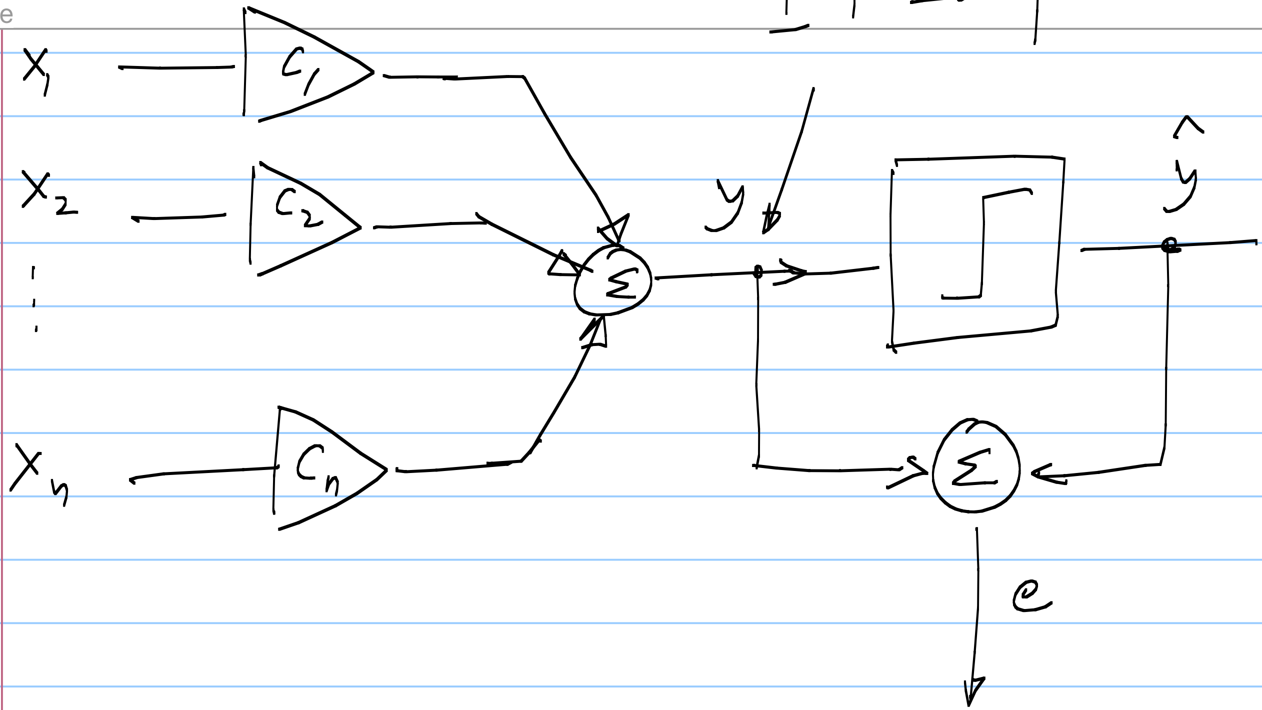
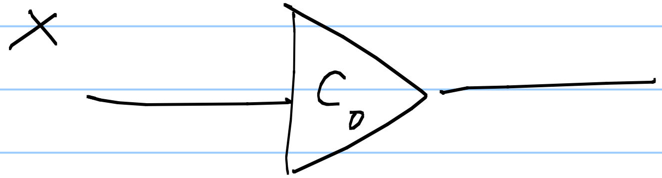
# Adaptation

$\pm 1$ ~~samples~~ values if equalization is perfect.

