Note Title 25-09-2007

LMS adaptation.

$$y = \sum_{k=0}^{N-1} c_k \, x[n-k]$$

* Need to adjust c_k to minimize e^2; $e = y - \hat{y}$

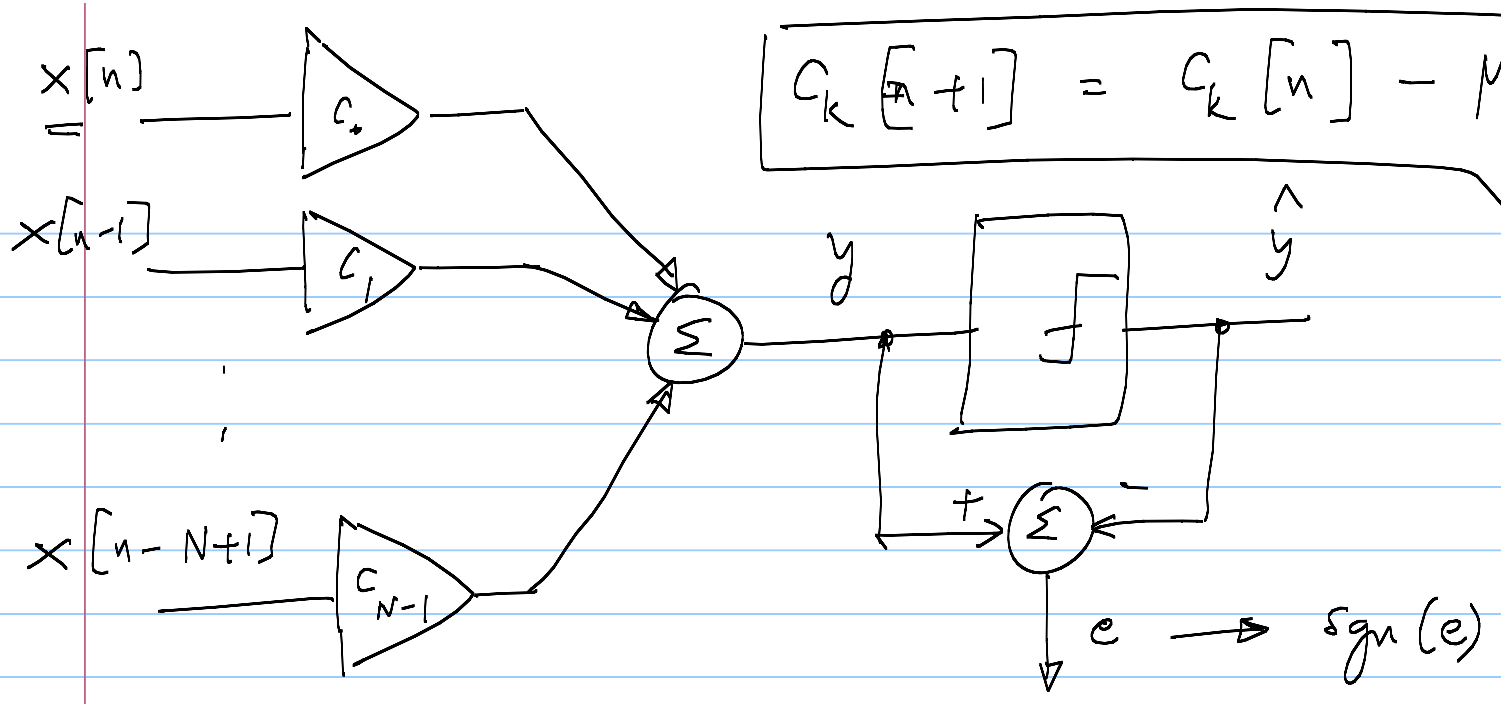
* $$c_k[n+1] = c_k[n] - \mu \cdot \underline{e \cdot x[n-k]}$$

Simplifications.

$$c_k[n+1] = c_k[n] - \mu \cdot \mathrm{sgn}(e) \cdot \mathrm{sgn}(x[n-k])$$ Simplest to implement

