A Model-Agnostic Technique for Simulating Per-Element Distortion Contributions

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***Abstract*— The nonlinearity of an element can be altered while retaining the original operating point and first-order terms by appropriately combining two instances of the nonlinear element with complementary scaling factors for incremental voltages above the operating points. Per-element distortion contributions in a circuit can then be determined by altering the nonlinear terms by known factors and simulating the output distortion in each case. This technique can be used in a standard circuit simulator with the appropriate nonlinear device models but requires no knowledge of the device model details on the part of the circuit designer. The technique is demonstrated by applying it to a common source amplifier with a nonlinear load and a two stage fully differential opamp.**

I. Motivation

Distortion due to nonlinearity and noise due to inherent randomness are the most important disturbances in signal processing circuits. Circuits must be designed such that these are kept below certain specified levels. It would be convenient to determine individual contributions of these disturbances from different blocks or components to the overall output so that the circuit can be suitably optimized. Determining noise contributions from individual components is routinely done in standard circuit simulators. For distortion though, no such facility is available. The total output distortion can however be determined easily by running a transient or periodic steady state analysis with nonlinear device models.

To determine the distortion or noise contribution of each device, one needs to know the equivalent nonlinear distortion or noise source in each component, and the transfer function from that source to the output. The difficulty in resolving individual distortion contributions is that, unlike in the case of noise, it is not straightforward to calculate the equivalent distortion source at the device level. If the nonlinear device were described by a Taylor or Volterra series in the port variables, the nonlinear source would consist of the higher order terms in the series. In practice, the device models are a lot more complicated and not described in closed form.

In this paper, we present a technique that bypasses both these steps of explicitly computing the distortion source of the device or the transfer functions to the output. It is shown that by running multiple simulations of the total output distortion of a circuit with slightly changed nonlinear characteristics in the relevant element, the contribution of the element to the output distortion can be determined. *Obtaining a device with changed nonlinear characteristics is based on elementary circuit theory and requires no knowledge of the device model.*

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In the next section, we review previously available techniques for determining individual distortion contributions. In Section III, we show how to synthesize a new nonlinear element which has the same operating point and linear characteristics, but different nonlinear characteristics. In Section IV, we show how to use this element with scalable nonlinear terms to obtain individual distortion contributions. In Section V the proposed technique is verified by applying it to several examples. Section VI concludes the paper.

II. Existing methods for determining individual contributions to distortion

In case of a cascade of open loop stages, one can simulate the distortion of the stages individually to determine their contributions. In cases where a stage is significantly loaded by the following one, it is harder to isolate the contributions. For closed loop systems, which are frequently used for low distortion applications, this method is not applicable.

The probing method described in [1] successively evaluates higher order nonlinear contributions by injecting additional nonlinear sources to the linear equivalent circuit. This is further developed or simplified for analog integrated circuits in, e.g., [2], [3], [4], [5] for analysis and to gain insight into distortion behavior of circuits. All of these are based on Taylor or Volterra series descriptions of the circuit components, which, as pointed out earlier, are not readily available, and have to be extracted by the designer. The specific question of systematically determining per-element distortion contributions is addressed in [6]. This describes an algorithm that can be used in a simulator to determine per-element contributions and cannot be used by a circuit designer running a conventional SPICE-like simulator.

[7] circumvents the extraction of nonlinear device models by using an appropriate multi-sine excitation with which one can determine the equivalent additional distortion source of each element. These sources are used in conjunction with small-signal transfer functions determined by ac/noise analysis to identify the distortion contribution from that particular element. This is essentially a missing tone test, and it may not be easy to relate this to conventional single-tone harmonic distortion and two-tone intermodulation distortion tests. It also involves choosing an appropriate multi-tone input signal which entails additional labor.