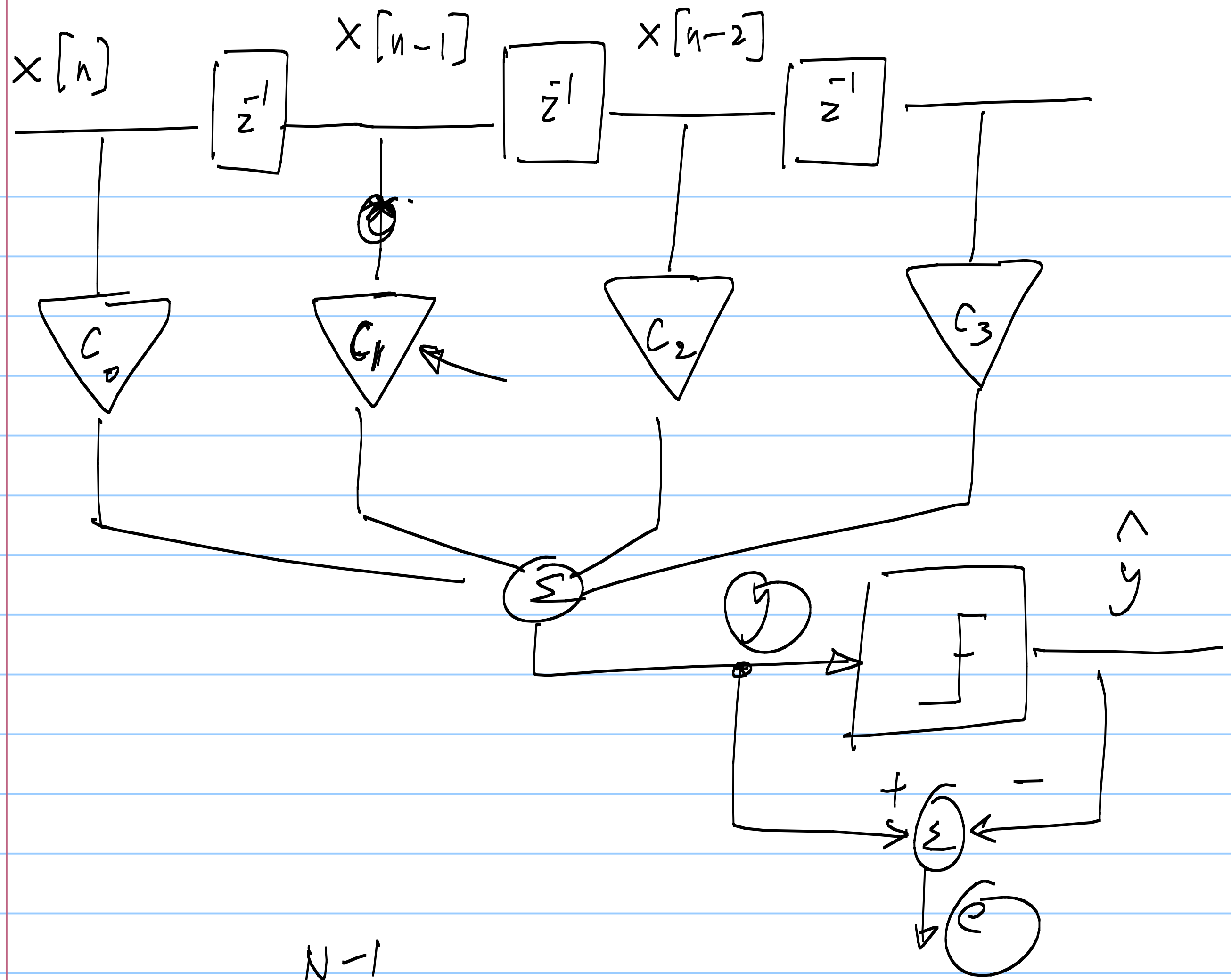



$$\frac{\mu}{2} \cdot \frac{d}{dC_0} e^2$$

$$2e \cdot \frac{de}{dC_0}$$




$$C_0[k+1] = C_0[k] - \mu \cdot e[k]\, x[k]$$




$$y = \sum_{k=0}^{N-1} c_k \, x[n-k] \; ; \qquad e = y - \hat{y}$$

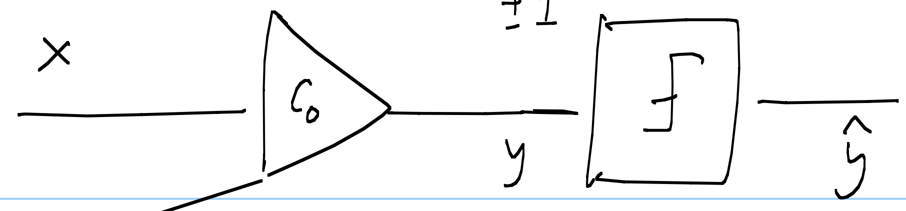
$$\frac{de}{dc_k} = x[n-k] \qquad \frac{d}{dc_k} \cdot e^2 = 2e \cdot \frac{de}{dc_k}$$