$$c_k[n+1] = c_k[n] - \mu \cdot \mathrm{sgn}(e) \cdot x[n-k]$$

$$c_k[n+1] = c_k[n] - \mu \cdot e \cdot \mathrm{sgn}(x[n-k])$$



$$\boxed{c_k[n+1] = c_k[n] - \mu \,\mathrm{sgn}(e[n])\, \mathrm{sgn}(x[n-k])}$$

If $e > 0$ $\{\Rightarrow y$ is too large$\}$

reduce the contribution of all branches

i.e. if $x[n-k] > 0$ reduce c_k by μ

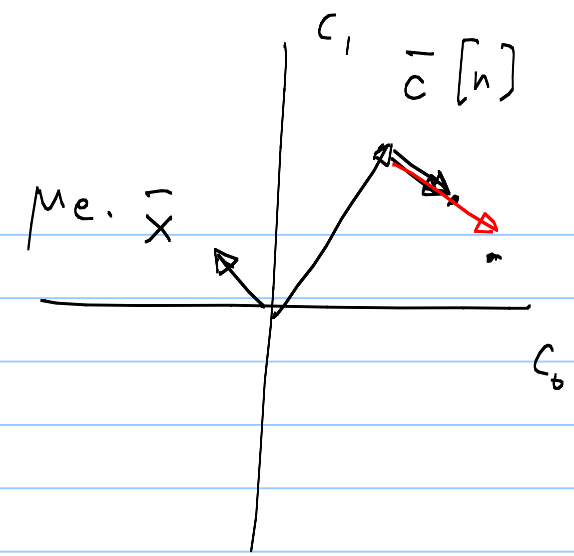
$x[n-k] < 0$ increase c_k by μ

$$c_k[n+1] = \left. c_k[n] - \mu \cdot x[n-k] \cdot e[n] \right] \checkmark$$

$$c_k[n] - \mu\, \mathrm{sgn}\left(x[n-k]\right) \cdot e[n]$$

$$c_k[n] - \mu\, x[n-k] \cdot \mathrm{sgn}\left(e[n]\right)$$

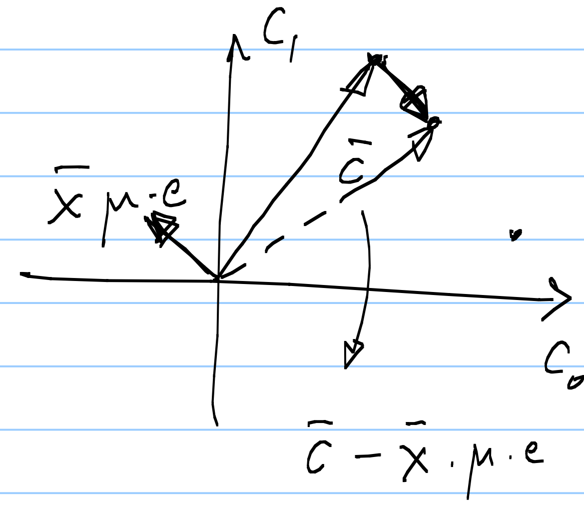
$$c_k[n] - \mu\, \mathrm{sgn}\left(x[n-k]\right) \cdot \mathrm{sgn}\left(e[n]\right)$$



$$\bar{c}[n+1] = \underline{\underline{\bar{c}[n]}} - \overbrace{\underbrace{\mu \cdot e[n]\ \underline{\underline{\bar{x}}}}}$$

✓

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_{N-1} \end{bmatrix} \qquad \begin{bmatrix} x[n] \\ x[n-1] \\ \vdots \\ x[n-N+1] \end{bmatrix}$$



$$\bar{c}[n+1] = \bar{c}[n] - \mu e \cdot \bar{w} \qquad \text{Full LMS}$$

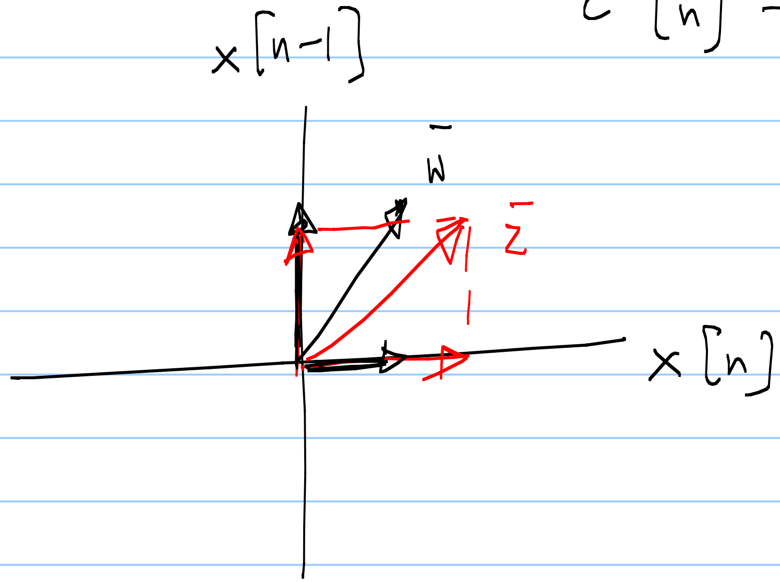
$$\bar{c}[n] - \mu \cdot \mathrm{sgn}(e) \cdot \bar{w}$$

$$\bar{w} = \begin{bmatrix} x[n] \\ x[n-1] \\ \vdots \\ x[n-N+1] \end{bmatrix}$$

$$\bar{c}[n+1] = \bar{c}[n] - \mu e \left[\bar{z} \right]$$

$$\bar{c}[n] - \mu \cdot \mathrm{sgn}(e) \cdot \bar{z}$$

$$\bar{z} = \begin{bmatrix} \mathrm{sgn}(x[n]) \\ \mathrm{sgn}(x[n-1]) \\ \\ \mathrm{sgn}(x[n-N+1]) \end{bmatrix}$$

