Automatic Tuning of Time Constants in Continuous-Time Delta–Sigma Modulators

Shanthi Pavan and Nagendra Krishnapura

Abstract—**We describe a digital technique for estimating and correcting time constant shifts in continuous-time delta–sigma modulators. The proposed method is based on the principle that the in-band gain and the out-of-band performance of a modulator are related. If the modulator output is denoted as $v(n)$, we show that the variance of $p(n) \equiv v(n) - v(n-1)$ is a good indicator of the modulator RC time constants. An alternative indicator, which is easier to implement in hardware is proposed. Simulation results demonstrating the effectiveness of the proposed techniques are given.**

***Index Terms*—Continuous-time $\Sigma\Delta$ modulators (CT-DSM), noise transfer function (NTF).**

I. Introduction

CONTINUOUS-TIME $\Sigma\Delta$ modulators (CT-DSM) are attractive low-power alternatives to their discrete-time counterparts. The linearized block diagram of a generic modulator is shown in Fig. 1. The loop filter is implemented using CT circuitry, with sampling occurring at the filter output. Systematic variations in the loop filter time constants (due to process shifts) can be modelled by the frequency scaling parameter k_p. For the nominal process corner, $k_p = 1$. The modulator input and quantizer output are denoted by u and v respectively. $e(n)$ represents the quantization noise, assumed to be additive. The usual modulator design procedure is as follows [1].

1) A suitable prototype discrete-time noise transfer function (NTF) is identified. The choice of NTF depends on the desired quantization noise and the number of levels in the quantizer.
2) The CT loop filter $H_c(s)$ is synthesized so that the impulse response from v to y in Fig. 1 matches the corresponding discrete-time filter response. Relations that compute the CT filter from the discrete-time (DT) filter taking into account the digital–analog converter (DAC) pulse shape are given in [1].

When *all* the time constants in the loop filter scale by the same factor $1/k_p$, the loop filter transfer function becomes $H_c(s/k_p)$, so that the resulting NTF is no longer the desired one. If k_p deviates significantly from unity, the modulator can become unstable. It is thus seen that some way of automatically tuning the loop filter time constants is necessary so that the modulator performance is guaranteed in the face of gross *RC* variations due to process shifts. The tuning process has two aspects: 1) estimating the deviation in time constants of the loop filter from the nominal value; 2) adjusting the loop filter components in a manner as to bring the time constants close to the nominal value. The error tolerable depends on the nominal NTF chosen. For the NTFs used in practice, simulations show that a $\pm 5\%$ deviation of the time constants from their nominal values is usually acceptable.

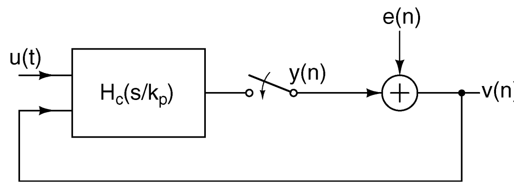

Fig. 1. Linearized model of a CT $\Sigma\Delta$ modulator.

Earlier work addressing this problem has heavily borrowed from the literature on automatic tuning of CT filters. The authors of [2] include a replica *RC*-oscillator along with the modulator circuitry. This oscillator is made with the same type of resistors and capacitors used in the loop filter. The capacitors are realized as digitally programmable arrays. The frequency of the oscillator is indicative of the time constants of the loop filter. A digital engine determines the right tuning code to be applied to the capacitor array in the oscillator, so that the difference between the desired and achieved time constant is small. The same code is applied to the capacitors in the loop filter. In [3], the gain of a replica integrator at a given frequency is used to estimate the time constants of the loop filter. A feedback loop adjusts the integrator unity gain frequency by varying the transconductance of the G_m–C operational transconductance amplifier (OTA) integrator. The two references cited above are examples of "indirect tuning" of the loop transfer function.

Direct tuning of the time constants has also been reported. The authors of [4] measure the variance of the (discrete-time) modulator output *after* decimation and use this information to calibrate a modulator using hybrid integrators. [5] uses the same idea in a cascaded CT modulator. Since a second-order NTF is employed in [5], the modulator is quite tolerant of *RC*-shifts. The aim of tuning in that work is to reduce the in-band noise due to mismatch in the transfer functions of the analog and digital filters in the cascaded design.