$$c_k[n+1] = c_k[n] - \mu \cdot e[n] \cdot x[n-k]$$

At every ~~instan~~ sampling instant

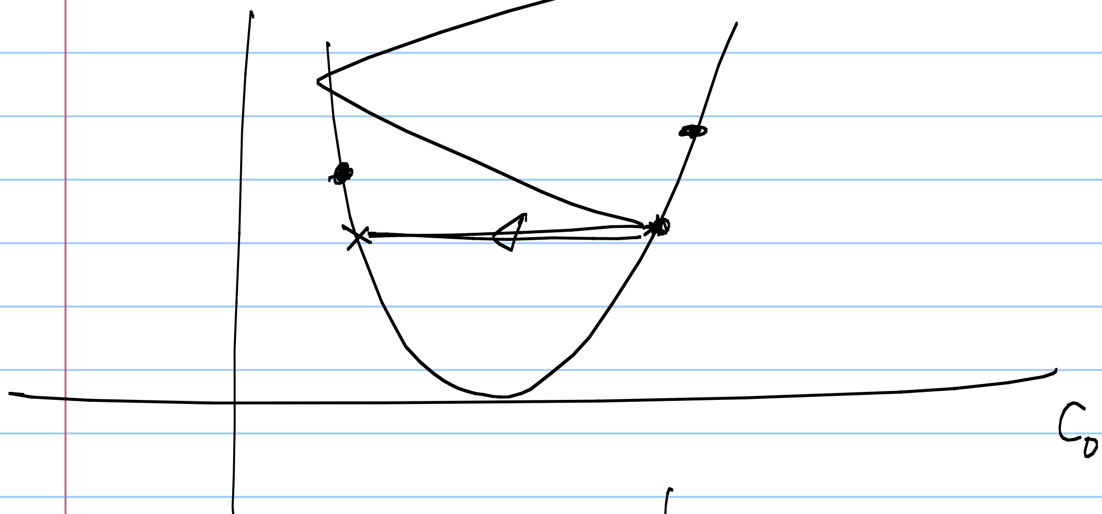
Update $c_k$

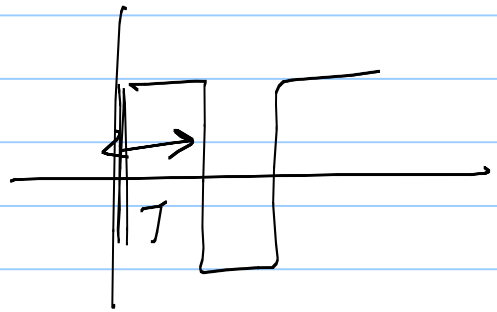
$$c_k - \mu e \cdot x_k$$

