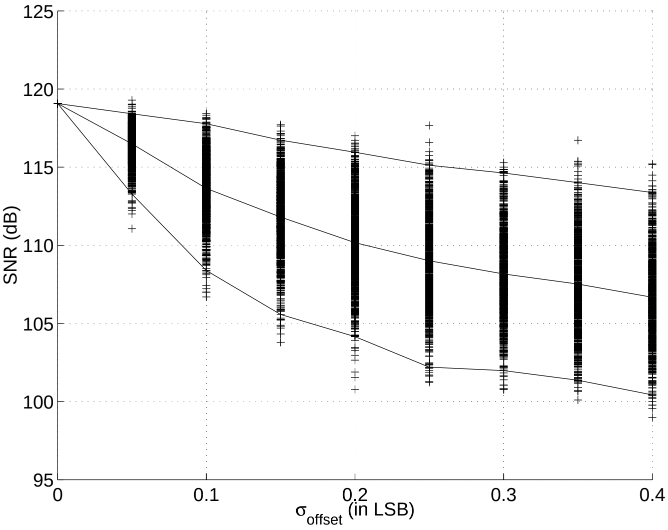


Comparator

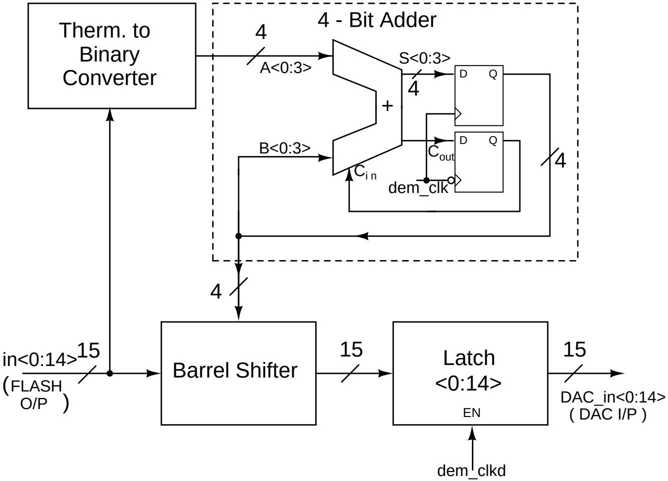


Vdda
Lb
Vrefp
Vcm
L
La
C_b
LC
ip
im
X
Y
M1
M4
M2
M3
C^2MOS
op
om
LR
Ld
Vrefm
Vcm
gnda
(a)
Ld
L , La
(b)
LC
LR
T_d

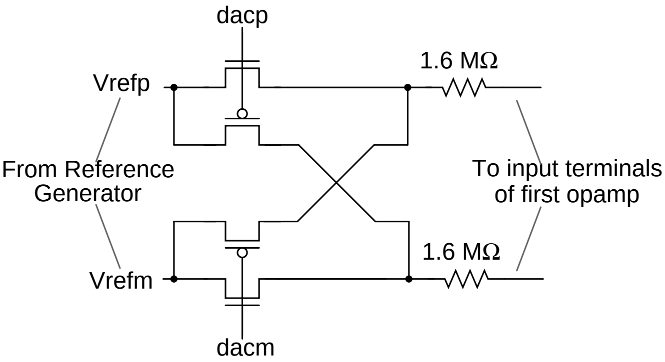
Effect of Random Offset in the Comparators



