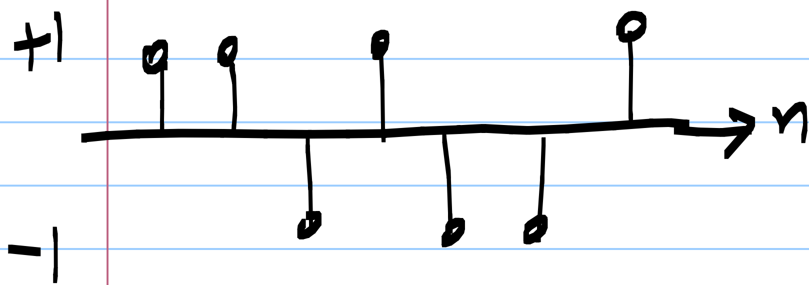
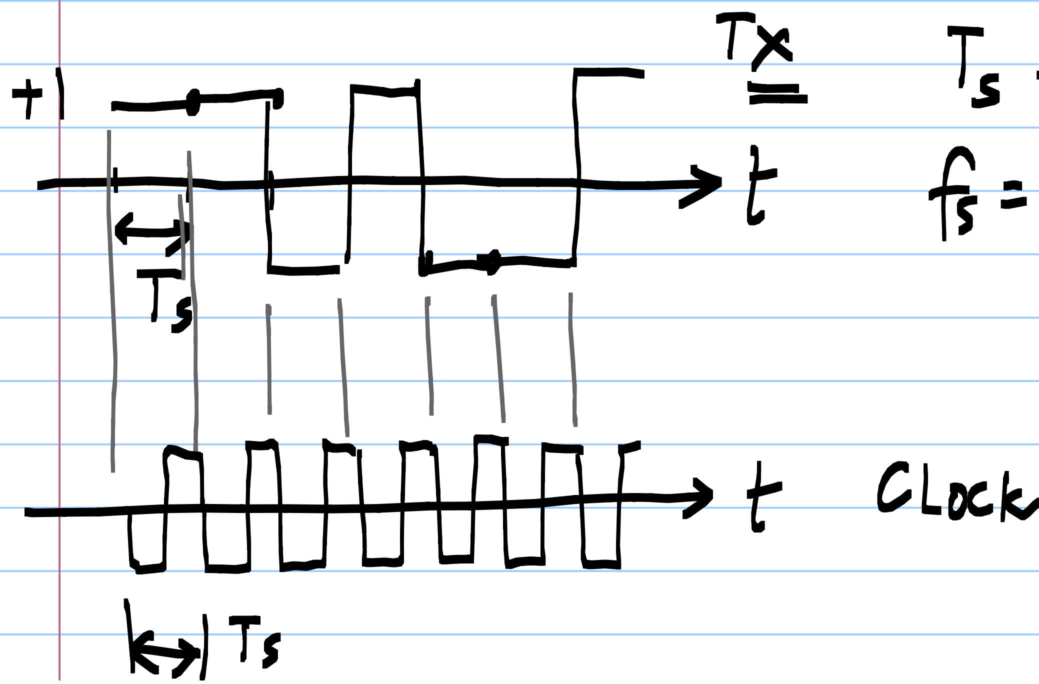
EE6322: VLSI Broadband communication circuits

Note Title 1/13/2020



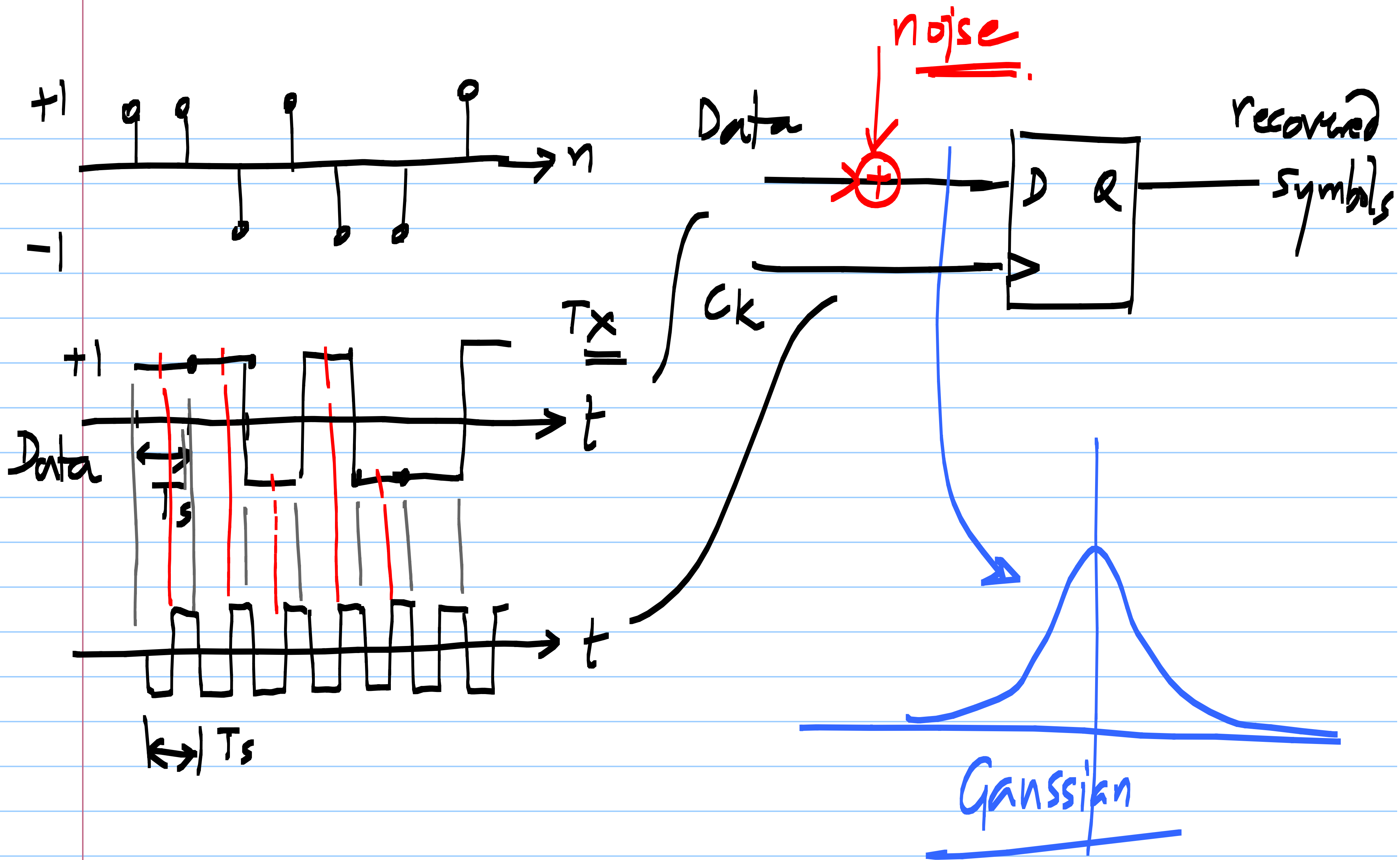
Symbol : 2 values, (1 bit)