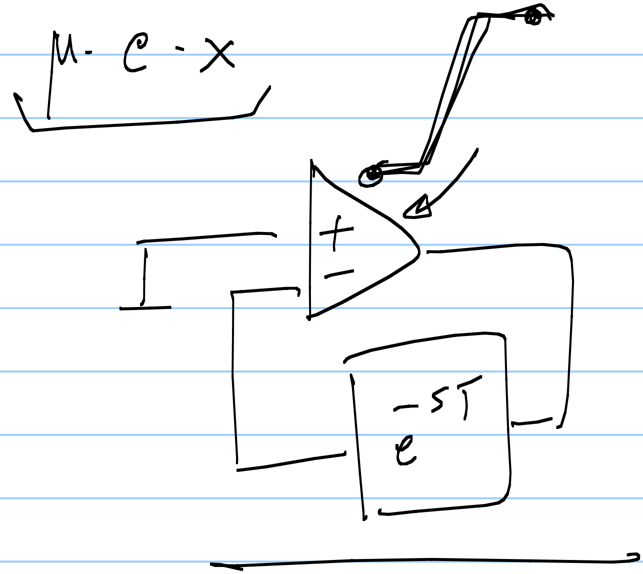


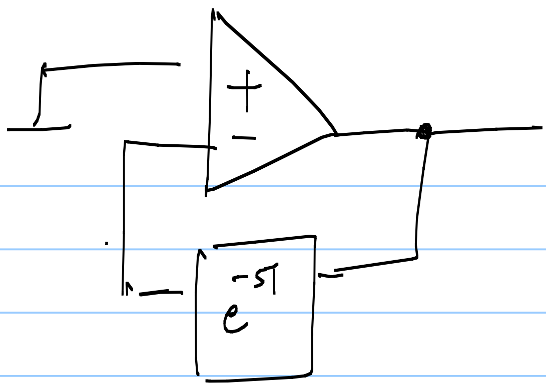

$$e = y - \hat{y}$$

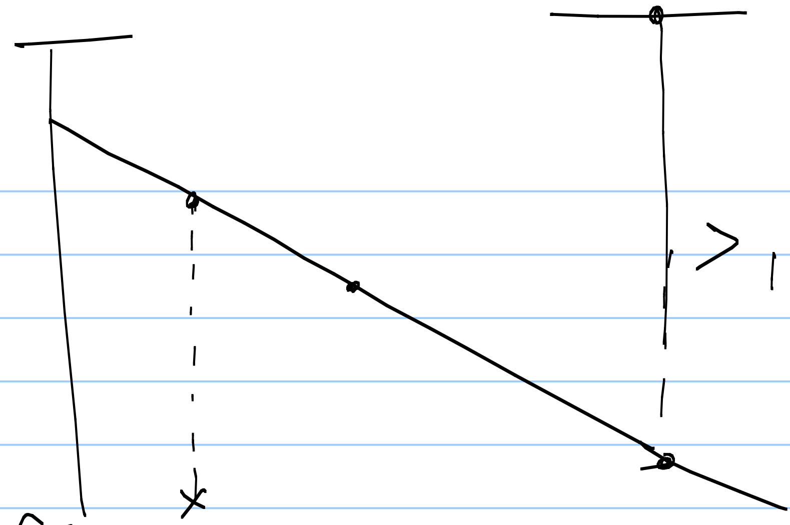
$$e^2 = (c_0 x - \hat{y})^2 = c_0^2 x^2 - 2 c_0 \cdot x \cdot \hat{y} + \hat{y}^2$$