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III. OBTAINING A DEVICE WITH SCALED NONLINEARITY

The proposed technique is based on substituting the nonlinear element in the circuit by another nonlinear element whose operating point and first order behavior are the same but whose nonlinearity is different. For simplicity, the principle is

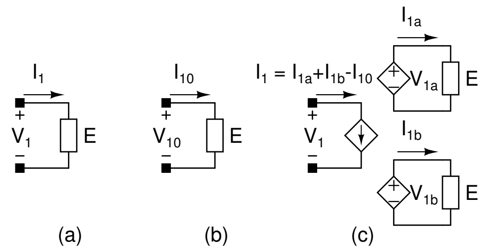

Fig. 1. (a) Nonlinear element E, (b) Operating point, (c) Nonlinear one port element constructed from two instances of E driven by V_{1a} and V_{1b}.

first illustrated with a memoryless one port element. Fig. 1(a) shows a nonlinear element E with a current-voltage relationship $I_1 = f(V_1)$. Fig. 1(b) shows the same element E at a certain operating point (V_{10}, I_{10}). Defining incremental voltage v_1 and current i_1 respectively as $v_1 = V_1 - V_{10}$ and $i_1 = I_1 - I_{10}$ and expanding the nonlinear relationship in a Taylor series around the operating point, we get

$$I_1 = f(V_{10}) + \left.\frac{df}{dV_1}\right|_{V_{10}} v_1 + \frac{1}{2!}\left.\frac{d^2 f}{dV_1^2}\right|_{V_{10}} v_1^2 + \frac{1}{3!}\left.\frac{d^3 f}{dV_1^3}\right|_{V_{10}} v_1^3 + \ldots \tag{1}$$

The first term is the operating point, the second term is the linear part, and successive terms are nonlinearities.

Now consider the one port in Fig. 1(c) which is constructed from two instances of elements E driven with voltages V_{1a} and V_{1b}. These voltages are related to V_1, the voltage across the one port, as follows:

$$V_{1a} = V_{10} + a_1 (V_1 - V_{10}), V_{1b} = V_{10} + (1 - a_1)(V_1 - V_{10}) \tag{2}$$

where a_1 is a scaling factor. In other words, the two copies of E experience differently scaled versions of the incremental voltage $v_1 = (V_1 - V_{10})$ applied to the one port. The current I_1 in the new one port element is defined as $I_1 = I_{1a} + I_{1b} - I_{10}$. Using Taylor series expansions for $I_{1a} = f(V_{1a})$ and $I_{1b} = f(V_{1b})$ we get

$$\begin{aligned} I_1 \quad = \quad & f(V_{10}) + \left.\frac{df}{dV_1}\right|_{V_{10}} v_1 + \left(a_1^2 + (1-a_1)^2\right) \frac{1}{2!}\left.\frac{d^2 f}{dV_1^2}\right|_{V_{10}} v_1^2 \\ & + \left(a_1^3 + (1-a_1)^3\right) \frac{1}{3!}\left.\frac{d^3 f}{dV_1^3}\right|_{V_{10}} v_1^3 + \ldots \end{aligned} \tag{3}$$

It is clear from equations (1) and (3) that the nonlinear one port in Fig. 1(c) has the same operating point and linear terms as the one port in Fig. 1(a), but scaled nonlinear terms. The N^{th} order term in the series is scaled by $a_1^N + (1-a_1)^N$.

This reasoning can be easily extended to two or more ports. Fig. 2(a) shows a two port E with voltage V_1, V_2 and current I_1, I_2. Fig. 2(b) shows the operating point condition. Fig. 2(c) shows a new two port network constructed from two

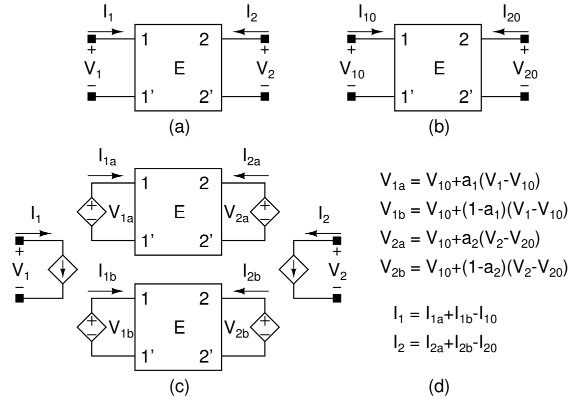

Fig. 2. (a) Nonlinear two port E, (b) Operating point, (c) Nonlinear two port constructed from two instances of E driven by $V_{1a,2a}$ and $V_{1b,2b}$.

instances of E which receive scaled versions of the incremental voltages above the operating point. The voltages applied to the two ports and the port currents are given by the relationships in Fig. 2(d). Using similar reasoning as with the one port, it is clear that I_1 and I_2 of the composite two port network in Fig. 2(c) consist of the same operating point and first order terms as in the original two port in Fig. 2(a), but have scaled higher order terms. Table I lists the scaling factors for second and third order terms in I_1 and I_2. The pattern for higher order terms is obvious.