In this brief, we present a direct tuning technique for time constants in a low-pass multibit single-loop CT-DSM. The method is entirely digital without any analog components. Like in [4] and [5], it is insensitive to dc offset in the modulator. However, it operates directly on the modulator output without recourse to the decimated sequence. The operating principle and performance of this scheme form the subject of the rest of the brief, which is organized as follows. In Section II, the Bode sensitivity integral is used to give an intuitive explanation of the effect of

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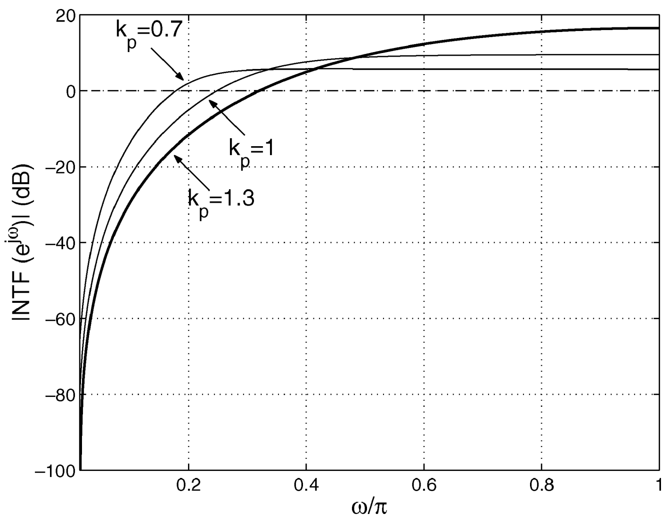

Fig. 2. Plot of $|\mathrm{NTF}|$ in dB—the area below the 0 dB line is equal to the area above the line. As k_p increases, improved in-band performance of the NTF is obtained at the cost of degraded response out of band and viceversa.

systematic time constant variation in CT-DSMs. Denoting the modulator output sequence by $v(n)$, we observe that the standard deviation of $p(n) \equiv v(n) - v(n-1)$ depends on the time constants in the loop filter. An alternative time-constant estimator, which is easier to implement in hardware is proposed. Section III gives simulation results for a third-order CT-DSM that confirm the efficacy of the proposed time constant estimators. Section IV describes a possible digital implementation of the time-constant tuning system. Conclusions are given in Section V.

II. Time-Constant Estimation in CT-DSMs

To correct systematic time-constant variations in the loop filter, a technique to estimate the deviation of the time constants from their nominal values is necessary. To gain intuition on what happens to the NTF of a CT-DSM when k_p changes (recall that the loop filter has a transfer function $H_c(s/k_p)$), we draw the reader's attention to the Bode Sensitivity Integral [6]. This integral, in the context of a $\Delta\Sigma$ modulator, mandates that if the NTF is minimum phase, the area of $\log|\mathrm{NTF}(e^{j\omega})|$ below zero equals the area above zero. This means that better in-band NTF performance can only be obtained at the expense of a high value of $|\mathrm{NTF}|$ at high frequencies. When $k_p > 1$, the loop filter becomes faster. The low frequency gain of the discrete-time equivalent loop filter increases, thereby decreasing the in-band quantization noise. This should result in an increased magnitude of the NTF at high frequencies. This is indeed the case, as illustrated with a third-order NTF in Fig. 2. The nominal NTF ($k_p = 1$) is maximally flat with an out-of-band gain of 3 (a 4-bit quantizer is assumed, which facilitates the use of an NTF with a large out-of-band gain). When $k_p = 1.3$ ($1/RC$ is 30% higher), it is seen that there is a significant out-of-band peaking of the NTF. For $k_p = 0.7$, the in-band quantization noise is higher than the nominal case, while the out-of-band gain is smaller than 3.