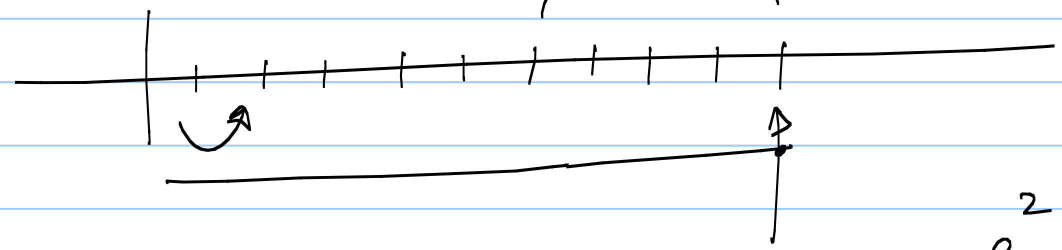


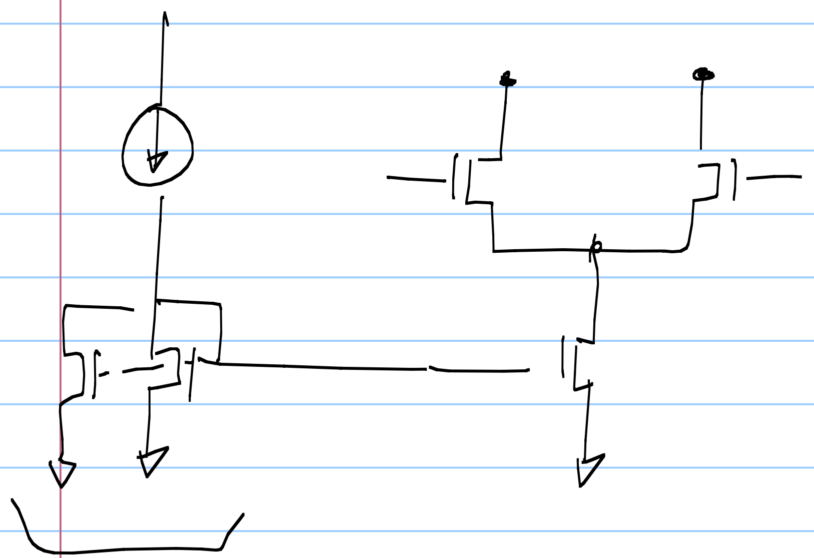
$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} \begin{matrix} \checkmark \\ \checkmark \end{matrix}$$

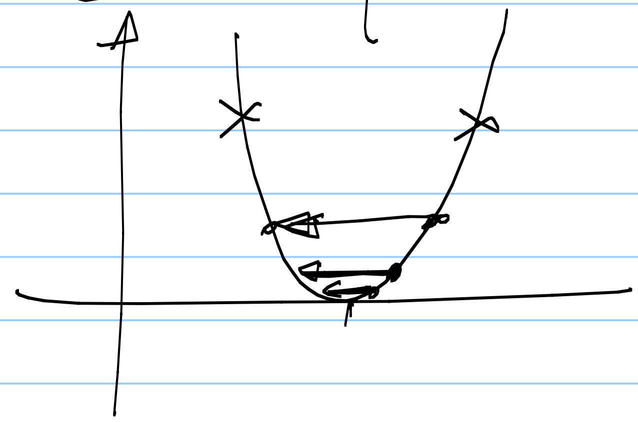
$$\begin{bmatrix} \mathrm{sgn}(x[n]) \\ \mathrm{sgn}(x[n-1]) \end{bmatrix} \begin{matrix} \checkmark \\ \checkmark \end{matrix}$$

$$\bar{C}[n+1] = \bar{C}[n] - \mu \cdot e \cdot \bar{X}$$

$$\begin{bmatrix} x[n] \\ x[n-1] \\ x[n-2] \end{bmatrix}$$







$$C_k[n] - \mu \cdot \mathrm{sgn}(e) \cdot \mathrm{sgn}(x[n-k])$$

$$\Sigma$$

LMS adaptation:

* Move the coeff. in the direction of the gradient.
* sign-sign LMS to simplify coefficient update computation
* Coefficient update at a lower rate than data rate to save power

* Average the product $\text{sgn}(e[n])\,\text{sign}(x[n-k])$ before updating c_k