TABLE I

SCALING FACTORS FOR NONLINEAR TERMS OF THE TWO PORT

Second order	Scaling factor	Third order	Scaling factor
v_1^2	$a_1^2 + (1-a_1)^2$	v_1^3	$a_1^3 + (1-a_1)^3$
$v_1 v_2$	$a_1 a_2 + (1-a_1)(1-a_2)$	$v_1^2 v_2$	$a_1^2 a_2 + (1-a_1)^2(1-a_2)$
v_2^2	$a_2^2 + (1-a_2)^2$	$v_1 v_2^2$	$a_1 a_2^2 + (1-a_1)(1-a_2)^2$
		v_2^3	$a_2^3 + (1-a_2)^3$

In an M port network, one would need M scaling factors $a_1, \ldots a_M$ to scale the incremental port voltages $v_1, \ldots v_M$. The N^{th} order nonlinear term in the scaled network will be of the form $\prod_{k=1}^{M} v_k^{l_k}$ where $0 \le l_k \le N$. The scaling factor for this term would be $\prod_{k=1}^{M} a_k^{l_k} + \prod_{k=1}^{M} (1-a_k)^{l_k}$. Though Taylor series expressions are used above for simplicity, the method is equally applicable to nonlinearity with memory. An N^{th} order term of the Volterra series of an M port network in which v_k appears l_k times $(0 \le l_k \le N)$ will be scaled by the same factor $\prod_{k=1}^{M} a_k^{l_k} + \prod_{k=1}^{M} (1-a_k)^{l_k}$.

IV. DETERMINING AN ELEMENT'S CONTRIBUTION

Fig. 3(a) shows a circuit with a nonlinear two-port element E whose contribution to distortion has to be determined. For clarity of discussion, we consider a single sinusoidal input voltage V_s, a voltage V_{out} as the output, and a two port E which has nonlinear terms only up to the second order. But the technique is general and works in the same way for multi-tone

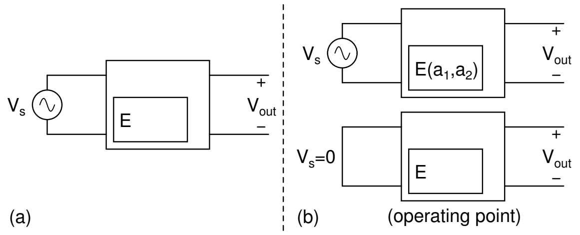

Fig. 3. (a) Original circuit with element E, (b) Circuit with E replaced by its scaled version and a copy of the original circuit at the operating point.

inputs, currents instead of voltages, and multi-port nonlinear elements with higher orders of nonlinear terms.

First, the distortion H_{out}^{0} of the circuit in Fig. 3(a) is simulated. H_{out}^{0} stands for any distortion component of interest in the output V_{out}—harmonic distortion of a given order, or the total harmonic distortion, or in case of multi-tone excitation, the relevant intermodulation component(s). H_{out}^{0} could be the frequency domain representation (Fourier transform magnitude and phase) of the distortion components or the time domain distortion waveform. Here we will assume the former.

Then, the schematic in Fig. 3(b) is generated from the original schematic. It consists of the circuit with the nonlinear element E replaced by its scaled version with scaling factors a_1, a_2. The scaled element E requires the operating point information. Therefore, a copy of the original circuit in quiescent condition is included in the schematic from which the operating point information is extracted[1]. The distortion $H_{out}^{a1,a2}$ is simulated in this scaled circuit.