The above discussion can be interpreted in the time domain. Fig. 3 shows the output of a CT-DSM for different values of k_p,

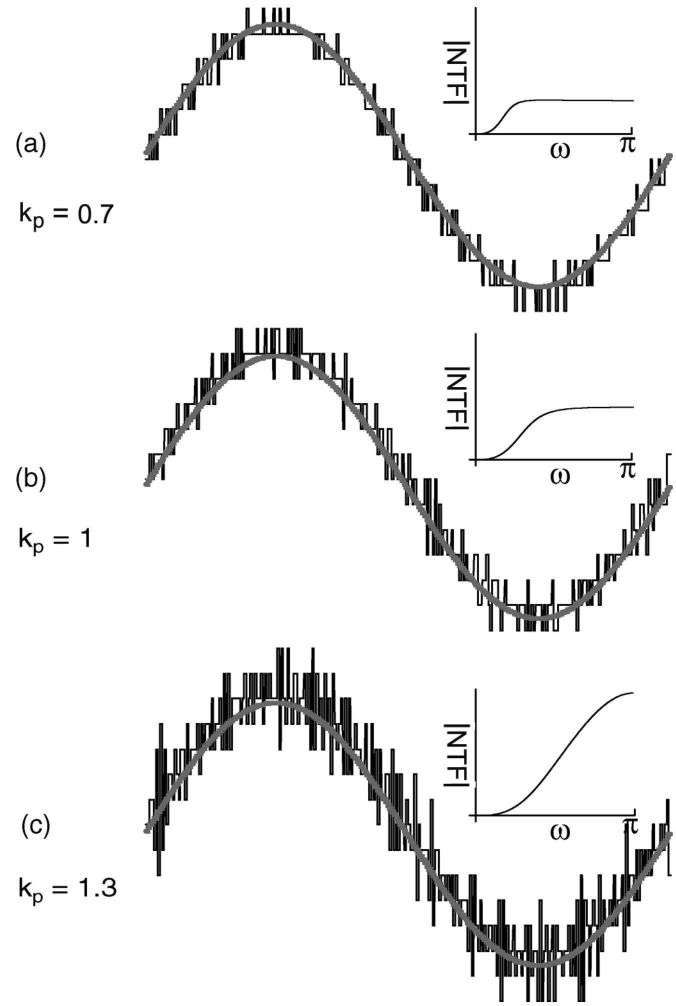

Fig. 3. Time-domain output stream of a 4-bit $\mathrm{CT} - \Sigma\Delta$ modulator when: (a) loop filter is slow, $k_p = 0.7$. The out-of-band gain of the NTF is smaller than the nominal value, and the modulator output "wiggles" less frequently than the nominal case; (b) nominal loop filter, $k_p = 1$. The out-of-band gain is 3; (c) loop filter is fast, $k_p = 1.3$. Notice the significant increase in the out-of-band gain, resulting in more frequent, larger amplitude "wiggles."

when the modulator is excited by a sinusoidal input. The corresponding NTF magnitudes are also plotted as insets. When $k_p = 1.3$, the large out-of-band gain in the NTF results in more "wiggling" of the analog–digital converter (ADC) output sequence around the input signal, since the quantization noise is amplified to a larger extent, as seen in Fig. 3(c). However, for $k_p = 0.7$, the modulator output sequence jumps by a lesser extent compared to the nominal case, as seen by comparing Figs. (3a) and (b). These observations suggest that the variance of the quantization noise is an indicator of k_p. Since the modulator output contains the input in addition to the shaped quantization noise, some means of eliminating the (slowly varying) input is necessary. One way is to monitor the first difference of the modulator output $p(n) \equiv v(n) - v(n-1)$. In the frequency domain

$$\begin{aligned} P(e^{j\omega}) &= U(e^{j\omega})\mathrm{STF}(e^{j\omega})(1 - e^{-j\omega}) \\ &\quad + E(e^{j\omega})\mathrm{NTF}(e^{j\omega})(1 - e^{-j\omega}) \\ &\approx E(e^{j\omega})\mathrm{NTF}(e^{j\omega})(1 - e^{-j\omega}) \end{aligned} \tag{1}$$

where STF is the signal transfer function.