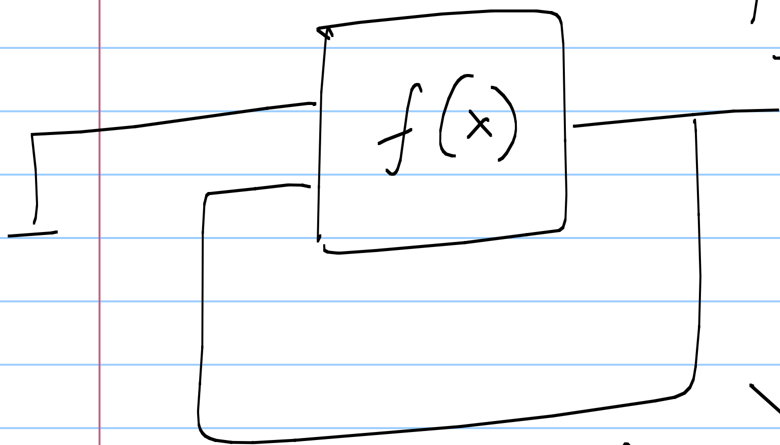
input

$g(x) = 0$

$[g(x) + x] - x = 0$

$f(x) = x$

$f(x) - x = 0$

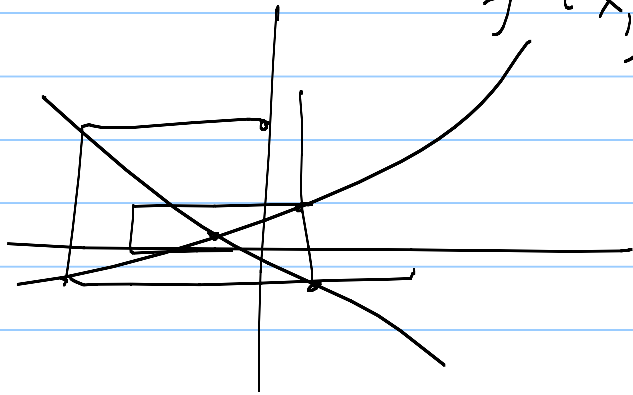


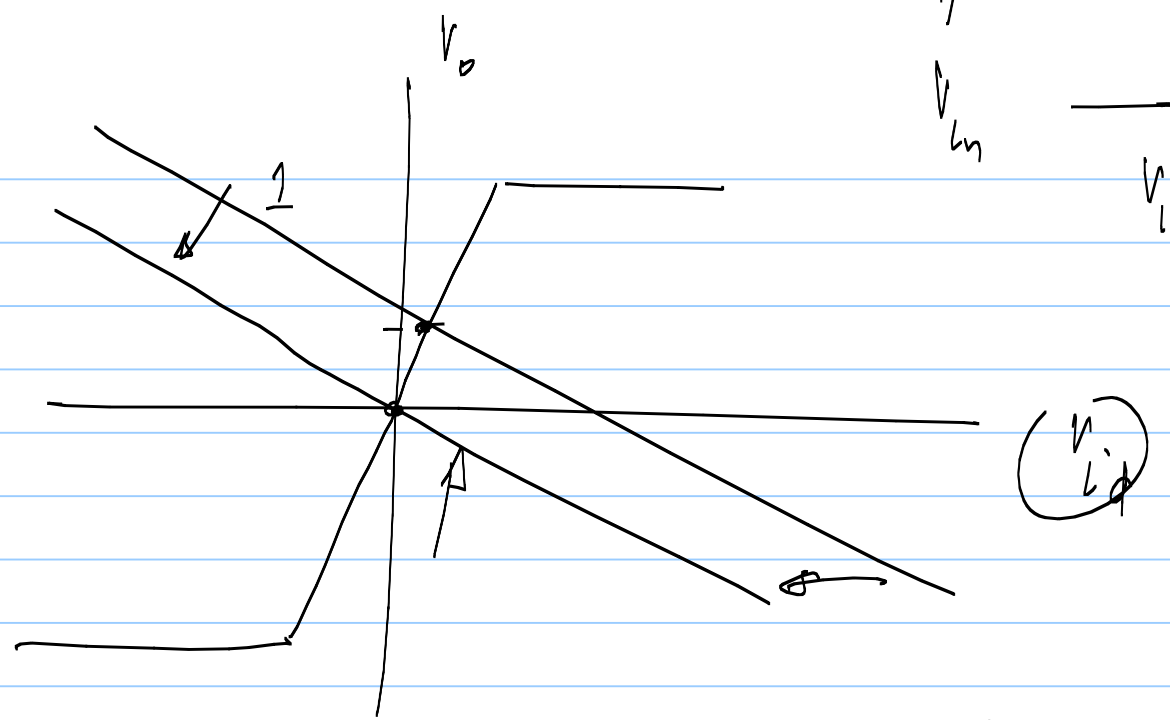

$x_0$

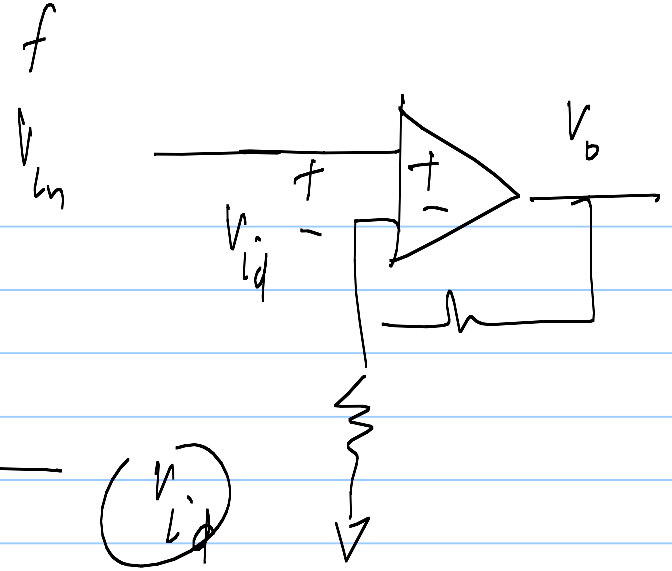
$f(x_0) = x_1$

$f(x_1) = x_2$

f






$$v_{id} = v_{in} - v_o \cdot \frac{R_1}{R_1 + R_2}$$

**Coefficient update:**

$$c_k[n+1] = c_k[n] - \mu \cdot e[n] \cdot x[n-k]$$

Product: $\mu \cdot e[n] \cdot x[n-k]$ $\qquad \mu \cdot \text{sgn}(e) \cdot \text{sgn}(x)$

