Introducing Negative Feedback with an Integrator as the Central Element

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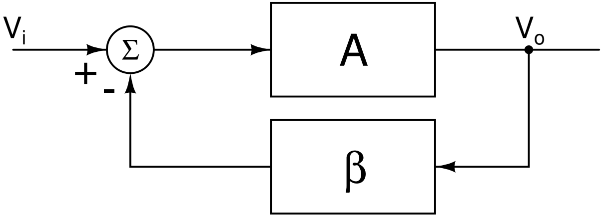
Motivation

- Intuition before full blown analysis
- Synthesis instead of ad-hoc introduction
- Time domain reasoning/analysis
  - More intuitive
  - Exact analysis difficult for complex systems
- Frequency domain analysis
  - More abstract
  - Can handle complex systems easily

Outline

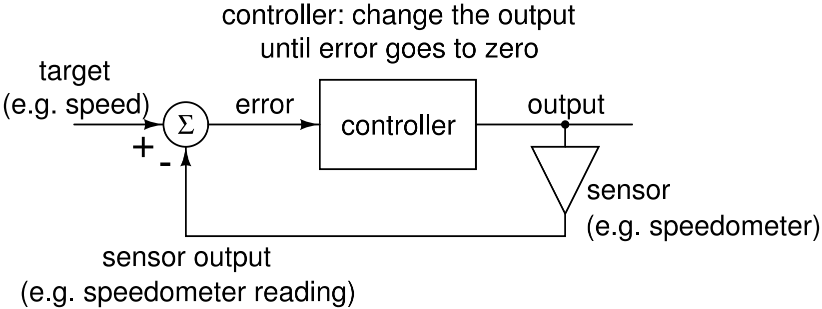
- Traditional introduction to negative feedback systems
- Integrator as controller in a negative feedback system
- Intuition and analysis in the time domain
- Pedagogical advantages of the proposed introduction
- Conclusions

Traditional introduction to negative feedback systems



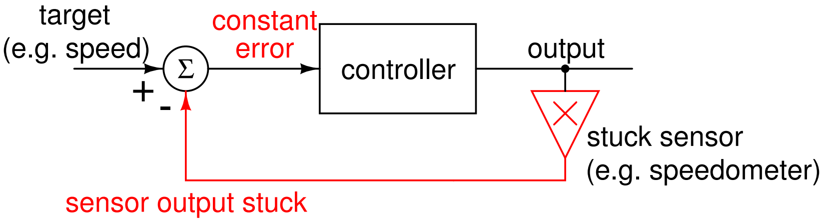
- Algebraic system—cannot explain evolution over time
- Unstable with arbitrarily small loop delay
  - Ideal delay T_d in the loop $\Rightarrow$ oscillations with a period $2T_d$
- Real systems have non-zero delay and don't respond instantaneously

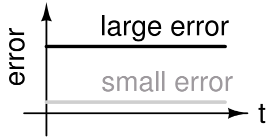
Intuitive understanding of negative feedback systems

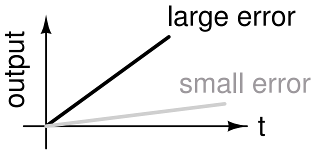


- Compare the sensed output to the target (desired output)
- *Continuously* change the output until the output approaches the target