$\therefore$ Bitrate = symbol rate



T_s : symbol interval

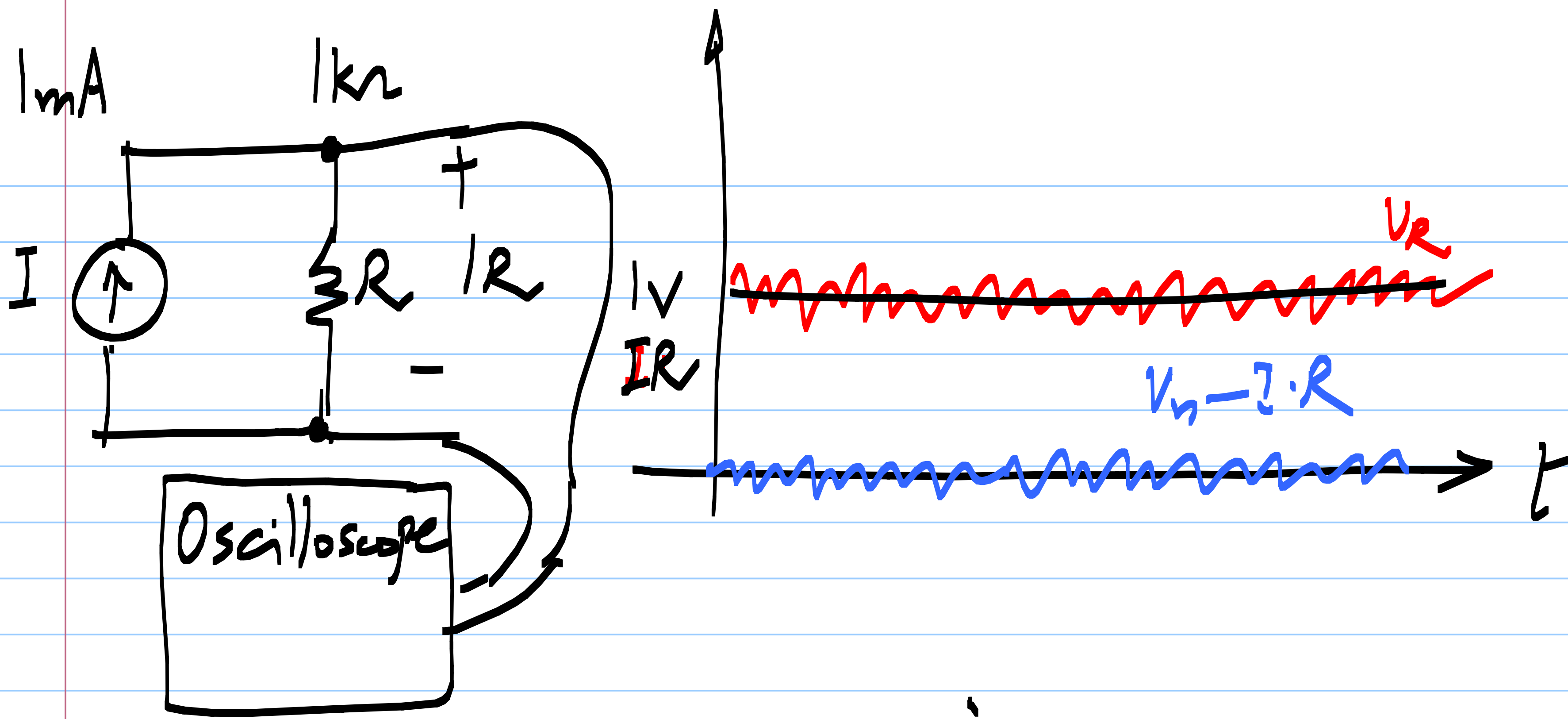
$f_s = \frac{1}{T_s}$ = symbol rate



+1
-1
n
Data
t
Ts
Tx
Data
noise
D
Q
Ck
recovered symbols
Gaussian

Noise: random process:

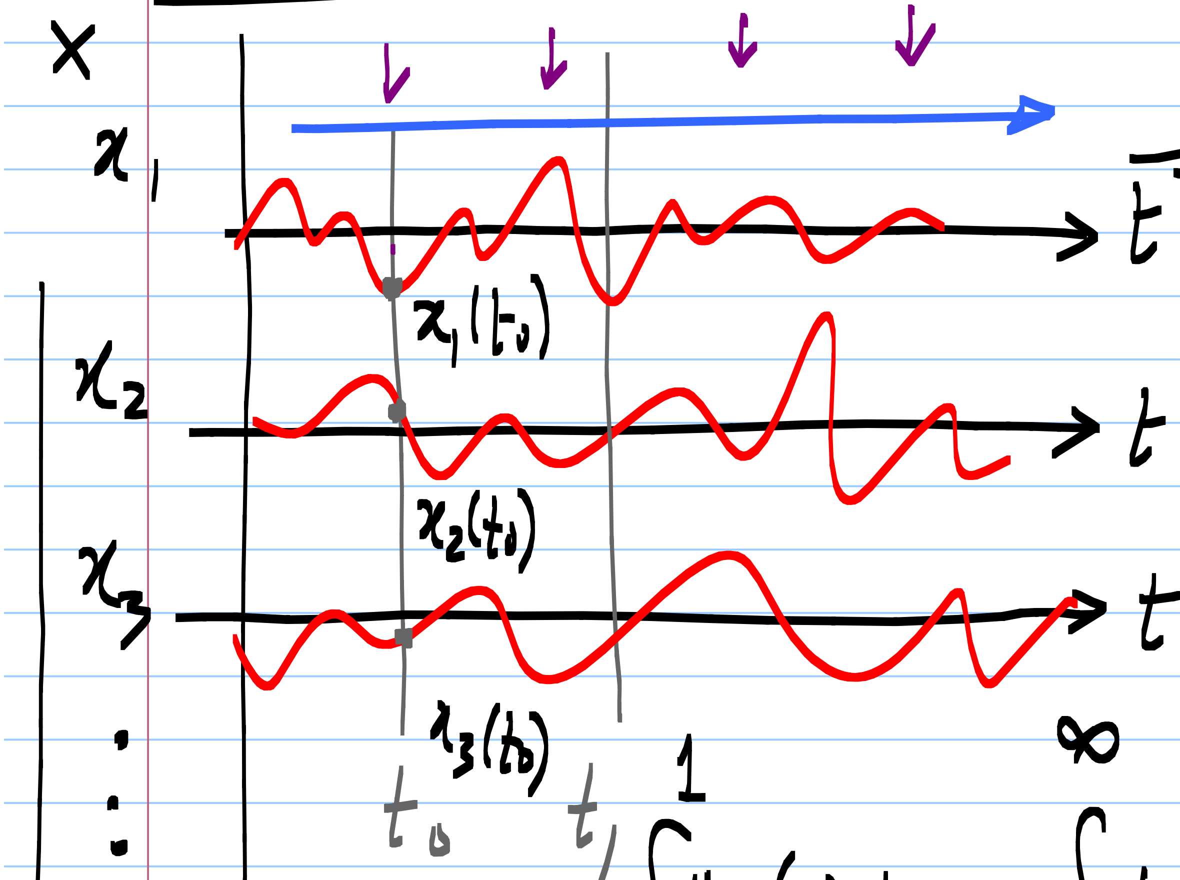
- Individual outcomes cannot be predicted
- Statistics can be found
  - over an ensemble
  - over time

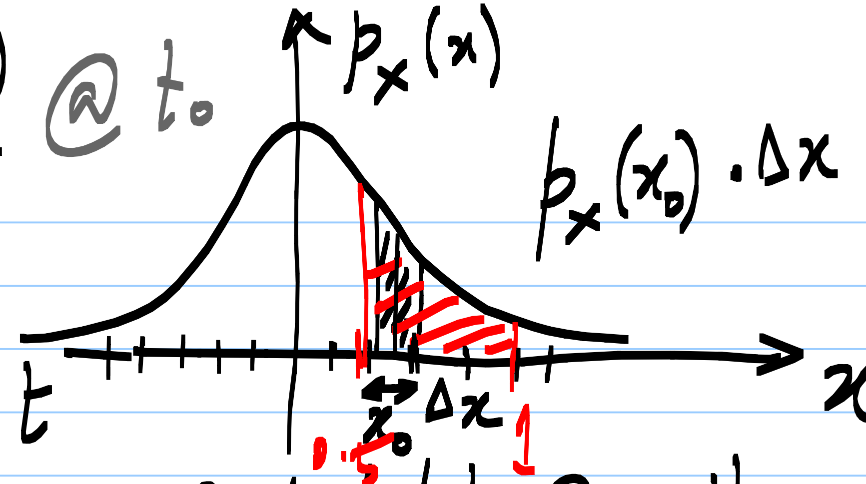


1mA
1kΩ
I
R
+
IR
−
Oscilloscope
1V
IR
V_R
$V_n - I \cdot R$
t

Noise (Random process)

@ t_0





$p_x(x_0) \cdot \Delta x$

Probability density

n.

0.5 & 1

Probability that

$0.5 \le x < 1$

$$\int_{0.5}^{1} p_x(x)\,dx$$

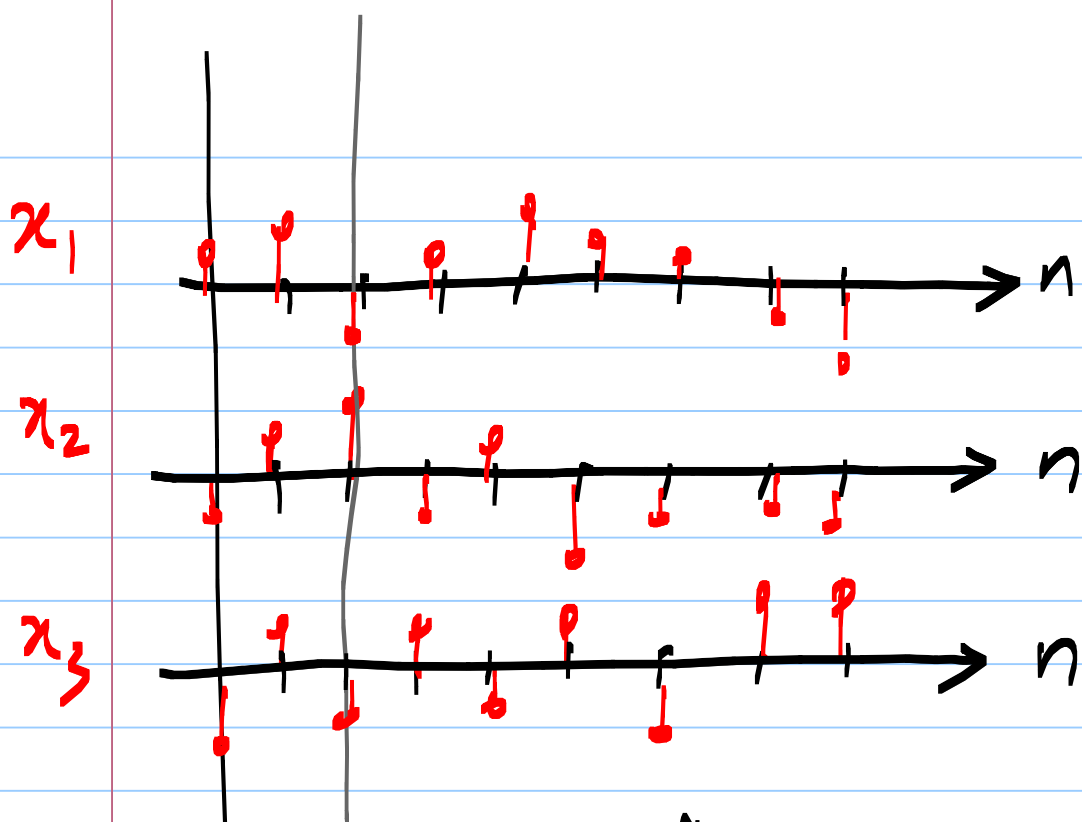
$$\int_{-\infty}^{\infty} p_x(x)\,dx = 1$$