Let the distortion contributed to the output in the original circuit (Fig. 3(a)) by the v_1^2, $v_1 v_2$, and v_2^2 terms of the element E be denoted by $H_{v_1^2}$, $H_{v_1 v_2}$, and $H_{v_2^2}$ respectively. Let H_{rest} denote the distortion contributed by the rest of the circuit. Then, the total distortion H_{out}^{0} in the original circuit is given by

$$H_{out}^{0} = H_{v_1^2} + H_{v_2^2} + H_{v_1 v_2} + H_{rest} \tag{4}$$

When the nonlinear element E is scaled by a_1, a_2, the distortion contributed by the v_1^2, $v_1 v_2$, and v_2^2 terms will be scaled as shown in Table I. The distortion contributed by the rest of the circuit is not changed (This assumption holds when the elements are weakly nonlinear and the distortion is small). The output distortion $H_{out}^{a_1,a_2}$ is therefore:

$$\begin{aligned} H_{out}^{a1,a2} \quad = \quad & \left(a_1^2 + (1-a_1)^2\right) H_{v_1^2} + \left(a_2^2 + (1-a_2)^2\right) H_{v_2^2} \\ & + \left(a_1 a_2 + (1-a_1)(1-a_2)\right) H_{v_1 v_2} + H_{rest} \end{aligned} \tag{5}$$

Simulating the circuit in Fig. 3(b) for three different combinations of a_1, a_2, yields us four equations—(4) and three cases of (5)—from which the four unknowns $H_{v_1^2}, H_{v_2^2}, H_{v_1 v_2}$, and H_{rest} can be determined.

If one is interested only in the total contribution from second order terms $H_{v_1^2, v_2^2, v_1 v_2} = H_{v_1^2} + H_{v_2^2} + H_{v_1 v_2}$ of E and not in individual contributions from each second order term, one can set $a_2 = a_1$. This results in

$$H_{out}^{a1,a1} \quad = \quad \left(a_1^2 + (1-a_1)^2\right) H_{v_1^2, v_2^2, v_1 v_2} + H_{rest} \tag{6}$$

In this case, the circuit in Fig. 3(b) needs to be simulated only for one value of a_1 to can determine $H_{v_1^2, v_2^2, v_1 v_2}$ and H_{rest}.

In the above, we assumed that the element E has nonlinear terms only up to the second order. If the element E has nonlinear terms up to N^{th} order, N circuit simulations are required to determine the total contribution from each of the $N-1$ nonlinear terms. If it is further required to resolve the distortion into separate terms in each order, $N(N+3)/2-1$ simulations with distinct combinations of a_1, a_2 are required. The number of significant nonlinear terms has to be initially determined by trial and error. Apriori knowledge that some terms are insignificant (e.g. even order terms in a fully differential two port) can reduce the number of simulations.

V. Examples

A. Common source amplifier with a resistive load

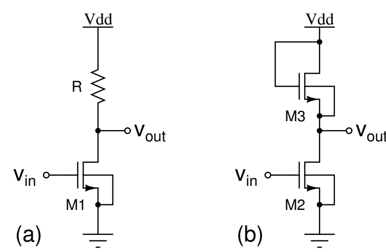

Fig. 4. Common source amplifier with (a) resistive load (b) diode connected load.

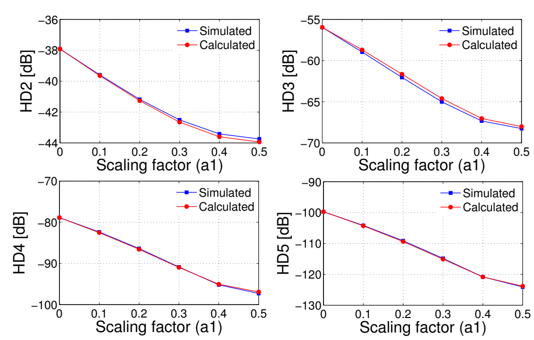

Fig. 5. Output distortion components in Fig. 4(a) with scaling.

Fig. 4(a) shows a common source amplifier with a resistive load. This circuit has a single nonlinear component, the transistor, and is used to verify that components of different order scale as described in the previous section. For input amplitudes such that there is negligible compression, a nonlinear

[1]For convenience, the technique is illustrated with a duplicated circuit for the operating point. But this duplication is not essential. As an alternative, the operating point could be simulated first and appropriate information could be fed to the scaled network E. Alternatively, one could, in transient simulation, initially deactivate the input source and set the scaling factors to unity. This yields the operating point information of the original circuit. After a certain delay, these values could be sampled and held and fed to the scaled network, the input signal activated, and the scaling factors set to the desired values.

term of a given order contributes only to a harmonic of the same order. Fig. 5 shows the output distortion components for a 20 mV peak input signal for different values of a_1. Good agreement is seen between the expected scaling factor $a_1^N + (1 - a_1)^N$ and that obtained from simulation.