Nature of the controller

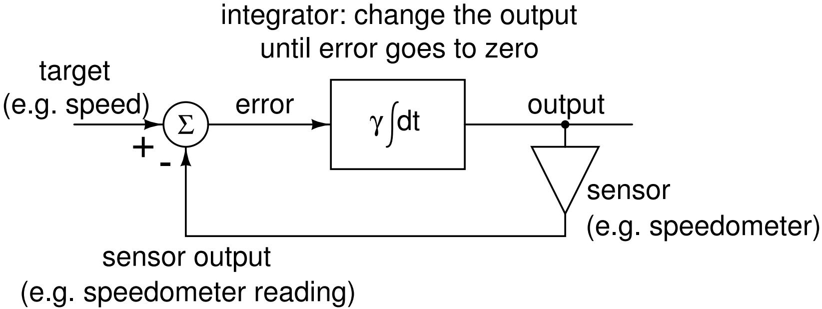




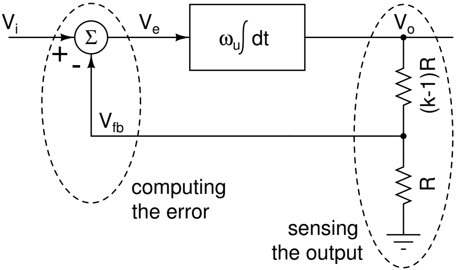


- Controller integrates the error

Negative feedback system with an integrator

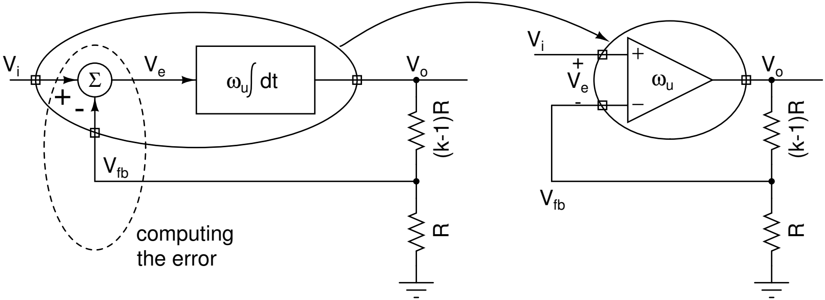


Negative feedback amplifier

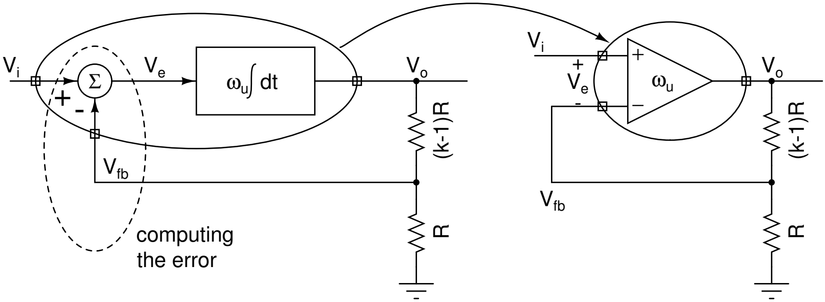


- Need the output V_o to be gain k times the input V_i
- Compare V_o/k to V_i and integrate the error
- Steady state when $V_o = kV_i$ for constant V_i

Opamp for implementing a negative feedback amplifier



Time domain behavior with constant/step inputs



$$\frac{1}{\omega_u}\frac{dV_o}{dt} = V_i - \frac{V_o}{k}$$

$$V_o(t) = kV_p\left(1 - \exp\left(-\frac{\omega_u}{k}t\right)\right)$$

- Time constant k/ω_u
- Asymptotically reaches $V_o = kV_i$ or $V_{fb} = V_i$

Relation to frequency domain analysis

$$\text{Loop gain } L(s) = \frac{\omega_u}{ks} = \frac{\omega_{u,loop}}{s}$$

Frequency domain:

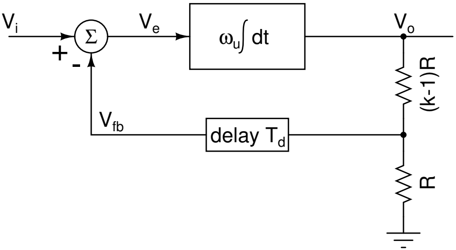
- Unity loop gain frequency $\omega_{u,loop}$
- Significant negative feedback up to $\omega_{u,loop} \Rightarrow$ nearly ideal behavior up to $\omega_{u,loop}$ (Closed loop Bandwidth)

$$\tau_{loop} = \frac{1}{\omega_{u,loop}}$$

Time domain:

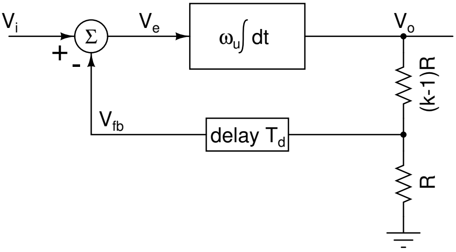
- Unit step response of the loop gain $= t/(1/\omega_{u,loop}) = t/\tau_{loop}$
- Closed loop response time constant $= 1/\omega_{u,loop} = \tau_{loop}$

Negative feedback amplifier with delay in the loop



- Reacts to past output ⇒ Don't know target has been reached
- Possibility of overshoots or unbounded oscillation
- Unaffected if the integrator's output doesn't change significantly over T_d

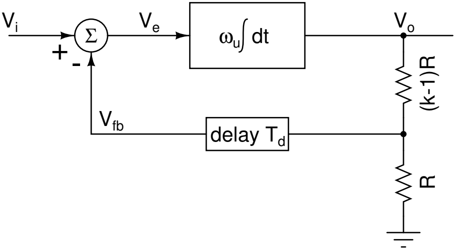
Negative feedback amplifier with delay in the loop



$$\frac{1}{\omega_u}\frac{dV_o(t)}{dt} = V_i - \frac{V_o(t - T_d)}{k}$$

Delay differential equation

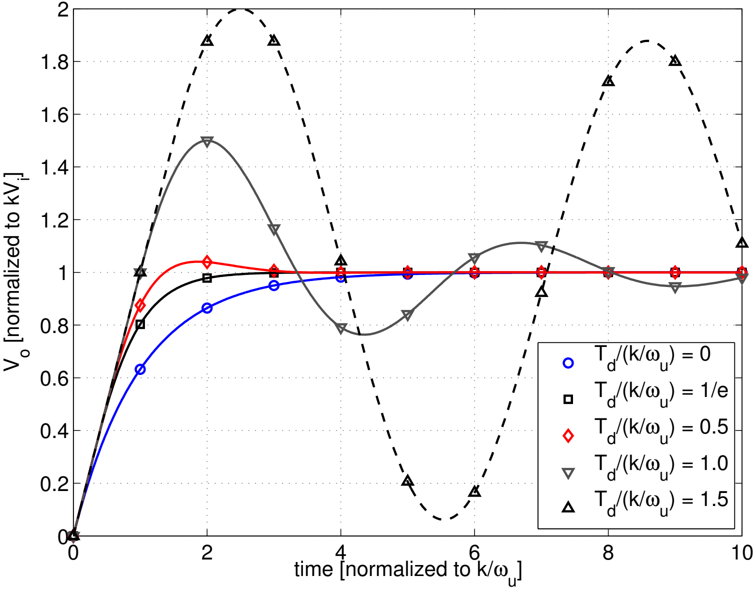
Negative feedback amplifier with delay in the loop