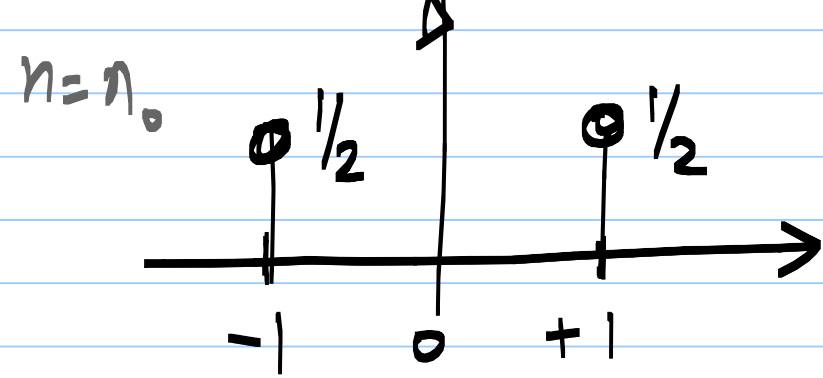
Deterministic signals

$x(t) = \exp(-2t)$

$\sin(50t)$

values known exactly {at all t}



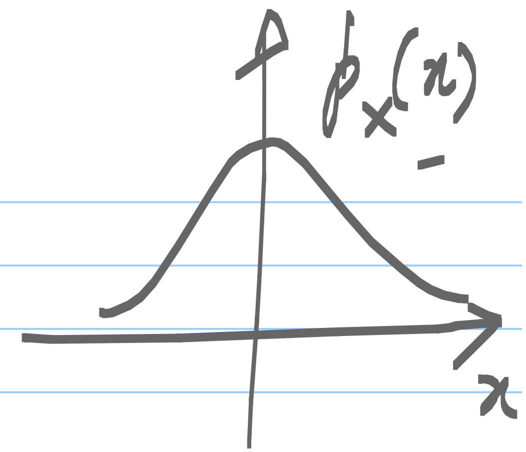


$x_1(n_0)$

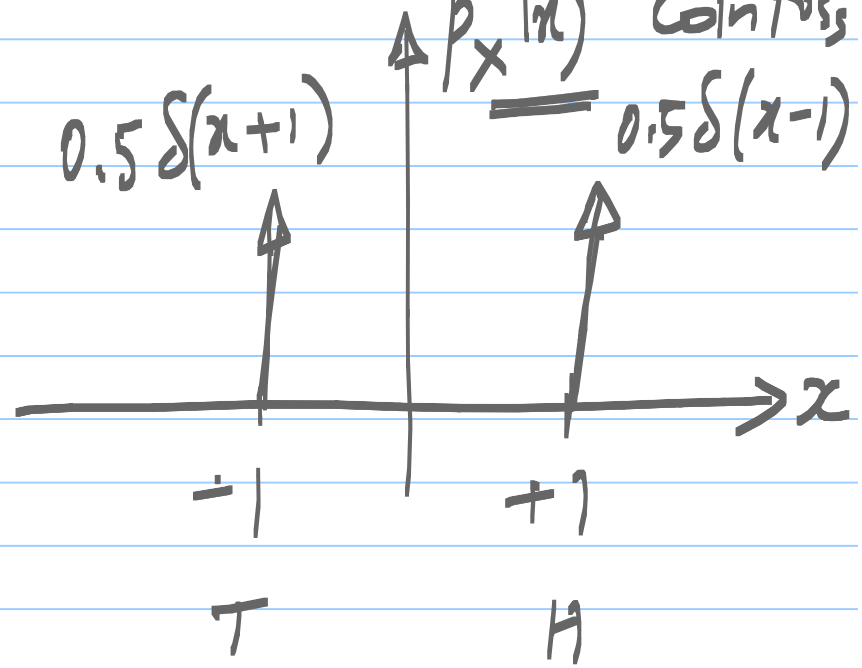
$x_2(n_0)$

$\vdots$

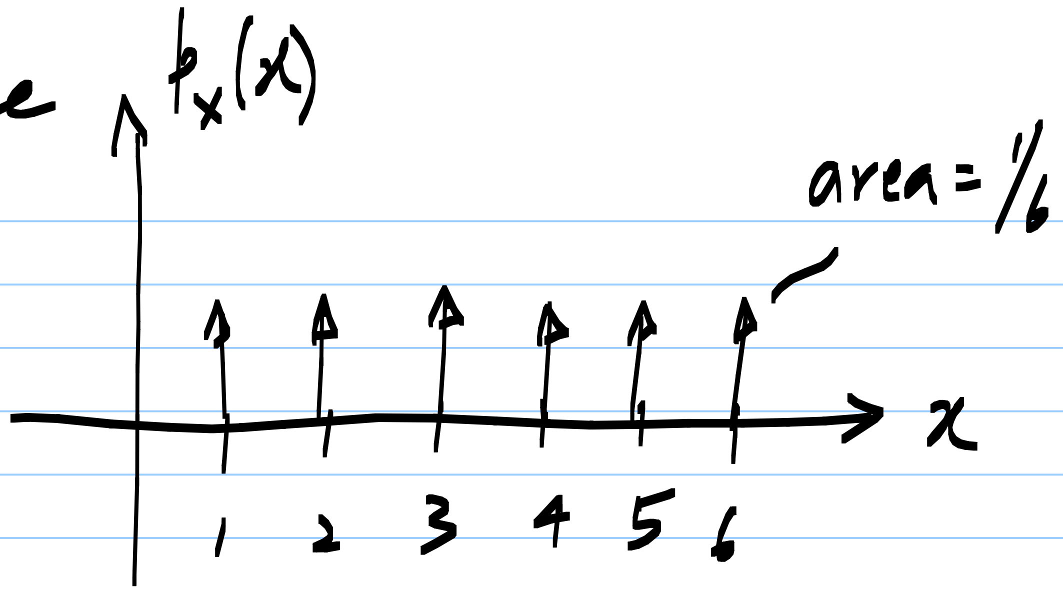
$x_N(n_0)$



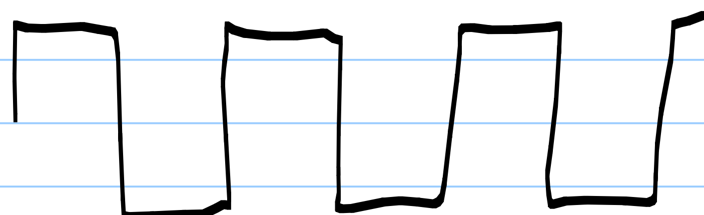
coin toss



Dice



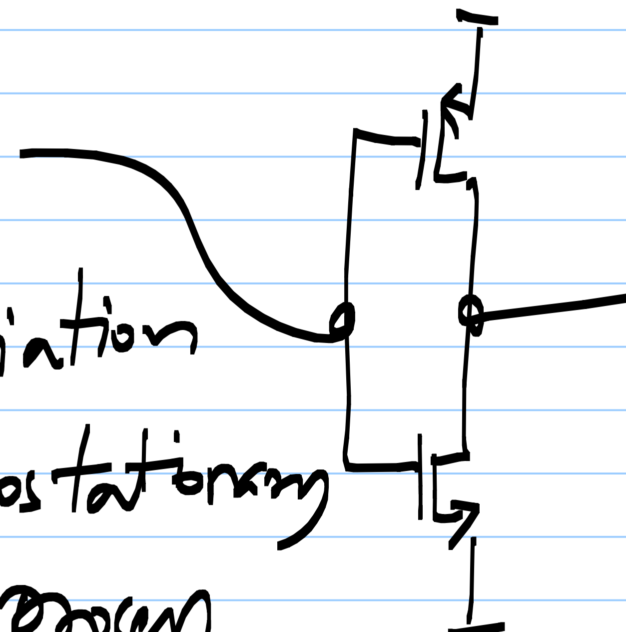
probability distribution (density function) same at all t. — stationary process

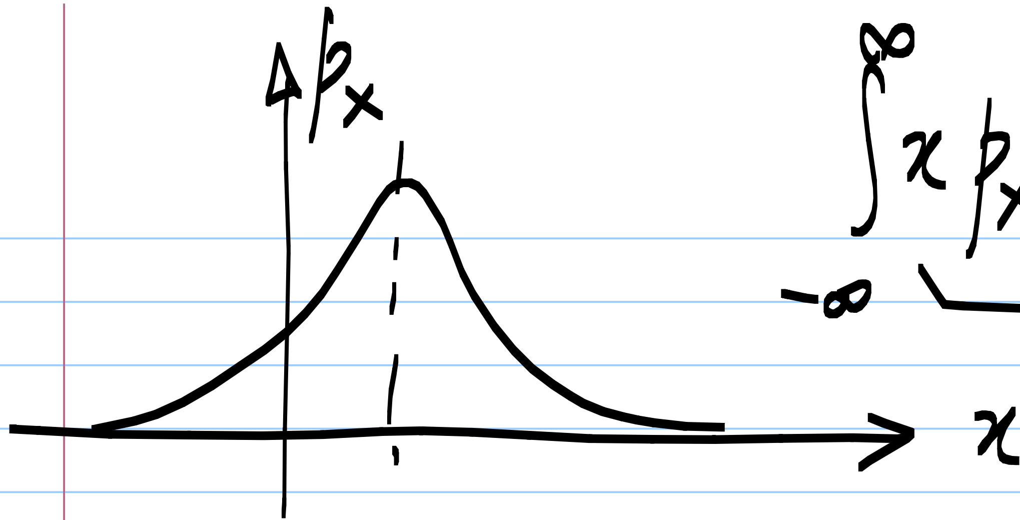


t_1 t_2 t_3

p_{x,t_1} p_{x,t_2} p_{x,t_3}

periodic variation of p_x : Cyclostationary process





$$\int_{-\infty}^{\infty} x \, p_x(x) \cdot dx = \underline{\underline{E[x]}}$$

Expectation

Mean

$$Y = X - E_x$$

$$y = x - E(x)$$

$$\int_{-\infty}^{\infty} y \, p_y(y) \cdot dy = 0$$

Variance:

$$\int_{-\infty}^{\infty} [x - E(x)]^2 \cdot p_x(x)\, dx : \sigma_x^2$$

$$\sqrt{} = \sigma_x : \text{standard deviation}$$

Mean: $E[x]$

Variance: $E[(x - E(x))^2]$

$$E[\underbrace{F(x)}] : \int_{-\infty}^{\infty} \underbrace{F(x)} \cdot p_x(x) \cdot dx$$