B. Common source amplifier with a MOS transistor load

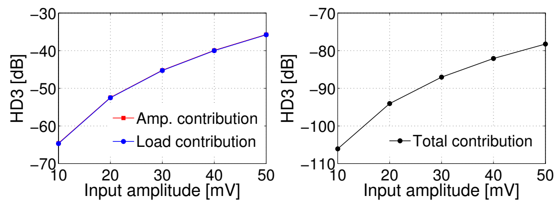

Fig. 6. Distortion contributions in Fig. 4(b).

Fig. 4(b) shows a common source amplifier with an nMOS diode connected load. Since the load is a replica of the amplifier, nonlinearities should cancel. The amplifier device is scaled by $a_1 = 0.01$ and the distortion contributions from the amplifier and the load are calculated using the method in Section IV. Fig. 6 shows the total (third harmonic) distortion and the contributions from each device. It can be seen that the contributions from the two components are almost equal in magnitude. Simulation results show that these contributions have opposite phase, which leads to cancellation. This can also be seen from the fact that the total output distortion is $\sim$40 dB below the contribution from either device. Extracting the contributions using a different value of the scaling factor $a_1 = 0.02$ yields the same results.

C. Opamp in closed loop

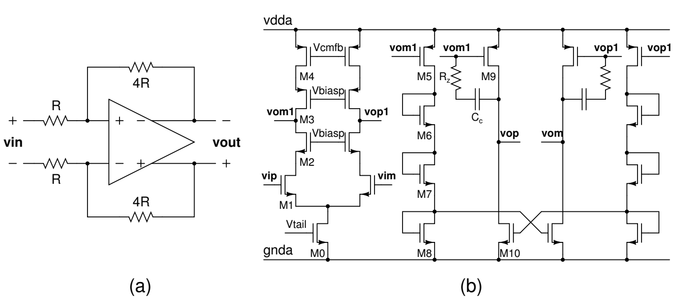

Fig. 7. (a) 4× inverting amplifier (b) Circuit diagram of the opamp in [8].

Fig. 7(a) shows an inverting amplifier of gain 4 with a bandwidth of the $\sim$3 MHz. Fig. 7(b) shows the opamp used in the amplifier [8]. Distortion of the second stage is extracted by scaling M_{5-10} by the same factor $a_1 = 0.01$. Similarly, contributions from different sections of the circuit are extracted by scaling all the transistors in the corresponding section. Fig. 8 shows the original distortion(third harmonic), contributions from different stages, and the corresponding sum versus input frequency for a 10 mV peak input signal. It can be seen that the second stage and the common mode feedback circuitry for the first stage are the major contributors to the output distortion.

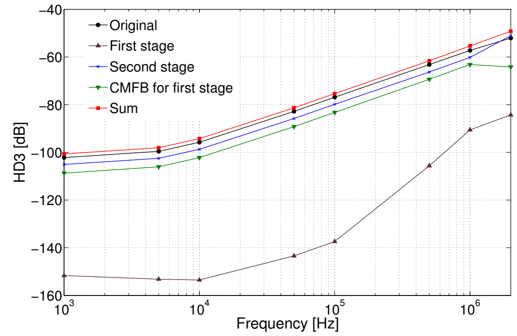

Fig. 8. Distortion versus frequency for the amplifier in Fig. 7.

VI. Conclusions

A method has been proposed by which a circuit designer can conveniently determine distortion contributions from different elements and from different terms of each element without going into device model details. It does not require the extraction of Taylor or Volterra series models for individual circuit elements. The technique is demonstrated by applying it to a common source amplifier with linear and nonlinear loads and a closed loop amplifier.

Since the modified element is based on instances of the original element, and not an abstracted model, the proposed technique also allows one to determine distortion contributions with process or temperature variations. This is in contrast to [2], [3], [4], [5] where the Taylor/Volterra model at each corner has to be determined. The technique can be applied to blocks (e.g. stages of an opamp, or the entire opamp) as well as to individual transistors to produce a hierarchical listing of distortion contributions. Because of its ease of use, the proposed technique can serve as a convenient tool for optimizing distortion or investigating the robustness of distortion cancellation schemes.

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