* $x[n]$ in digital form $\Rightarrow$ $y$ is also digital

$$e = (y - \hat{y})$$

Digital multiplier.

* Analog multipliers to implement $e[n] \cdot x[n-k]$

Simplify coeff. updating:

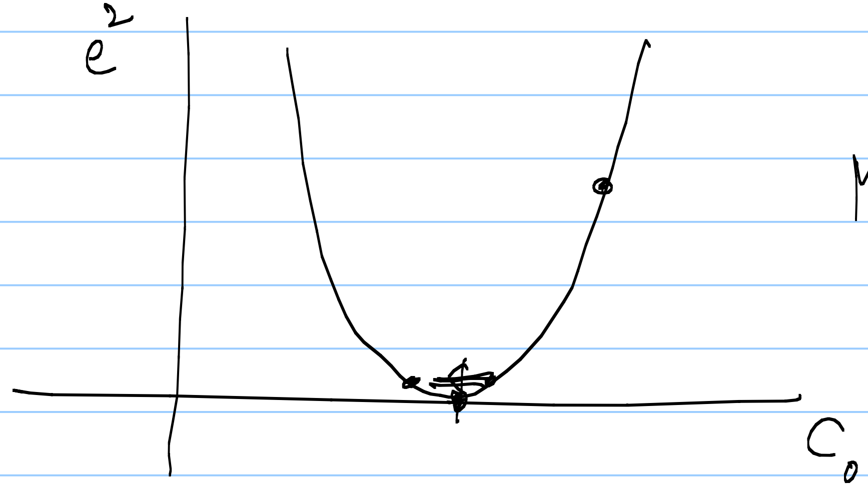
Simplified coeff. updating:

~~sign~~ Quantize $e[n]$ & $x[n-k]$ to 1 bit

$$C_k[n+1] = C_k[n] - \mu \, \text{sgn}(e[n]) \cdot \text{sgn}(x[n-k])$$

Gradient of $e$ wrt $C_k$

Sign-sign LMS.




$\mu \cdot e \cdot x$