Expectation

Zero mean: $E[x^2]$

$$\bar{x}_1 : \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x_1(t)\,dt$$

Time average
dc value

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x_1^2(t)\,dt$$

Mean-squared value

Zero-mean, stationary processes,

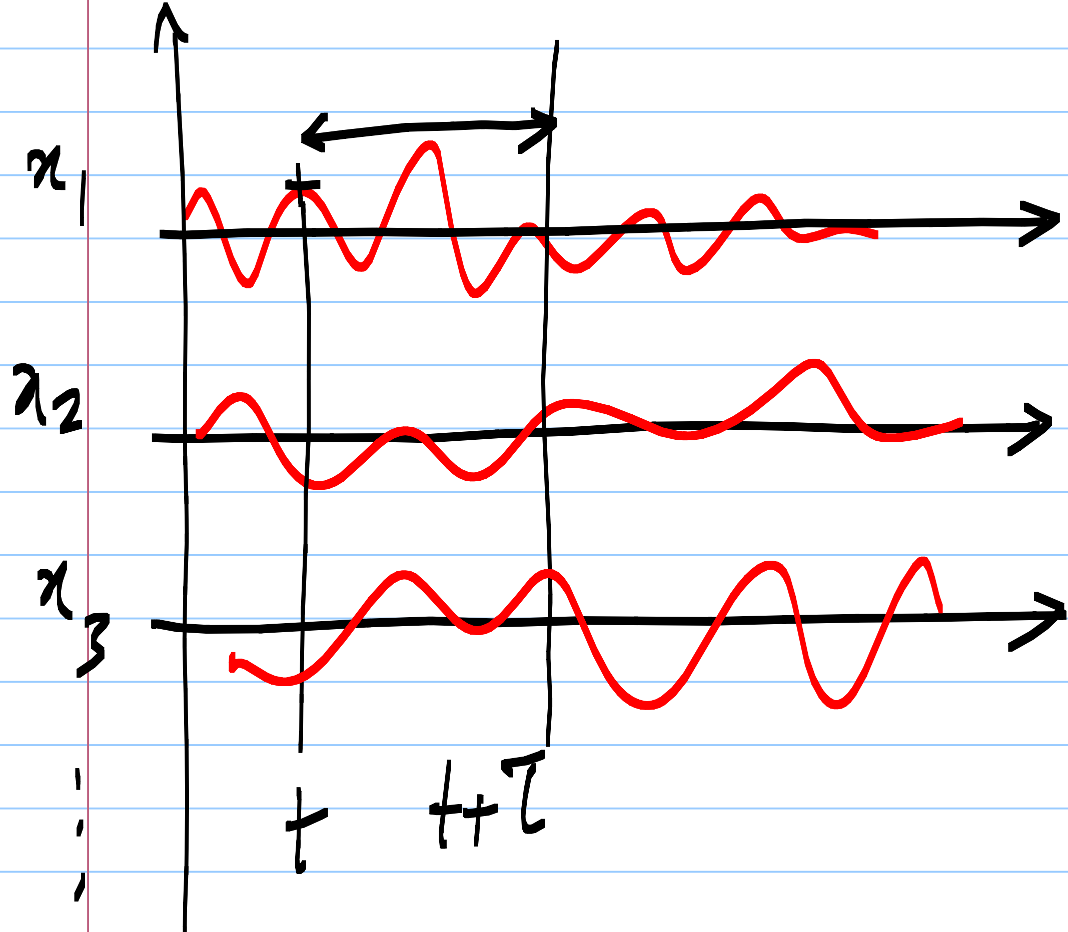
Variance = Mean-squared value

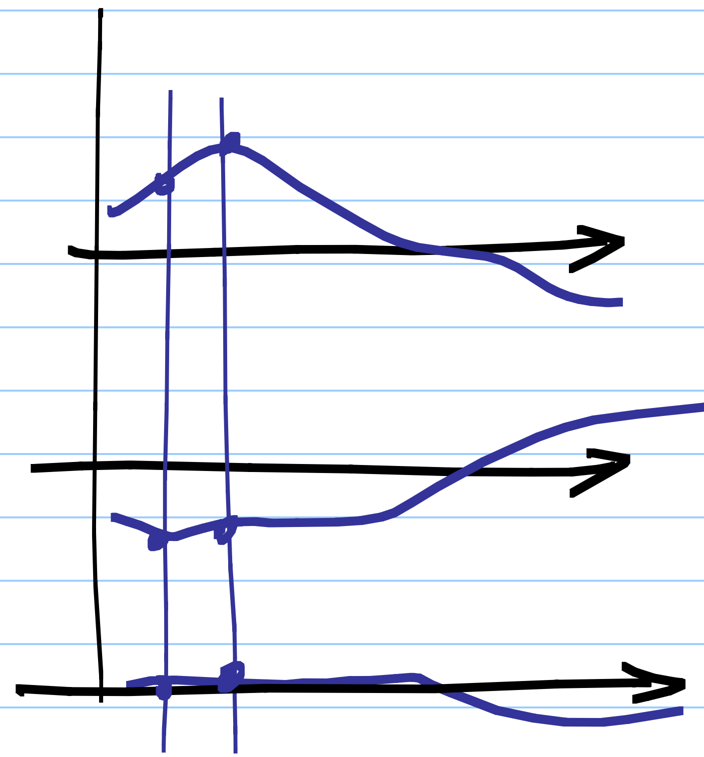
Ergodic processes

Autocorrelation

$$R_x(\tau) = E\left[x(t)\, x(t+\tau)\right]$$

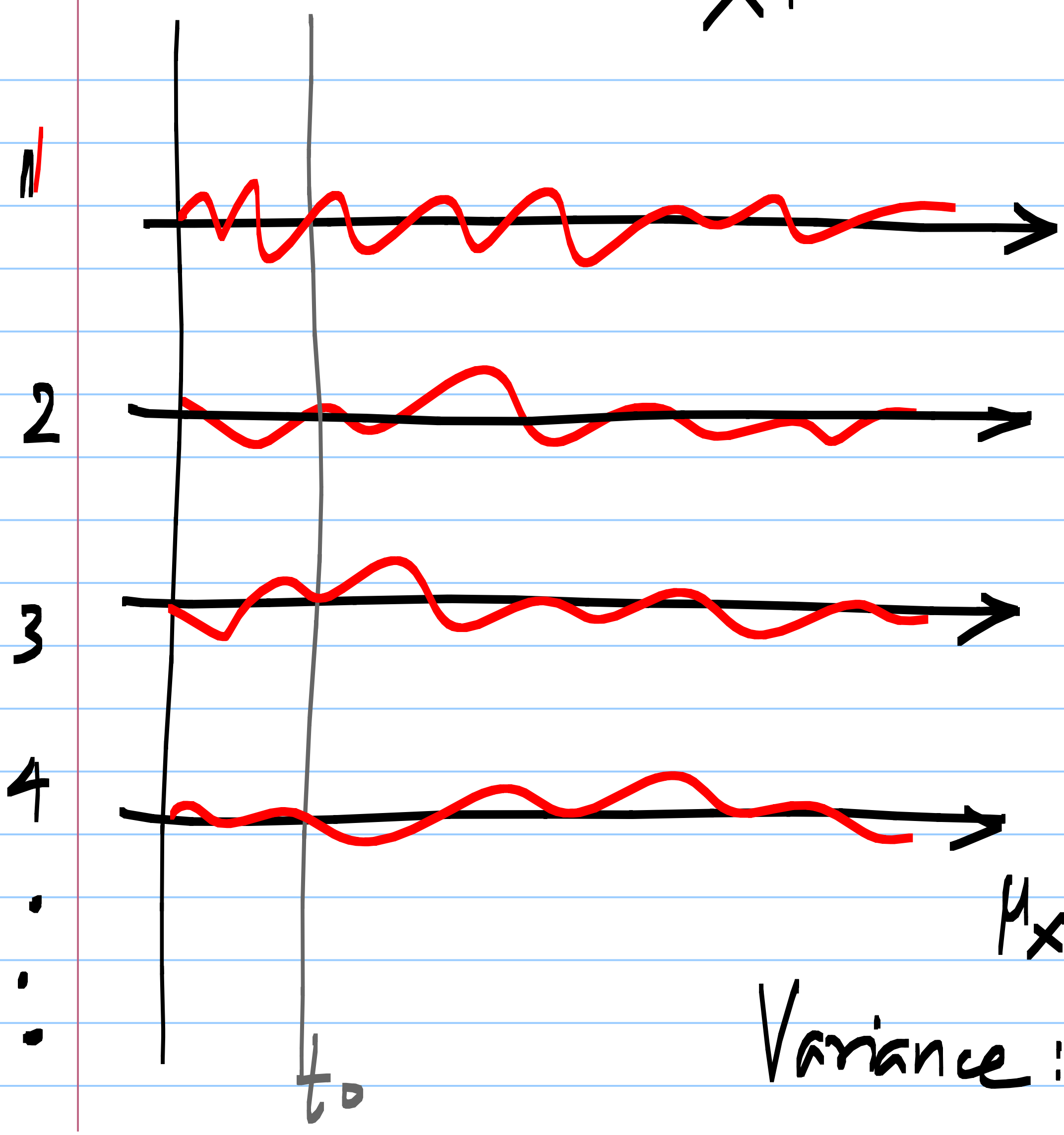
$$\int_{-\infty}^{\infty} x(t)\cdot x(t+\tau)\, p_x(x)\, dx$$





Random process

X:



Probability density function:

$p_X(x)$

Probability of X being in $(x_0, x_0+\Delta x)$ $= p_X(x_0)\cdot \Delta x$