- $T_d/\tau_{loop} \leq 1/e (= 0.367)$: No overshoot
- $1/e < T_d/\tau_{loop} < \pi/2$: Overshoot + ringing
- $\pi/2 < T_d/\tau_{loop}$: Unstable

In practice we need a “well behaved” response (limited overshoot)

Negative feedback amplifier with delay in the loop



Negative feedback amplifier with delay in the loop

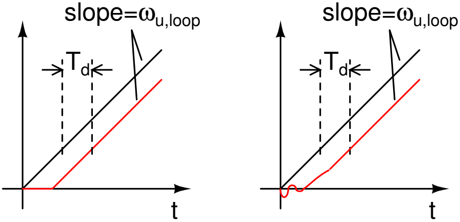
% Overshoot	0	1	2	4	10	20
T_d/τ_{loop}	$\leq 1/e$ (0.367)	0.445	0.465	0.5	0.585	0.695

Fixing the stability problem in presence of delay

- Stability governed by the ratio of T_d to τ_{loop}
- Reduce T_d: Faster circuit/technology
- Increase $\tau_{loop} \Rightarrow$ Decrease $\omega_{u,loop}$: Slower integration

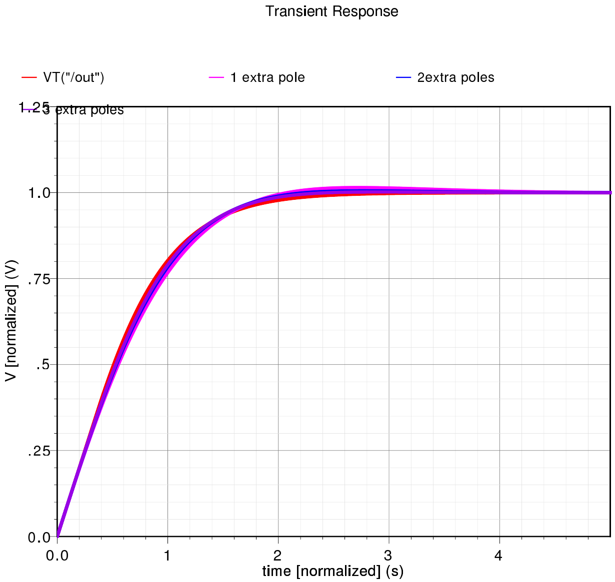
Delays in circuit implementation—parasitic poles and zeros

$$\text{Loop gain } L(s) = \underbrace{\frac{\omega_{u,loop}}{s}}_{\text{Ideal}} \cdot \underbrace{\frac{\prod_{k=1}^{M}(1+s/z_k)}{\prod_{k=2}^{N}(1+s/p_k)}}_{\text{Parasitic}}$$



Unit step response of $L(s)$ is a ramp of slope $\omega_{u,loop}$ (same as ideal) with a delay $T_d = \sum_{k=1}^{N} 1/p_k - \sum_{k=1}^{M} 1/z_k$

Closed loop response with eqiuvalent delay



Advantages of this formulation

- Synthesis from common experience of negative feedback based adjustment in the time domain
- Intuition and key results obtained from time domain reasoning
  - Exponential settling
  - Possibility of ringing, overshoot, and instability

Advantages of this formulation

- Traditional viewpoint
  - Memoryless amplifier (loop gain) in the ideal case
  - Frequency dependence as non-ideal feature
- Proposed viewpoint
  - Integrator in the ideal case (∞ dc gain)
  - Finite dc gain due to non-ideal implementation
- $\omega_{u,loop}$ more fundamental characteristic of the negative feedback loop than dc loop gain
  - Increasing $\omega_{u,loop}$ requires higher power
  - Increasing dc loop gain indirectly influences power
- Loop gain of all feedback systems has integrator-like behavior over some frequency range
  - Nyquist plot should enter the unity circle near the negative imaginary axis
  - Bode plot should have -20 dB/decade slope near the unity gain frequency

Advantages of this formulation

- Clear why fastest negative feedback systems are slower than fastest open loop systems
- Leads to commonly used opamp and phase locked loop topologies

Suggested course outline

- **Negative feedback with integrating controller**
- **Opamp for computing and integrating error**
- **Time domain analysis with delay**
- Laplace transform stability analysis, Nyquist criteria
- **Relation to time domain analysis results**
- Synthesis of opamp, PLL topologies

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