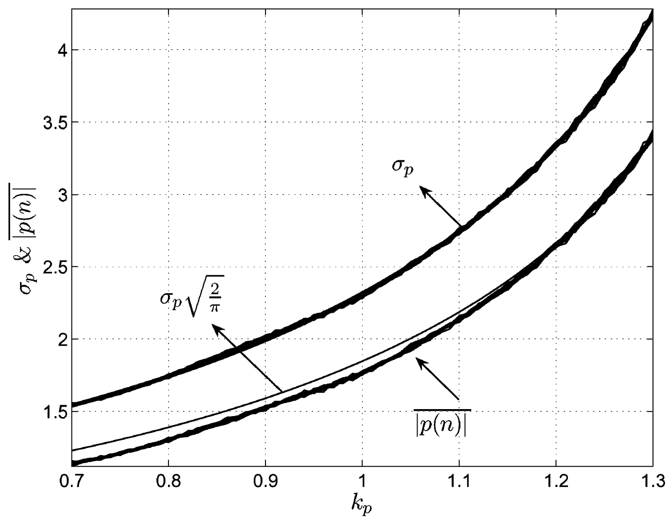

Fig. 4. σ_p and $\overline{|p(n)|}$ as a function of k_p for random bandlimited inputs. The quantizer step $\Delta = 2$.

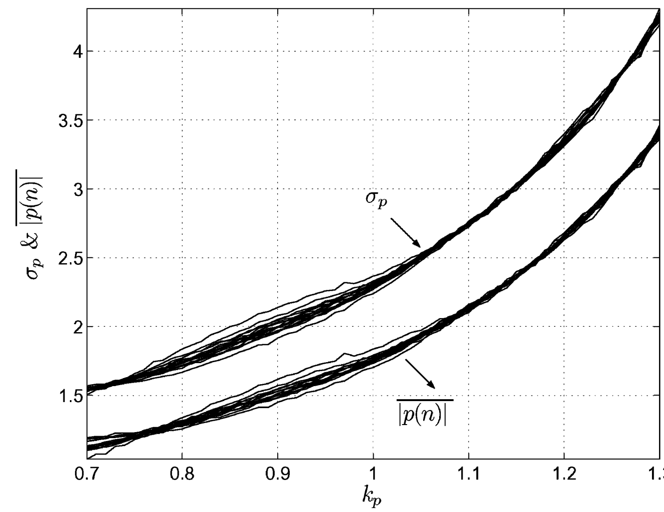

Fig. 5. σ_p and $\overline{|p(n)|}$ as a function of k_p for sinusoidal inputs with random amplitudes.

From (1), it is seen that the variance of $p(n)$ is related to the variance of the shaped quantization noise. The differencing operation involved in the generation of $p(n)$ further accentuates the high frequency gain of the NTF. Thus, σ_p is expected to increase with k_p. Note that σ_p does not depend on dc offset in the modulator. Using (1) and the usual linear assumption on quantization error, we get

$$\sigma_p^2 = \frac{\Delta^2}{12\pi} \int_0^\pi \left|\mathrm{NTF}(e^{j\omega})(1 - e^{-j\omega})\right|^2 d\omega \qquad (2)$$

where Δ is the step size of the internal quantizer.

An alternative measure of the variation of $p(n)$ is $\overline{|p(n)|}$—the mean of the absolute value of the first difference of the modulator output. This is easier to compute than σ_p, since squaring is avoided. Fig. 4 shows σ_p and $\overline{|p(n)|}$ for 16 "busy" bandlimited input waveforms for k_p varying from 0.7–1.3 for a third-order 15-level modulator with a nominal out-of-band gain of 3. An OSR of 64 is assumed, with $\Delta = 2$. The averages are obtained using 2^{15} samples of $v(n)$. The analytically calculated σ_p from (2) is also plotted in Fig. 4, but is not visible as it is very close to the simulated σ_p. Note that σ_p and $\overline{|p(n)|}$ increase with k_p.

Fig. 5 shows the computed σ_p and $\overline{|p(n)|}$ for sinewave inputs with 16 random amplitudes. We see good agreement with the results obtained with random inputs. Larger variability for small k_p can be attributed to tonal behavior of the modulator, since the out-of-band gain is low.

A. σ_p *Versus* $\overline{|p(n)|}$

In this subsection, we point out an interesting relationship between σ_p and $\overline{|p(n)|}$. Note that $p(n)$ is obtained by passing the quantization noise $e(n)$ through a filter with a transfer function $\mathrm{NTF}(z^{-1})(1 - z^{-1})$. As is the usual practice, $e(n)$ is assumed to be uniformly distributed and uncorrelated from sample to sample. From the above, $p(n)$ is seen to be a linear combination of uncorrelated random variables. By the central limit theorem, the distribution function of $p(n)$ tends to a Gaussian.