$E[X]$

Mean: μ_X $= \int_{-\infty}^{\infty} x \cdot p_X(x)\, dx$

Variance: $E[(X - E[X])^2] = \sigma_X^2$

Stationary: Mean & variance: same at all 't'

Wide-sense

Ergodicity: Mean $E[x] = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$

{Ensemble} $\qquad$ time average

$\sqrt{\ }$: Standard deviation σ_x

Variance $E\left[(x - E(x))^2\right] = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(x(t) - \bar{x}\right)^2 dt$

σ_x^2 [Ensemble]

Mean-squared value $\xrightarrow{\sqrt{\ }}$ rms

Auto correlation: (Zero mean)

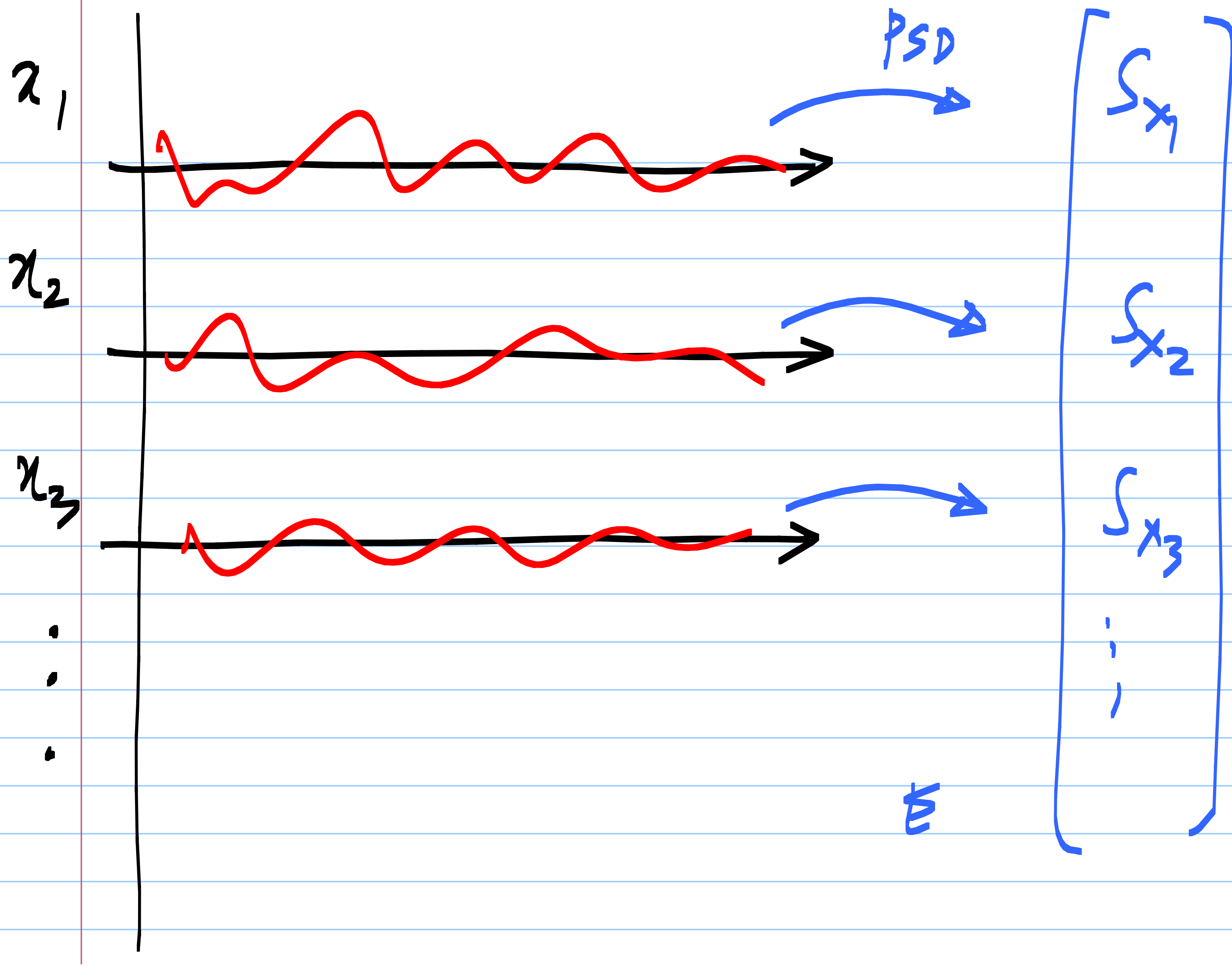
$$R_X(t, \tau) = E\left[X(t) \cdot X(t+\tau)\right]$$