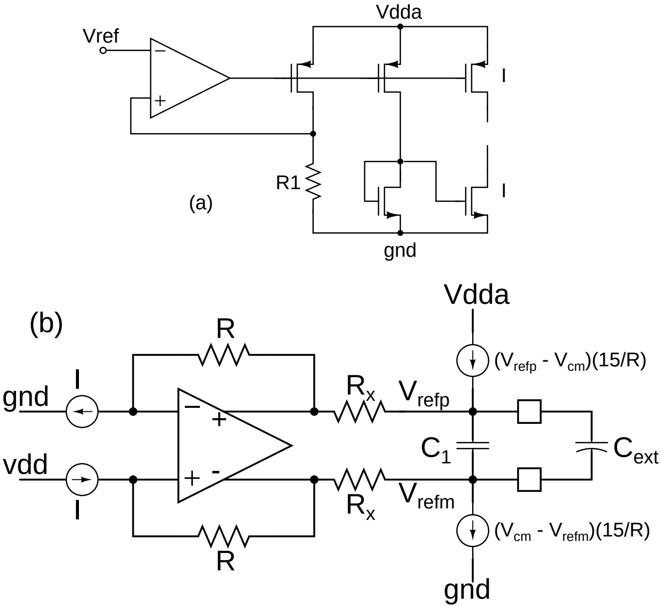
Digital Backend



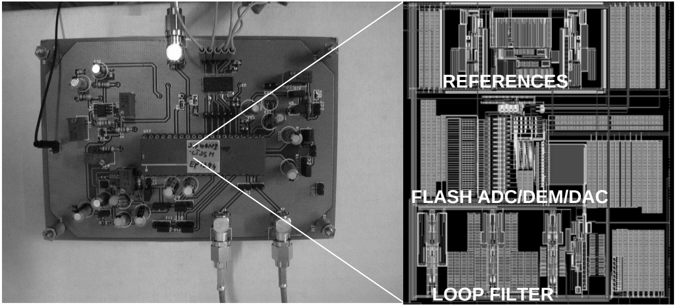
Unit DAC Resistor



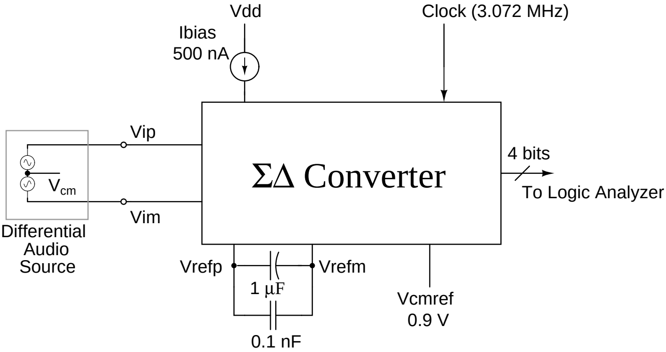
Reference Generation Circuitry



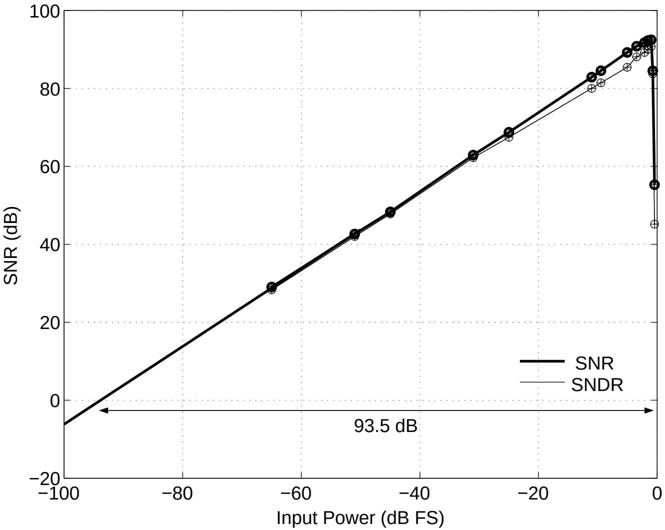
Test Setup and Die Layout



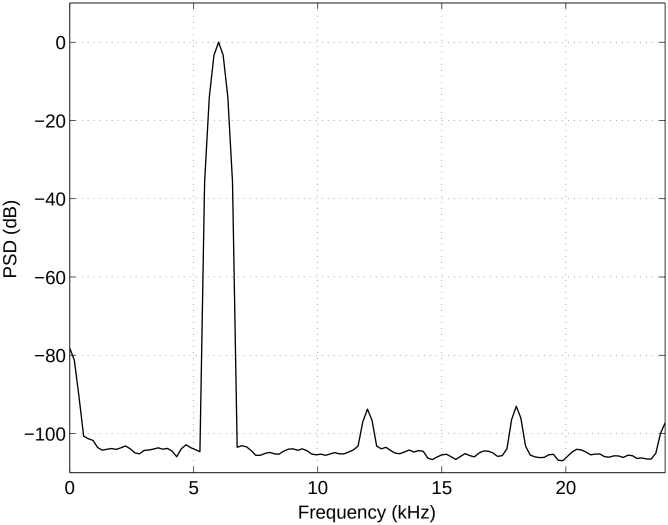
Test Setup Schematic



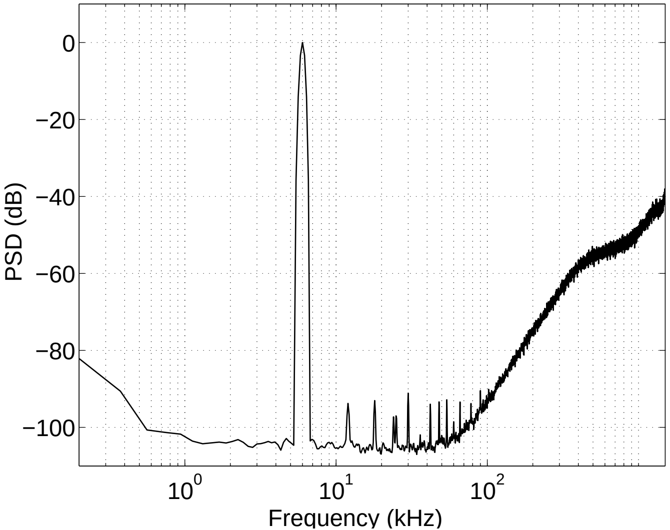
Measured Dynamic Range



In Band Spectrum



Out of Band Spectrum



Performance Summary

Table: Summary of Measured ADC performance.

Signal Bandwidth/Clock Rate	24 kHz / 3.072 MHz
Quantizer Range	3 $V_{pp,diff}$
Input Swing for peak SNR	-1 dBFS
Dynamic Range/SNR/SNDR	93.5 dB/92.5 dB/90.8 dB
Active Area	0.72 mm^2
Process/Supply Voltage	0.18 μm CMOS/1.8 V
Power Dissipation (Modulator)	90 μW
Power Dissipation (Modulator and Reference Buffers)	121 μW
Figure of Merit(DR/SNR)	0.049 pJ/level, 0.054 pJ/level

Some References ...

- *Delta-Sigma Data Converters: Theory, Design and Simulation*
  S. Norsworthy, R. Schreier and G. Temes, *IEEE Press*
  The Yellow Bible of $\Delta\Sigma$ ADCs
- *Understanding Delta-Sigma Data Converters*
  R. Schreier and G. Temes, *IEEE Press*
  The Green Bible of $\Delta\Sigma$ ADCs
  Both the above are essential reading !

Some References ...

- *Theory, Practice, and Fundamental Performance Limits of High-Speed Data Conversion Using Continuous-Time Delta-Sigma Modulators*
J. Cherry, *Ph.D Dissertation, Carleton University.*
Excellent reading on continuous-time Delta-Sigma modulator design.
- *A Power Optimized Continuous-time $\Delta\Sigma$ ADC for Audio Applications*
S. Pavan, N. Krishnapura et. al, *IEEE Journal of Solid State Circuits, February 2008.*
Detailed description of the case study discussed in this tutorial.