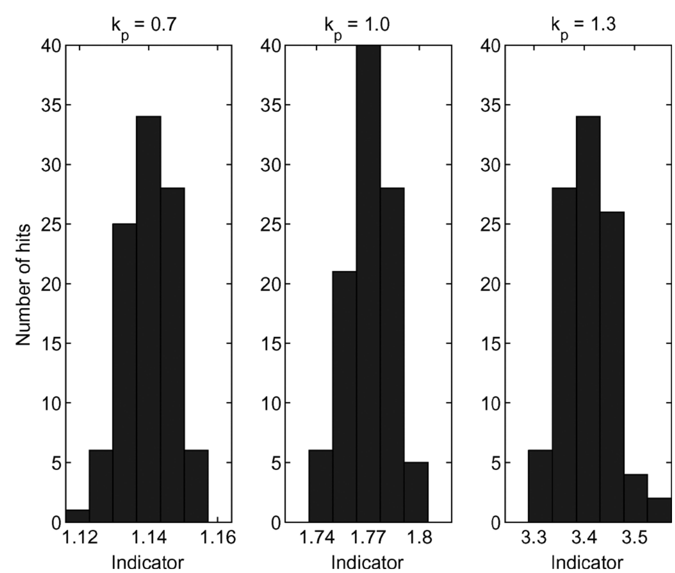

Fig. 6. Distribution of $\overline{|p(n)|}$ for a worst case 1.8% random mismatch in *RC* time constants—the three histograms are for different values of systematic k_p deviation from the nominal value: $k_p = 0.7$, $k_p = 1$, and $k_p = 1.3$. The number of trials is 100.

Recall that for a Gaussian random variable $X = N(0, \sigma^2)$, $E(|X|) = \sigma\sqrt{2/\pi}$. Hence, $\overline{|p(n)|} \approx \sigma_p\sqrt{2/\pi}$. $\sigma_p\sqrt{2/\pi}$ is compared with $\overline{|p(n)|}$ in Fig. 4, and good agreement is seen. Apparently, the Gaussian approximation of the quantization noise is more accurate for higher values of k_p [1].

Up to now, the discussion in this brief assumed that *all* resistors and capacitors differ from their nominal values by the *same* factor. In practice, apart from a systematic shift in *RC* values, mismatch among components will cause relative values of time constants to vary in a random fashion. This would also modify the modulator NTF. One potential issue with the time constant estimation techniques proposed earlier in this section, therefore, is the variability of σ_p and $\overline{|p(n)|}$ with random resistor and capacitor mismatch. Fig. 6 shows histograms of $\overline{|p(n)|}$ for a worst case 1.8% random mismatch in *RC* time constants for three process corners. It is seen that random variations in the NTF

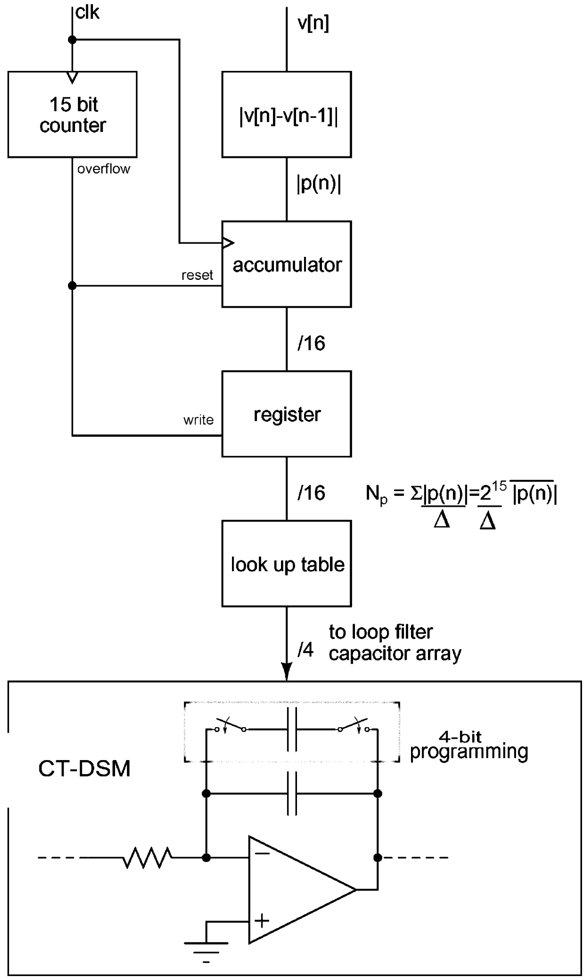

Fig. 7. Hardware implementation of time-constant tuning.

cause $\overline{|p(n)|}$ to vary by a few percent around the nominal value. Further, the variation is larger when the loop filter is "fast." This makes sense, as the NTF has a higher out-of-band gain when all the *RC* time constants are small, resulting in greater sensitivity to parameter variations.