$$R_X(\tau) = E\left[X(t) \cdot X(t+\tau)\right] = \int_{-\infty}^{\infty} p_X(x) \cdot x(t) \, x(t+\tau) \cdot dx$$

$$R_X(0) = \sigma_X^2$$ Variance

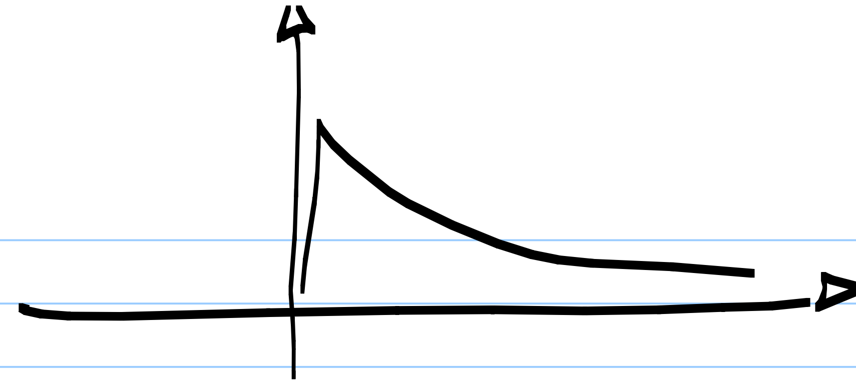
F.T

$$\int_{-\infty}^{\infty} R_X(\tau) \cdot \exp(-j2\pi f \tau) \cdot d\tau = \underbrace{S_X(f)}_{\text{Power spectral density}}$$



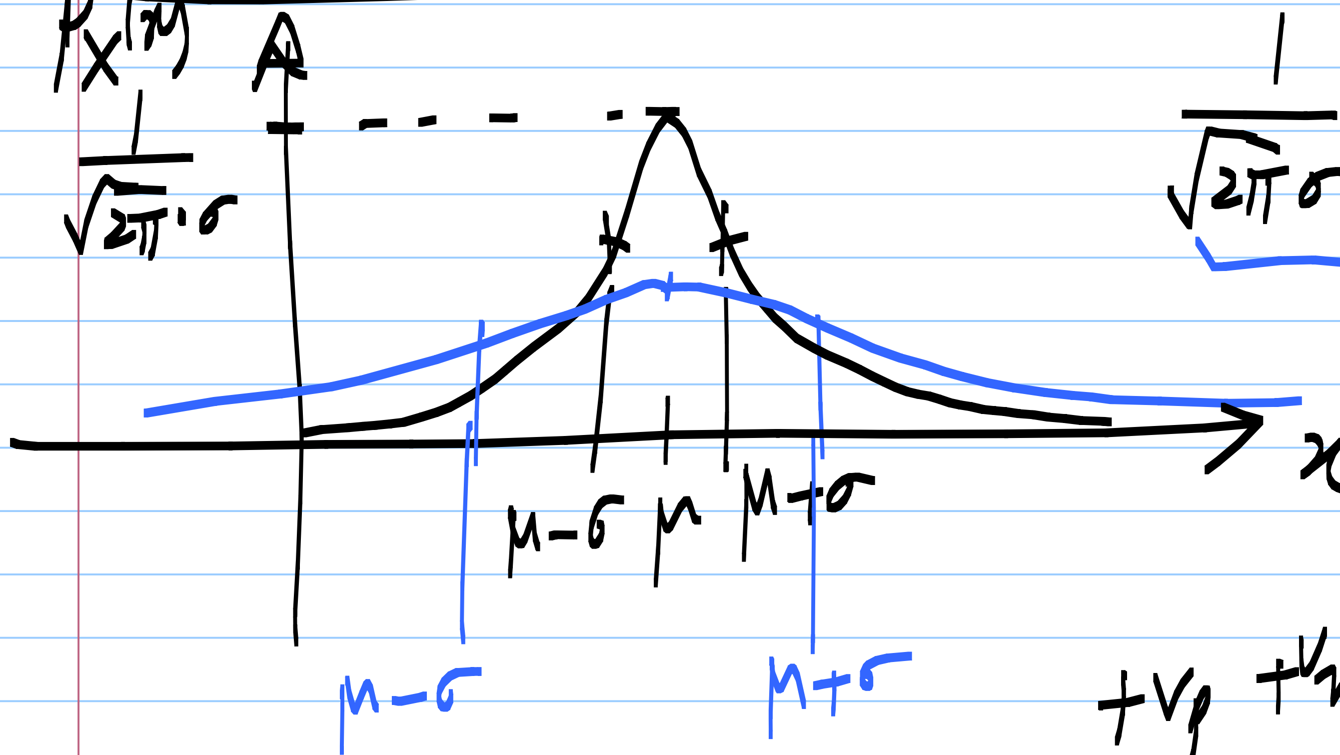
x_1
x_2
x_3
PSD
S_{x_1}
S_{x_2}
S_{x_3}
E

$$x(t) = \exp(-at) \cdot u(t)$$



$$X(f) \quad \left(\frac{1}{a + j2\pi f} \right)$$

Gaussian distribution:



$$p_X(x) = \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$N(\mu, \sigma)$

$+V_p \; +V_n$

$-V_p \; +V_n$