III. Hardware Implementation

In this work, we assume that the loop filter is implemented using active-*RC* techniques [7], [8]. This does not lead to any loss of generality. Fig. 7 shows a possible hardware implementation of the *RC* time constant tuning system. The loop filter time constants are assumed to be tunable by means of a 4-bit digitally controlled capacitor array (variable from 0.7 to 1.3 times the nominal value) [2]. Filter time constants, can in principle, also be adjusted using other techniques (e.g a tunable resistor array). At startup, the capacitors in the loop filter are set to their nominal values. $|p(n)| = |v(n) - v(n-1)|$ is computed from the modulator output $v(n)$ and digitally accumulated. After 2^{15} cycles, the accumulated value is stored in the register, and the accumulator is reset. The stored value is $N_p = (1/\Delta)\sum_0^{2^{15}-1}|p(n)| = (2^{15}/\Delta)\overline{|p(n)|}$. Simulations show that $\overline{|p(n)|}$ is less than 3.5 for k_p in the range (0.7, 1.3). Therefore, 16 bits are required in the accumulator to represent N_p. From simulations of the $\Delta\Sigma$ modulator, the variation of $\overline{|p(n)|}$ with k_p, the relative error in loop filter *RC* time constant, is known (Fig. 4). Thus, from the measured $\overline{|p(n)|}$, the actual k_p can be determined. The capacitors on chip need to be multiplied by k_p to restore the *RC* time constants of the loop filter to their nominal values. A lookup table can therefore be formed between N_p and the settings of the capacitor array. Although 16 bits are used for accumulation, the lookup table input need only be about 6-bit wide to set the values of loop filter capacitors which are controlled by a 4-bit setting.

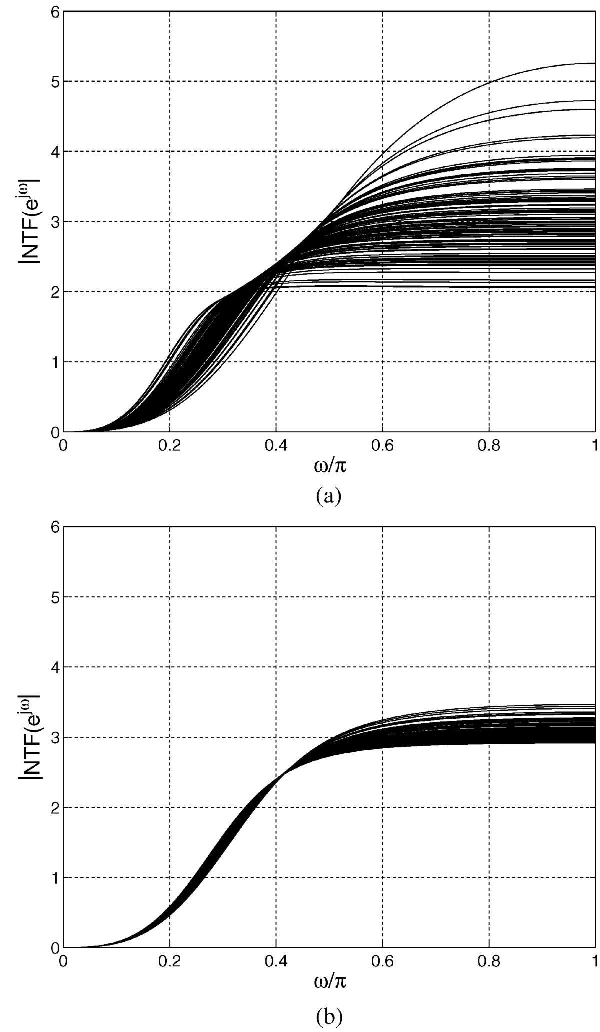

Fig. 8. Monte-Carlo simulations of NTF magnitude responses—(a) before tuning and (b) after tuning. The variations in NTF are greatly suppressed with residual variation being that due to 4-bit quantization of the loop filter capacitor array. 100 trials with $\pm 3\sigma_{k_p} = \pm 30\%$.

Fig. 8(a) shows the NTF magnitudes with variations in *R* and *C* of the loop filter without correcting for the latter's variations. Fig. 8(b) shows the NTF magnitudes when the capacitor value is set using a 4-bit control as described in the previous paragraph. As seen, the variation in NTF magnitude response is suppressed greatly. The residual variation is because of quantizing the capacitor tuning to 4 bits, and is well within acceptable limits.

On an integrated circuit, this tuning system can be enabled at startup, and once the correct settings for the capacitors in the loop filter are determined, they can be frozen. The convergence rate depends on the clock period. For a 3.072 MHz clock

(24-kHz signal bandwidth with an OSR of 64), the convergence period is approximately 11 μs (2^{15} cycles). At high clock rates, power consumption can be reduced at the expense of convergence speed by operating the accumulators at a lower rate—only the first-order differencing (computation of $v(n) - v(n-1)$) needs to operate at the full rate.

From the discussion in the previous section, it was seen that the estimated value of k_p was quite insensitive to the type of signal (e.g sinewave or a random "busy" waveform) at the modulator input. In the event that the input contains strong out-of-band signals, the proposed tuning technique can be run at startup, with a very low frequency signal generated on chip (e.g—a ramp). Note that the precision of this signal is not at all important, it serves to make the quantizer input "busy."

A. *Comparison With Other Tuning Techniques*

The most prevalent tuning methods (for example [2], [3]) for time constants in CT-DSMs seem to be based on the master-slave principle, as stated in the introduction. Since time-constant estimation involves the design of a replica oscillator (or integrator), a significant part of the loop is analog in nature. Our proposed technique is fully digital and can be easily migrated to new technologies. Digital implementation also offers several interesting possibilities—if temperature drifts of time constants are sufficiently low, the tuning loop can be run off-chip (in software)—this is possible since only the digital output sequence of the modulator is necessary. One aspect of our technique is that it is considerably slower, since it involves processing a large number of modulator output samples. However, speed of these tuning techniques is not a serious issue, since the correction is usually done at start-up.

The proposed technique compares favorably to the digital schemes proposed in [4] and [5]. Note that these papers address a different (but related) problem—that of matching the analog filter transfer function to the tunable digital noise cancelling filter in a cascaded modulator. In these works, no attempt is made to restore the time constants of the CT loop filter (although it seems quite straightforward to do so)—the digital filter coefficients are modified instead. The estimation technique used in [4] & [5] computes the variance of the decimated sequence. [4] is a simulation study, while the time-constant estimator in [5] is implemented off-chip in software. It seems as if computing the variance of the decimated sequence is expensive (since high resolution multipliers and adders are required). These techniques also require that there be no in-band signal, thereby necessitating the removal of the analog–digital converter (ADC) from the signal path.

Our technique works with an in-band signal, so the ADC can process the signal while the *RC*-product is being estimated. Since the modulator output is used (rather than the decimated sequence), the tuning process takes a smaller time. No complex operations, like squaring, are necessary. The proposed technique is based on intuition from a linear model of the SDM, which is appropriate when only a multibit quantizer is employed in the loop. Simulations indicate satisfactory performance with quantizer resolutions beyond 2 bits. This is not overly restrictive, since multibit operation is beneficial in several other ways [1].

IV. Conclusion

Systematic time-constant variations due to process shifts can significantly impair the performance of CT-DSMs. The variance of the high pass filtered output stream of the modulator was seen to be an indicator of the time constants (or k_p) of the loop filter. An alternative indicator, the average of $|p(n)| \equiv |v(n) - v(n-1)|$, which is easier to implement in digital hardware was proposed. Either of the above k_p estimators could be used in conjunction with a look-up table to program the capacitor values in the loop filter to suppress the variations in the modulator NTF due to process shifts. The tuning technique does not use any extra analog components. Simulation results demonstrating the effectiveness of the proposed technique were given.

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